SYDNEY GRAMMAR SCHOOL

2010 Trial Examination

FORM VI
MATHEMATICS 2 UNIT

Tuesday 3rd August 2010

General Instructions
- Reading time — 5 minutes
- Writing time — 3 Hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- A list of standard integrals is provided at the end of the examination paper.
- All necessary working should be shown in every question.
- Start each question in a new booklet.

Structure of the paper
- Total marks — 120
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection
- Write your candidate number clearly on each booklet.
- Hand in the ten questions in a single well-ordered pile.
- Hand in a booklet for each question, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place the question paper inside your answer booklet for Question One.

Checklist
- SGS booklets — 10 per boy
- Candidature — 85 boys

Examiner
SO

QUESTION ONE (12 marks) Use a separate writing booklet. **Marks**

(a) Find the value of $\dfrac{3{\cdot}6 \times 7{\cdot}4}{\sqrt{5{\cdot}6 + 2{\cdot}5}}$ correct to 2 significant figures. $\boxed{1}$

(b) Factorise $x^3 - 125$. $\boxed{1}$

(c) If $(\sqrt{7} - 3)(2\sqrt{7} + 2) = p + q\sqrt{7}$, find p and q. $\boxed{2}$

(d) Simplify $\dfrac{x}{3} - \dfrac{x+2}{4}$. $\boxed{2}$

(e) Solve $|x - 1| < 3$. $\boxed{2}$

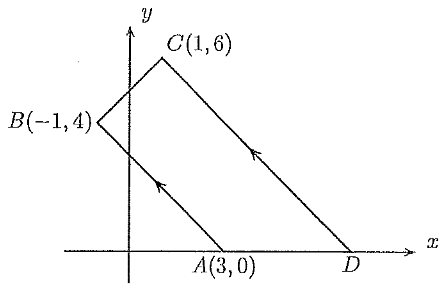
(f) Solve $\cos\theta = \dfrac{\sqrt{3}}{2}$, for $0 \le \theta \le 2\pi$. $\boxed{2}$

(g) Find the sum of the first 17 terms of the arithmetic series $3 + 11 + 19 + \cdots$. $\boxed{2}$

Exam continues next page ...

QUESTION TWO (12 marks) Use a separate writing booklet. **Marks**

(a)



In the diagram above, $ABCD$ is a trapezium with $AB \parallel DC$. The coordinates of A, B and C are $(3,0)$, $(-1,4)$ and $(1,6)$ respectively. D lies on the x-axis.

(i) Find the length of AB. $\boxed{1}$

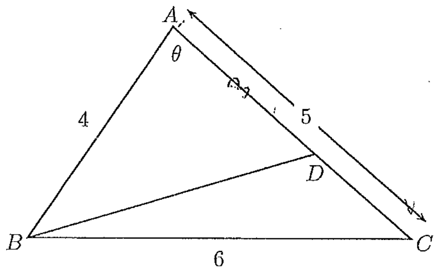
(ii) Find the gradient of AB. $\boxed{1}$

(iii) Find the equation of the line CD, and hence find the coordinates of D. $\boxed{2}$

(iv) Show that the perpendicular distance from A to the line CD is $2\sqrt{2}$ units. $\boxed{2}$

(v) Hence, or otherwise, calculate the area of the trapezium $ABCD$. $\boxed{2}$

(b)



In the diagram above $\angle BAC = \theta$ as shown.

(i) Find the exact value of $\cos\theta$. $\boxed{2}$

(ii) The point D lies on AC. Given that $AD = 3$, calculate the exact length of BD. $\boxed{2}$

Exam continues overleaf ...

QUESTION THREE (12 marks) Use a separate writing booklet. **Marks**

(a) Differentiate the following functions:

(i) $y = 3x^2 + \dfrac{1}{x}$ $\boxed{2}$

(ii) $y = 3(2x-5)^4$ $\boxed{2}$

(iii) $y = x\tan x$ $\boxed{2}$

(b) Find the equation of the tangent to the curve $y = \log_e x$ at $(e,1)$. $\boxed{2}$

(c) Find $\displaystyle\int \sec^2 \tfrac{1}{3}x\,dx$. $\boxed{2}$

(d) Evaluate $\displaystyle\int_0^1 \frac{4}{4x+1}\,dx$. $\boxed{2}$

Exam continues next page ...

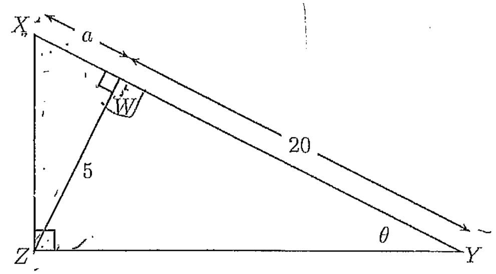
QUESTION FOUR (12 marks) Use a separate writing booklet. **Marks**

(a) Consider the parabola $x^2 = 4(y-2)$.

(i) Write down the coordinates of the vertex. $\boxed{1}$

(ii) Find the coordinates of the focus. $\boxed{1}$

(b)



The diagram above shows $\triangle XYZ$ which is right-angled at Z. The interval ZW is perpendicular to XY. Let $\angle WYZ = \theta$.

(i) Show that $\angle WZX = \theta$. $\boxed{1}$

(ii) Hence prove that $\triangle WZX$ is similar to $\triangle WYZ$. $\boxed{2}$

(iii) Let $XW = a$. If $WZ = 5$ and $WY = 20$, find a. $\boxed{1}$

(c) Let α and β be the roots of $3x^2 - 4x - 2 = 0$.

(i) State the value of $\alpha\beta$. $\boxed{1}$

(ii) Find $\dfrac{5}{\alpha} + \dfrac{5}{\beta}$. $\boxed{2}$

(d) The second term of a geometric series is 270 and the fifth term is 80.

(i) Find the common ratio and the first term of the series. $\boxed{2}$

(ii) Find the limiting sum of the series. $\boxed{1}$

Exam continues overleaf ...

QUESTION FIVE (12 marks) Use a separate writing booklet. **Marks**

(a) Consider the curve $y = x^3 - 3x + 2$.

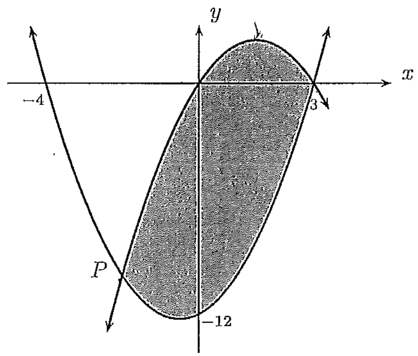
(i) Find the coordinates of the stationary points and determine their nature. $\boxed{3}$

(ii) Find any points of inflexion. $\boxed{2}$

(iii) Sketch the curve, showing the stationary points and any points of inflexion. $\boxed{2}$

(iv) For what values of x is the curve concave down? $\boxed{1}$

(b)



The graphs of the functions $y = x^2 + x - 12$ and $y = -x^2 + 3x$ are shown in the diagram above. They intersect at $(3, 0)$ and at P.

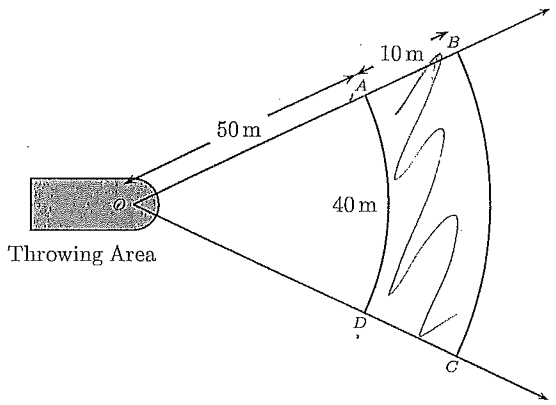
(i) By solving simultaneously, show that P has x-coordinate -2. $\boxed{1}$

(ii) Calculate the area of the shaded region. $\boxed{3}$

Exam continues next page ...

QUESTION SIX (12 marks) Use a separate writing booklet. **Marks**

(a)



The diagram above shows the javelin competition area at an athletic stadium. The circular arcs AD and BC have centre O. The arc AD has length 40 metres and radius 50 metres.

(i) Calculate the size of $\angle AOD$ in radians. **2**

(ii) The worst throw of the day landed on the arc AD and the best throw of the day landed on the arc BC. If $AB = 10$ metres, calculate the area of the region $ABCD$ in which all the other throws landed. **2**

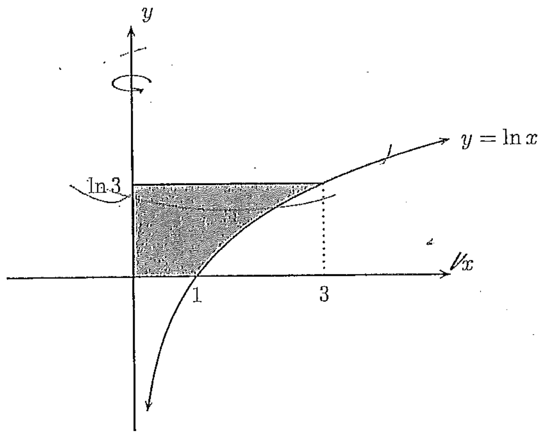
(b) Julian's house is being overrun with Black European cockroaches. Assume that without intervention the population P of cockroaches grows exponentially according to the equation $P = Ae^{kt}$, where A and k are constants, and t is the time in days. When Julian leaves for a holiday there are 50 cockroaches in his house. After ten days the cockroach population has increased to 275.

(i) Show that $P = Ae^{kt}$ satisfies $\dfrac{dP}{dt} = kP$. **1**

(ii) Find the exact value of k. **2**

(iii) When the cockroach population exceeds 2000, the house will be declared an area of infestation. Julian returns from his holiday after 3 weeks. Will he discover an infestation when he arrives home? **2**

(c) **3**

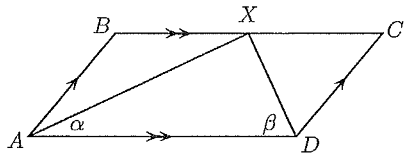


In the diagram above the shaded region is bounded by the curve $y = \ln x$, the x-axis, the y-axis and the line $y = \ln 3$. Calculate the exact volume of the solid formed when the shaded region is rotated about the y-axis.

QUESTION SEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) Find the value of m such that $\displaystyle\int_{\frac{1}{2}}^{m} \frac{1}{x^2}\,dx = 1$. $\boxed{2}$

(b)



In the diagram above $ABCD$ is a parallelogram and X is on BC such that AX bisects $\angle BAD$ and DX bisects $\angle CDA$. Let $\angle XAD = \alpha$ and $\angle XDA = \beta$.

(i) Prove that $\triangle ABX$ is isosceles. $\boxed{2}$

(ii) Prove that $\angle AXD = 90^\circ$. $\boxed{2}$

(c) (i) Copy and complete the table for the function $y = x \sin x$, writing the y-values correct to four decimal places. $\boxed{1}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Use Simpson's rule with five function values to approximate $\displaystyle\int_0^{\pi} x \sin x\,dx$. Round your answer to two decimal places. $\boxed{2}$

(iii) Show $\dfrac{d}{dx}(\sin x - x\cos x) = x \sin x$, and hence find the exact value of $\displaystyle\int_0^{\pi} x \sin x\,dx$. $\boxed{2}$

(iv) Hence determine the percentage error in your approximation in part (ii). Write your answer correct to one decimal place. $\boxed{1}$

QUESTION EIGHT (12 marks) Use a separate writing booklet. **Marks**

(a) Given that $x = \frac{3}{4}$ is one root of the quadratic equation $mx^2 + 7x - m = 0$, find the other root. $\boxed{2}$

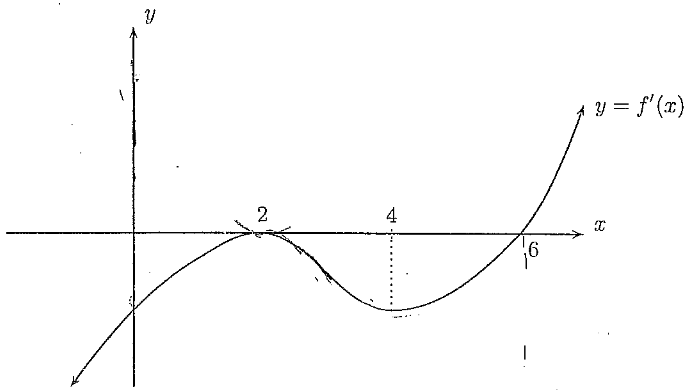
(b) Sophie has a toy that she uses to blow spherical bubbles. The rate of change of the volume V cm^3 of a bubble is given by

$$\frac{dV}{dt} = \frac{6t}{t^2+1} \text{ cm}^3/\text{s}.$$

(i) Find the equation for the volume V of a bubble t seconds after Sophie starts blowing. Assume that the initial volume of a bubble is zero. $\boxed{2}$

(ii) A bubble will burst when its radius exceeds 1·5 cm. Sophie takes a deep breath and blows a bubble. After how many seconds of blowing will it burst? Give your answer correct to one decimal place. $\boxed{3}$

(c)



The diagram above shows the graph of the gradient function $y = f'(x)$ of the function $y = f(x)$.

(i) For what values of x is the function $y = f(x)$ increasing? $\boxed{1}$

(ii) For what values of x is the curve $y = f(x)$ concave down? $\boxed{1}$

(iii) Given that $f(0) = 2$, draw a possible sketch of $y = f(x)$. $\boxed{3}$

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) Solve $2\sin^2\alpha - \cos\alpha + 1 = 0$, for $0 \le \alpha \le 2\pi$. [3]

(b) Solve $\log_6(x+3) + \log_6(x-2) = 2$. [3]

(c) A particle is moving along the x-axis . Its position at time t is given by $x = 5e^{-t}\sin t$.

(i) Show that its velocity is given by $v = 5e^{-t}(\cos t - \sin t)$. [1]

(ii) Where is the particle initially, and what is its initial velocity? [2]

(iii) At what time during the interval $0 \le t \le \pi$ is the particle stationary? [2]

(iv) Assuming that its acceleration at time t is $\ddot{x} = -10e^{-t}\cos t$, find the time during the interval $0 \le t \le \pi$ when the acceleration is zero. [1]

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

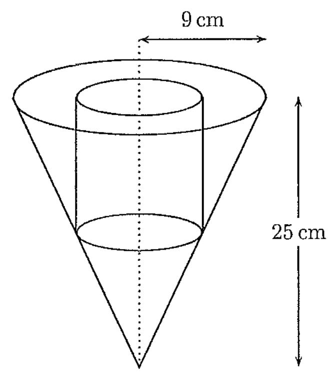
(a) Nick has found his dream home and needs to borrow \$700 000 from the bank to be able to purchase it. He has calculated that he is able to afford monthly repayments of \$6000 per month. The loan plus interest is to be repaid in equal monthly instalments of \$$M$ over 30 years. Reducible interest is charged at 9·6% per annum and is calculated monthly. Let \$$A_n$ be the amount owing after n months.

(i) Write down expressions for A_1 and A_2, and show that the amount owing after 3 months is given by $A_3 = 700\,000(1{\cdot}008)^3 - M(1 + 1{\cdot}008 + 1{\cdot}008^2)$. [3]

(ii) Hence show that $A_n = 700\,000(1{\cdot}008)^n - 125M(1{\cdot}008^n - 1)$. [1]

(iii) Calculate the monthly instalment \$$M$, correct to the nearest dollar, and determine whether Nick will be able to purchase his dream home. [2]

(b)



A cylinder is inscribed in a cone of radius 9 cm and height 25 cm.

(i) Show that the height h of the cylinder is given by [2]

$$h = \frac{25(9-r)}{r},$$

where r is the radius of the cylinder.

(ii) Find the volume V of the cylinder in terms of r. [1]

(iii) Hence find the maximum possible volume of the cylinder. [3]

END OF EXAMINATION

MATHEMATICS SOLUTIONS 2010

Question 1.

a) $9.36034\ldots$

$= 9.4$ ✓

b) $x^3 - 125$

$= (x-5)(x^2+5x+25)$ ✓

c) $\text{LHS} = (\sqrt{7}-3)(2\sqrt{7}+2)$

$= 14 + 2\sqrt{7} - 6\sqrt{7} - 6$ ✓

$= 8 - 4\sqrt{7}$

$\therefore p = 8$ and $q = -4$ ✓

(d) $\dfrac{x}{3} - \dfrac{x+2}{4}$

$= \dfrac{4x - 3(x+2)}{12}$ ✓

$= \dfrac{x-6}{12}$ ✓

(e) $|x-1| < 3$

$-3 < x-1 < 3$ ✓

$\therefore -2 < x < 4$ ✓

(f) $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{6}$ or $\dfrac{11\pi}{6}$ ✓✓ (1 mark if not in radians)

(g) $S_{17} = \dfrac{17}{2}[6 + 16(8)]$ ✓

$= 1139$ ✓

QUESTION 2

a)(i) $AB = \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{16+16}$

$= 4\sqrt{2}$ ✓

(ii) $m = \dfrac{4-0}{-1-3}$

$= -1$ ✓

(iii) $y - 6 = -(x-1)$

$= -x + 1$

$x + y - 7 = 0$ ✓

(or $y = -x + 7$)

cuts x-axis when $y = 0$

so D has coordinates $(7, 0)$ ✓

(iv) $p = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|3+0-7|}{\sqrt{1^2+1^2}}$ ✓

$= \dfrac{4}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= 2\sqrt{2}$ ✓

(v) $CD = \sqrt{(1-7)^2 + (6-0)^2}$

$= \sqrt{36+36}$

$= 6\sqrt{2}$ ✓

$\text{Area} = \frac{1}{2} \times 2\sqrt{2}(6\sqrt{2} + 4\sqrt{2})$

$= \sqrt{2} \times 10\sqrt{2}$

$= 20 \text{ units}^2$ ✓

(Accept other valid methods)

(b)(i) $\cos\theta = \dfrac{4^2+5^2-6^2}{2\times 4\times 5}$ ✓

$= \dfrac{16+25-36}{40}$

$= \dfrac{5}{40}$

$= \dfrac{1}{8}$ ✓

(ii) $BD^2 = 4^2 + 3^2 - 2\times 4\times 3\times \frac{1}{8}$

$= 16 + 9 - 3$

$= 22$

$BD = \sqrt{22}$ ✓

QUESTION 3

(a) (i) $\frac{dy}{dx} = 6x - \frac{1}{x^2}$ ✓✓

(ii) $\frac{dy}{dx} = 24(2x-5)^3$ ✓✓

(iii) $\frac{dy}{dx} = \tan x + x\sec^2 x$ ✓✓

b) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

at $x = e$, $m = \frac{1}{e}$ ✓

$y - 1 = \frac{1}{e}(x - e)$

$y = \frac{1}{e}x$ ✓

c) $\int \sec^2 \frac{1}{3}x \, dx$

$= \frac{\tan \frac{1}{3}x}{\frac{1}{3}} + c$ ✓

$= 3\tan\frac{1}{3}x + c$ ✓

d) $\int_0^1 \frac{4}{4x+1} dx$

$= [\ln(4x+1)]_0^1$ ✓

$= \ln 5 - \ln 1$

$= \ln 5$ ✓

QUESTION FOUR

(a) (i) $(0,2)$ ✓

(ii) $(0,3)$ ✓

b) (i) $\angle WZY = 90 - \theta$ (angle sum of triangle WZY)

$\therefore \angle WZX = \theta$ (angle $XZY = 90°$) ✓

(ii) $\angle WYZ = \angle WZX$ (from part (i))

$\angle YWZ = \angle ZWX$ (right angles) ✓

$\therefore \Delta WZX \,|||\, \Delta WYZ$ (equiangular) ✓

(iii) $\frac{XW}{WZ} = \frac{WZ}{WY}$ (matching sides of similar triangles)

$\frac{a}{5} = \frac{5}{20}$

$\therefore a = \frac{5}{4}$ ✓

c) (i) $\alpha\beta = -\frac{2}{3}$ ✓

(ii) $\frac{5(\alpha+\beta)}{\alpha\beta}$ ✓ $= \frac{5\left(\frac{4}{3}\right)}{-2/3}$

$= 5 \times \frac{4}{3} \times \frac{-3}{2}$

$= -10$ ✓

d) (i) $ar = 270$ and $ar^4 = 80$

By solving simultaneously

$r^3 = \frac{8}{27}$

$\therefore r = \frac{2}{3}$ ✓

$a\left(^2/_3\right) = 270$ ✓

$\therefore a = 405$

(ii) $S_\infty = \frac{405}{1/3}$

$= 1215$ ✓

QUESTION FIVE

a) i) $\frac{dy}{dx} = 3x^2 - 3$ $\qquad \frac{d^2y}{dx^2} = 6x$

stationary at $\frac{dy}{dx} = 0$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1$ or $x = 1$

x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6
concavity	$\wedge$	$-$	$\vee$

when $x = -1$, $y = -1+3+2 = 4$ ✓ (* for both y-coordinates), $\frac{d^2y}{dx^2} < 0$ $\therefore (-1, 4)$ is a MAXIMUM turning point. ✓

when $x = 1$, $y = 1-3+2 = 0$, $\frac{d^2y}{dx^2} > 0$ $\therefore (1, 0)$ is a MINIMUM turning point. ✓

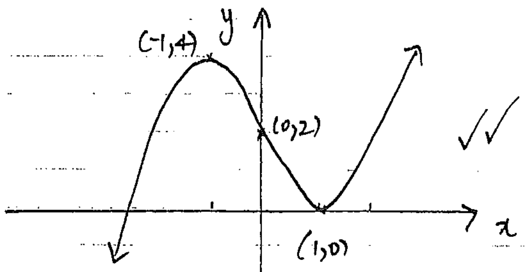
ii) Point inflexion at $\frac{d^2y}{dx^2} = 0$, and a concavity change.

$6x = 0$

$x = 0$, when $x = 0$, $y = 1-3+2 = 2$ ✓

Concavity changes (see above)

$\therefore (0, 2)$ is a point of inflexion ✓



✓✓

(iv) Concave down: $x < 0$ ✓

(b) (i) $x^2 + x - 12 = -x^2 + 3x$

$2x^2 - 2x - 12 = 0$

$x^2 + x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$ or 3

$\therefore$ P has x-coordinate -2. ✓

(ii) Area $= \int_{-2}^{3} -x^2 + 3x - (x^2 + x - 12)\,dx$

$= \int_{-2}^{3} (-x^2 + 3x - x^2 - x + 12)\,dx$

$= \int_{-2}^{3} (-2x^2 + 2x + 12)\,dx$ ✓

$= 2\int_{-2}^{3} (-x^2 + x + 6)\,dx$

$= 2\left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-2}^{3}$ ✓

$= 2\left[\left(-9 + \frac{9}{2} + 18\right) - \left(\frac{8}{3} + 2 - 12\right)\right]$

$= 2\left[\frac{27}{2} + \frac{22}{3}\right]$

$= 2\left(\frac{81 + 44}{6}\right)$

$= \frac{125}{3}$ units^2 ✓

(or $41\frac{2}{3}$ units^2)

QUESTION SIX

a)(i) $l = r\theta$

$40 = 50\theta$ ✓

$\theta = \frac{4}{5}$

$\therefore \angle AOD = \frac{4}{5}$ radians

ii) Area $ABCD = \frac{1}{2}OB^2\theta - \frac{1}{2}OA^2\theta$

$= \frac{1}{2} \times \frac{4}{5}(60^2 - 50^2)$ ✓

$= \frac{2}{5} \times 1100$

$= 440 \text{ m}^2$ ✓

b)(i) $P = Ae^{kt}$

$\frac{dP}{dt} = k \times Ae^{kt}$

$= kP$ ✓

(ii) $t=0,\ P=50$

$50 = Ae^0$

$\therefore A = 50$

$t=10,\ P=275$

$275 = 50e^{10k}$ ✓

$e^{10k} = \frac{11}{2}$

$\ln\left(\frac{11}{2}\right) = 10k$ ✓

$k = \frac{1}{10}\ln\left(\frac{11}{2}\right)$

iii) 3 weeks = 21 days, $t=21$

$P = 50e^{21k}$ ✓

$= 1793.6\ldots$ ✓

$\therefore$ Julian will not discover an infestation.

(c) $y = \ln x$

$e^y = x$

$x^2 = e^{2y}$

$V = \pi \int_{y_1}^{y_2} x^2\, dy$

$= \pi \int_0^{\ln 3} e^{2y}\, dy$ ✓

$= \pi \left[\frac{1}{2}e^{2y}\right]_0^{\ln 3}$ ✓

$= \frac{\pi}{2}\left(e^{2\ln 3} - e^0\right)$

$= \frac{\pi}{2}(9-1)$

$= 4\pi\ u^3$ ✓

QUESTION 7

a) $\int_{1/2}^{m} \frac{1}{x^2}\, dx = 1$

$\left[\frac{-1}{x}\right]_{1/2}^{m} = 1$ ✓

$-\frac{1}{m} + \frac{1}{1/2} = 1$

$-\frac{1}{m} + 2 = 1$

$m = 1$ ✓

b) $\angle BXA = \angle XAD = \alpha$ (Alternate angles, $AD \parallel BC$)

i) $\angle CXD = \angle ADX = \beta$ (Alternate angles, $AD \parallel BC$) ✓

$\therefore \angle AXD = 180 - (\alpha + \beta)$ (straight angle BXC)

AX bisects $\angle BAD$

so $\angle BAX = \alpha$ ($\angle BAD = 2\alpha$)

$\therefore \angle BAX = \angle BXA$

$\therefore \triangle BAX$ is isosceles (equal base angles) ✓

ii) DX bisects $\angle CDA$, so $\angle ADC = 2\beta$

$\angle BAD + \angle ADC = 180$ (cointerior angles, $AB \parallel CD$)

$2\alpha + 2\beta = 180$ ✓

$\alpha + \beta = 90$

$\therefore \angle AXD = 180 - 90$

$= 90°$ ✓

c) (i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	0.5554	1.5708	1.6661	0

✓

i) $\int_0^{\pi} x\sin x \doteq \frac{\pi/2}{6}\left(0+4\times 0.5554+1.5708\right)+\frac{\pi/2}{6}\left(1.5708+4\times 1.6661+0\right)$ ✓

$\doteq \frac{\pi}{12}\left(3.7924+8.2352\right)$

$\doteq 3.1488\ldots$

$\doteq 3.15$ ✓

ii) $\frac{d}{dx}\left(\sin x - x\cos x\right) = \cos x - \left(\cos x - x\sin x\right)$ ✓

$= x\sin x$

$\int_0^{\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_0^{\pi}$

$= \sin\pi - \pi\cos\pi$

$= \pi$ ✓

iv) Error = approximation − exact value

$= 3.15 - \pi$

$= 0.008407\ldots$

% Error = $\frac{\text{error}}{\pi}\times 100$

$= 0.267\ldots$

$= 0.3\%$ ✓

QUESTION EIGHT

(a) $mx^2+7x-m=0$

$\alpha+\beta=-\frac{b}{a}$ $\qquad \alpha\beta=\frac{c}{a}$

$x=3/4$, $\frac{3}{4}\times\beta=-\frac{m}{m}$ ✓

$\beta=-\frac{4}{3}$ ✓

(b) (i) $\frac{dV}{dt}=\frac{6t}{t^2+1}$

$V=3\int\frac{2t}{t^2+1}\,dt$

$=3\ln(t^2+1)+C$ ✓

$t=0 \quad V=0 \quad C=0$

$\therefore V=3\ln(t^2+1)$ ✓

(ii) $V=\frac{4}{3}\pi r^3$, $r=1.5$.

$=\frac{4}{3}\times\pi\times\left(\frac{3}{2}\right)^3$

$=\frac{9\pi}{2}$ ✓

$\frac{9\pi}{2}=3\ln(t^2+1)$ ✓

$\ln(t^2+1)=\frac{3\pi}{2}$

$t^2+1=e^{\frac{3\pi}{2}}$

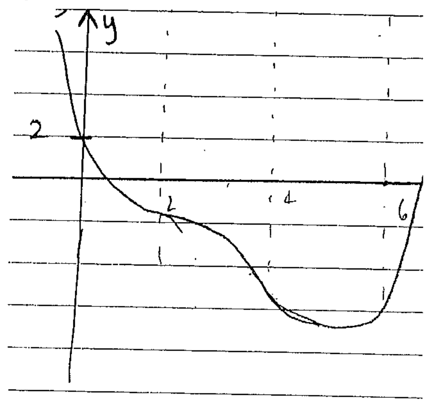
$t^2=e^{3\pi/2}-1$

$t\doteq 10.55$ ✓

(c) (i) $x>6$ ✓

(ii) $2<x<4$ ✓

(iii)



QUESTION NINE

a) $2\sin^2\alpha - \cos\alpha + 1 = 0$

$2(1-\cos^2\alpha) - \cos\alpha + 1 = 0$ ✓

$2 - 2\cos^2\alpha - \cos\alpha + 1 = 0$

$2\cos^2\alpha + \cos\alpha - 3 = 0$

$(2\cos\alpha + 3)(\cos\alpha - 1) = 0$ ✓

$\therefore \cos\alpha = -\frac{3}{2}$ or 1

$\therefore \alpha = 0, 2\pi$ ✓

b) $\log_6(x+3) + \log_6(x-2) = 2$

$\log_6[(x+3)(x-2)] = 2$

$(x+3)(x-2) = 36$ ✓

$x^2 + x - 6 = 36$

$x^2 + x - 42 = 0$

$(x-6)(x+7) = 0$

$x = 6$ or -7 ✓

But $\log_6(x+3)$ & $\log_6(x-2)$ must be positive

$\therefore x = 6$ is the only solution ✓

(c) $x = 5e^{-t}\sin t$

(i) $\frac{dx}{dt} = -5e^{-t}\sin t + 5e^{-t}\cos t$ ✓

$= 5e^{-t}(\cos t - \sin t)$

(ii) $t=0$, $x=0$, $v=5$ ✓✓

(iii) $\frac{dx}{dt} = 0$ when $\cos t - \sin t = 0$

$\frac{\sin t}{\cos t} = 1$ ✓

$\tan t = 1$

$\therefore t = \frac{\pi}{4}$ ✓

(iv) $\ddot{x} = -10e^{-t}\cos t$

acceleration is zero when $\cos t = 0$

i.e. $t = \frac{\pi}{2}$ ✓

QUESTION 10

a) (i) $r = 9.6 \div 12 = 0.8\%$ per month

$A_1 = 700\,000(1.008) - M$ ✓

$A_2 = A_1(1.008) - M$

$= (700\,000(1.008) - M)(1.008) - M$

$= 700\,000(1.008)^2 - M(1 + 1.008)$ ✓

$A_3 = A_2(1.008) - M$

$= [700\,000(1.008)^2 - M(1+1.008)](1.008) - M$

$= 700\,000(1.008)^3 - M(1 + 1.008 + 1.008^2)$ ✓

(ii) $A_n = 700\,000(1.008)^n - M(1 + 1.008 + \dots + 1.008^{n-1})$

$= 700\,000(1.008)^n - M\left(\frac{1.008^n - 1}{.008}\right)$

$= 700\,000(1.008)^n - 125M(1.008^n - 1)$ ✓

(iii) If repaid in 30 years, $A_{360} = 0$.

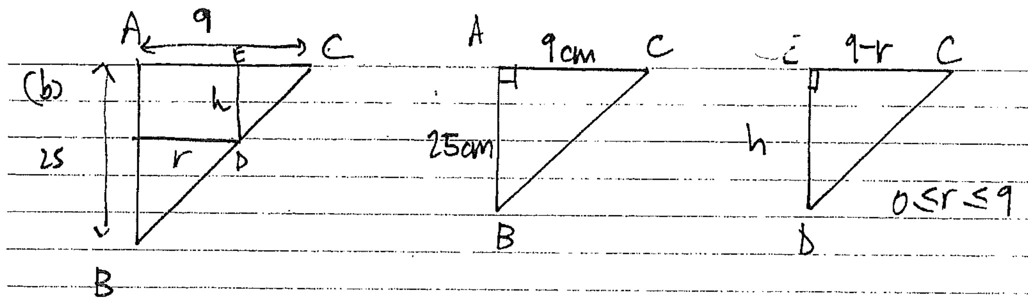
$700\,000(1.008)^{360} - 125M(1.008^{360} - 1) = 0$

$M = \frac{700000(1.008)^{360}}{125(1.008^{360} - 1)}$ ✓

$= 5937.119\dots$

$= \$5937$ (to the nearest dollar) ✓

$\therefore$ Nick will be able to afford his dream home.



$\Delta ABC \,|||\, \Delta EDC$ (equiangular)

$\frac{9}{25} = \frac{9-r}{h}$ (matching sides in similar triangles) ✓✓

$h = \frac{25(9-r)}{9}$

(ii) Using the result $h = \frac{25(9-r)}{r}$

$V = \pi r^2 h$

$= \pi \times r^2 \times \frac{25(9-r)}{r}$

$= 25\pi(9r - r^2)$ ✓

(iii) $\frac{dV}{dr} = 25\pi(9-2r)$ and $\frac{d^2V}{dr^2} = -50\pi$

stationary at $\frac{dV}{dr} = 0$, $9 - 2r = 0$
$r = 9/2$ ✓

at $r = 9/2$, $\frac{d^2V}{dr^2} < 0$ $\therefore$ a Maximum ✓

Max Volume $= 25\pi\left(9 \times \frac{9}{2} - \left(\frac{9}{2}\right)^2\right)$

$= 25\pi\left(81/2 - 81/4\right)$

$= \frac{2025\pi}{4}\ u^3$ ✓