WESTERN REGION

2010
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 2

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-8.
- All questions are of equal value.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

Question 1 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) Evaluate $\int_{\frac{\pi}{16}}^{\frac{\pi}{12}} \sec 4x \ \tan 4x \ dx$ **2**

b) Find $\int x \ln x \ dx$ **2**

c) Find $\int \frac{9x^3+9x^2+5x+4}{3x+1} dx$ **3**

d) i. Find constants a, b and c such that **2**

$$\frac{3x^2-2x-3}{(x^2+9)(x-3)} = \frac{ax+b}{x^2+9} + \frac{c}{x-3}$$

ii. Hence find $\int \frac{3x^2-2x-3}{(x^2+9)(x-3)} dx$. **2**

e) By making the substitution $t = \tan\frac{\theta}{2}$, evaluate $\int_0^{\frac{\pi}{2}} \frac{d\theta}{1+\sin\theta+\cos\theta}$ **4**

End of Question 1

Question 2 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) Given $A = 3+4i$ and $B = 1-i$, express the following in the form $x+iy$ where x and y are real numbers.

i. AB **1**

ii. $\frac{A}{iB}$ **2**

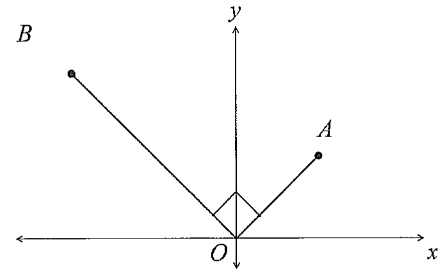
iii. $\sqrt{A}$ **3**

b) If $w = \sqrt{3} - i$,

i. Find the exact value of $|w|$ and $\arg w$. **2**

ii. Find the exact value of w^5 in the form $a+ib$ where a and b are real. **2**

c)



On the Argand diagram, OA represents the complex number $z_1 = x+iy$, $\angle AOB = \frac{\pi}{2}$ and the length of OB is twice that of OA.

i. Show that OB represents the complex number $-2y+2ix$. **1**

ii. Given that AOBC is a rectangle, find the complex number represented by OC. **1**

iii. Find the complex number represented by BA. **1**

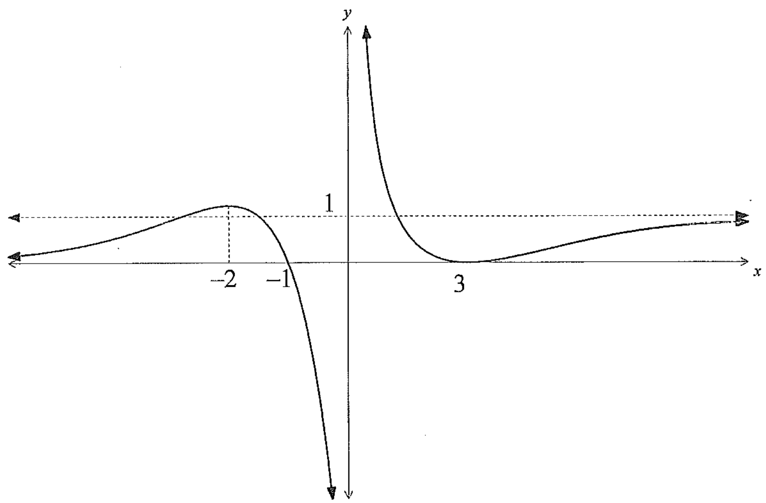
d) Sketch the region on an argand diagram where **2**

$|z-1| \le \sqrt{2}$ and $0 \le \arg(z+i) \le \frac{\pi}{4}$ both hold.

End of Question 2

Question 3 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a)



The diagram shows the graph of the function $y = f(x)$ which has asymptotes, vertically at $x = 0$ and horizontally at $y = 1$ for $x \geq 0$ and at $y = 0$ for $x \leq 0$.

Draw separate sketches of the following showing any critical features.

i. $y = \dfrac{1}{f(x)}$ **2**

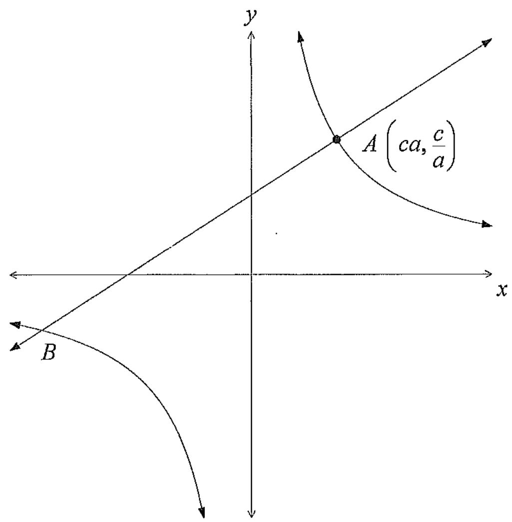
ii. $y = [f(x)]^2$ **2**

iii. $y = f'(x)$ **2**

Question 3 continues

Question 3 continued **Marks**

b)



The point $A\left(ca, \dfrac{c}{a}\right)$, where $a \neq \pm 1$ lies on the hyperbola $xy = c^2$. The normal through A meets the other branch of the curve at B.

i. Show that the equation of the normal through A is **2**

$$y = a^2 x + \frac{c}{a}\left(1 - a^4\right)$$

ii. Hence if B has coordinates $\left(cb, \dfrac{c}{b}\right)$, show that $b = \dfrac{-1}{a^3}$. **3**

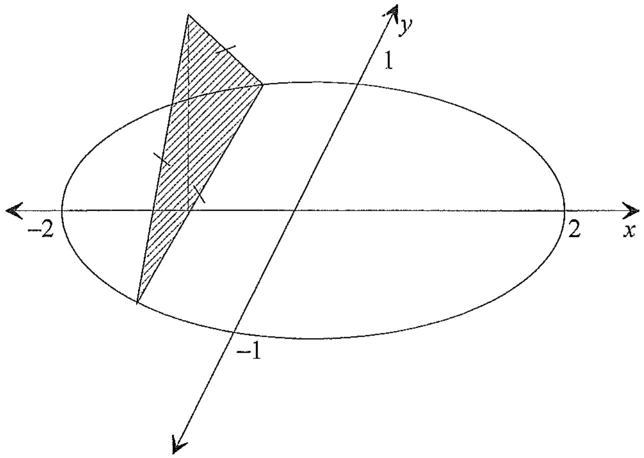
iii. If this hyperbola is rotated clockwise through 45°, show that the equation becomes **4**

$$x^2 - y^2 = 2c^2 .$$

End of Question 3

Question 4 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) A solid shape has as its base an ellipse in the XY plane as shown below. Sections taken perpendicular to the X-axis are equilateral triangles. The major and minor axes of the ellipse are 4 metres and 2 metres respectively.



i. Write down the equation of the ellipse. **1**

ii. Show that the area of the cross-section at $x = k$ is given by **2**

$$A = \frac{\sqrt{3}}{4}\left(4 - k^2\right).$$

iii. By using the technique of slicing, find the volume of the solid. **2**

b) The region enclosed by the curve $y = 5x - x^2$, the x axis and the lines $x = 1$ and $x = 3$ is rotated about the y axis. By using the method of cylindrical shells, find the volume of the solid so produced. **4**

Question 4 continues

Question 4 continued **Marks**

c) The roots of the equation $x^3 - 3x^2 + 9 = 0$ are α, β and γ.

i. Determine the polynomial equation with roots α^2, β^2 and γ^2. **2**

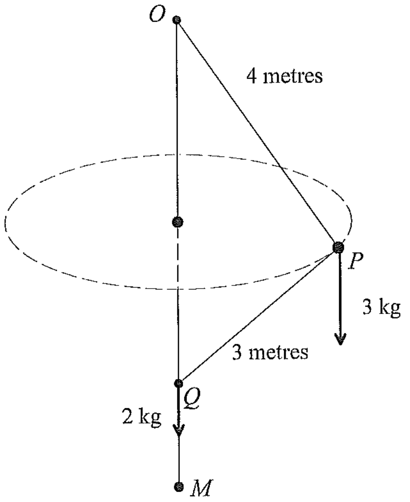
ii. Find the value of $\alpha^2 + \beta^2 + \gamma^2$ and hence evaluate $\alpha^3 + \beta^3 + \gamma^3$. **2**

d) Given that the polynomial $P(x)$ has a double root at $x = \alpha$, show that the polynomial $p'(x)$ will have a single root at $x = \alpha$. **2**

End of Question 4

Question 5 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a)



The above sketch shows a smooth vertical rod OM. Light inextensible strings OP and QP are attached to the rod at O and a mass of 3kg at P. At Q, a 2kg mass is free to slide on the rod. P is rotating in a horizontal circle about the rod.

When the distance OQ is 5 metres

i. Calculate the tension T_1 in PQ and T_2 in OP. (In terms of g) **3**

ii. Hence calculate the angular velocity of P to maintain this system. Give your answer correct to one decimal place. (Use $g = 10\ ms^{-2}$) **3**

b) By taking logarithms of both sides and then differentiating implicitly, verify the rule for differentiating the quotient $y = \dfrac{u(x)}{v(x)}$ is given by **2**

$$\frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

Question 5 continues

Question 5 continued **Marks**

c) i. Show that the recurrence (reduction) formula for **4**

$$I_n = \int \tan^n x\,dx \quad \text{is} \quad I_n = \frac{1}{n-1}\tan^{n-1} x - I_{n-2}.$$

ii. Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \tan^3 x\,dx$ **3**

End of Question 5

Question 6 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) A solid of unit mass is dropped under gravity from rest at a height of H metres. Air resistance is proportional to the speed (V) of the mass. (acceleration under gravity = g)

i. Write the equation for the acceleration of the mass. (Use k as the constant of proportionality) **1**

ii. Show that the velocity (V) of the solid after t seconds is given by **3**

$$V = \frac{g}{k}\left(1 - e^{-kt}\right)$$

iii. Bt using the fact that $\frac{d}{dx}\left(\frac{1}{2}V^2\right) = \ddot{x}$, show that **4**

$$x = \frac{g}{k^2}\left[\ln\frac{g}{g - kV} - \frac{kV}{g}\right].$$

b) Given $z = \cos\theta + i\sin\theta$, and using De Moivres' Theorem

i. Find an expression for $\cos 4\theta$ in terms of powers of $\cos\theta$. **3**

ii. Determine the roots of the equation $z^4 = -1$. **2**

iii. Using the fact that $z^n + \frac{1}{z^n} = 2\cos n\theta$, find an expression for $\cos^4\theta$ in terms of $\cos n\theta$. **2**

End of Question 6

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Question 7 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) i. Prove that $\cos[(k-1)\theta] - 2\cos\theta\cos k\theta = -\cos[(k+1)\theta]$. **1**

ii. Hence, using mathematical induction, prove that if n is a positive integer then **4**

$$1 + \cos\theta + \cos 2\theta + \ldots\ldots\ldots\ldots + \cos(n-1)\theta = \frac{1 - \cos\theta - \cos n\theta + \cos[(n-1)\theta]}{2 - 2\cos\theta}$$

b) A mass of 20kg hangs from the end of a rope and is hauled up vertically from rest by winding up the rope. The pulling force on the rope starts at 250N and decreases uniformly by 10N for every metre wound up. **3**

Find the velocity of the mass when 10 metres have been wound up.

(Neglect the weight of the rope and take $g = 10ms^{-2}$)

c) When a polynomial $P(x)$ is divided by $(x-1)$ the remainder is 3 and when divided by $(x-2)$ the remainder is 5. Find the remainder when the polynomial is divided by $(x-1)(x-2)$. **3**

d) Show that $\frac{x^4 + x^2 + 1}{x^2} \geq 3$ for all x. **2**

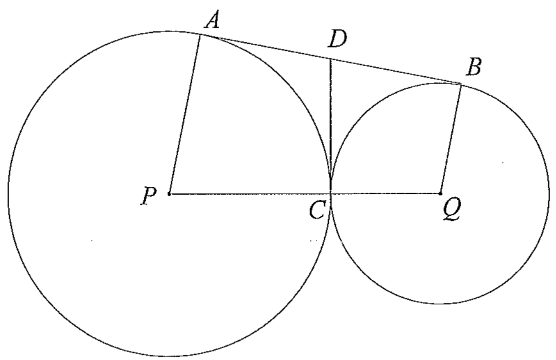
(Hint: Start from $(x^2 - 1)^2 \geq 0$)

e) If abc represents a three digit number (not the product of a, b and c), show that if $a + c = b$ then the number is divisible by 11. **2**
(a, b and c are positive integers)

End of Question 7

Question 8 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a)



In the diagram PCQ is a straight line joining the centres of the circles P and Q. AB and DC are common tangents.

i. Explain why PADC and CDPQ are cyclic quadrilaterals. **2**

ii. Show that $\Delta ADC \parallel\!| \Delta BQC$. **3**

iii. Show that PD || CB. **2**

b) Given $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

If $P = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta$

i. Prove that $P\sin\dfrac{\theta}{2} = \sin\dfrac{7\theta}{2}$. **3**

ii. Hence show that if $\theta = \dfrac{2\pi}{7}$ then **2**

$$P = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta = 0$$

iii. By writing P in terms of $\cos\theta$, prove that $\cos\dfrac{2\pi}{7}$ is a root of the Polynomial equation **3**

$$8x^3 + 4x^2 - 4x - 1 = 0$$

End of Examination

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Mathematics Extension 2

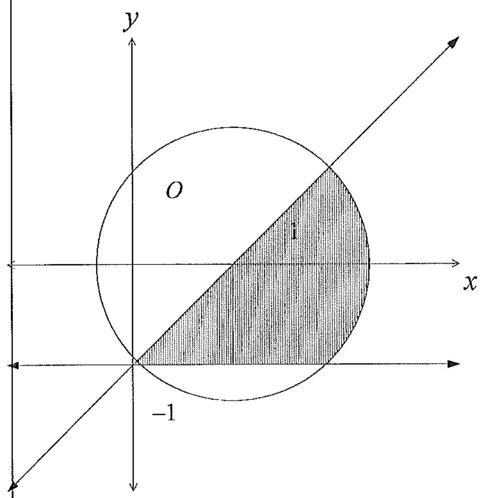
SOLUTIONS

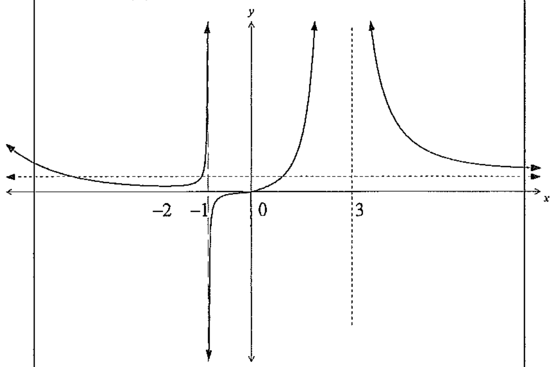
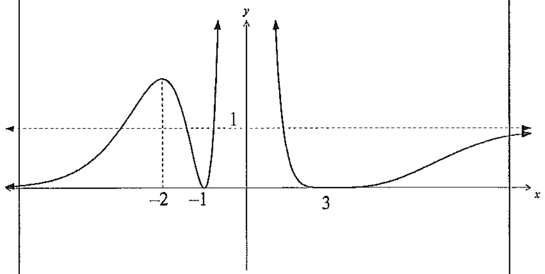
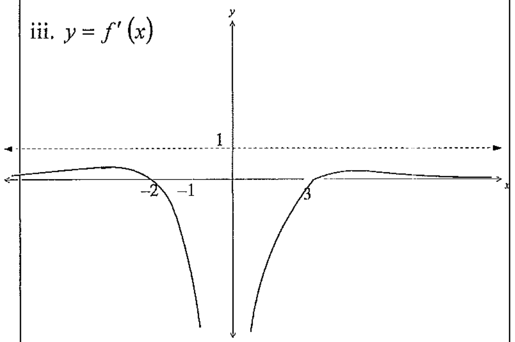
Question 1	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\int_{\frac{\pi}{16}}^{\frac{\pi}{12}} sec\,4x\,.tan\,4x\;dx=\left[\frac{sec\,4x}{4}\right]_{\frac{\pi}{16}}^{\frac{\pi}{12}}$	1	Use standard integral sheet
	$=\frac{1}{4}\left[sec\frac{\pi}{3}-sec\frac{\pi}{4}\right]$		
	$=\frac{1}{4}\left[2-\sqrt{2}\right]$	1	
	$=\frac{2-\sqrt{2}}{4}$		
b)	$\int x\ln x\,dx=\frac{1}{2}x^2\ln x-\int\frac{1}{2}x^2.\frac{1}{x}\,dx$	1	
	$=\frac{1}{2}x^2\ln x-\frac{1}{2}\int x\,dx$		
	$=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+c$	1	
c)	$\int\frac{9x^3+9x^2+5x+4}{3x+1}\,dx=\int\left(3x^2+2x+1+\frac{3}{3x+1}\right)dx$	2	By division
	$=x^3+x^2+x+ln\,(3x+1)+c$	1	
d)	i. $\frac{3x^2-2x-3}{(x^2+9)(x-3)}=\frac{ax+b}{x^2+9}+\frac{c}{x-3}$		
	$3x^2-2x-3=(ax+b)(x-3)+c(x^2+9)$	1	
	$x=3\to 18=18c$		
	$\therefore c=1$		
	$x=0\to -3=-3b+9$		Any fair method
	$\therefore\; b=4$		
	$x=1\to -2=(a+4)(-2)+10$		
	$=-2a+2$		
	$\therefore a=2$		
	$\therefore a=2,\,b=4,\,c=1$	1	
	ii. $\int\frac{3x^2-2x-3}{(x^2+9)(x-3)}\,dx=\int\left(\frac{2x+4}{x^2+9}+\frac{1}{x-3}\right)dx$		
	$=\int\left(\frac{2x}{x^2+9}+\frac{4}{x^2+9}+\frac{1}{x-3}\right)dx$	1	
	$=\ln(x^2+9)+\frac{4}{3}\tan^{-1}\frac{x}{3}+\ln(x-3)+c$	1	Fully simplified not needed
	$=\ln(x^2+9)(x-3)+\frac{4}{3}\tan^{-1}\frac{x}{3}+c$		

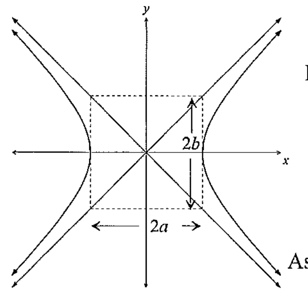
Question 1	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
e)	$\int_0^{\frac{\pi}{2}} \frac{d\theta}{1+\sin\theta+\cos\theta}$ if $t=\tan\frac{\theta}{2}$ $\frac{dt}{d\theta}=\frac{1}{2}\sec^2\frac{\theta}{2}$ $=\frac{1}{2}\left(1+\tan^2\frac{\theta}{2}\right)$ $=\frac{1}{2}(1+t^2)$	1	
	$\frac{d\theta}{dt}=\frac{2}{1+t^2}$ $d\theta=\frac{2\,dt}{1+t^2}$ $\theta=\frac{\pi}{2}, t=1$ $\theta=0, t=0$	1	
	$\int_0^1 \frac{\frac{2}{1+t^2}dt}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}=\int_0^1\frac{2\,dt}{1+t^2+2t+1-t^2}$	1	
	$=\int_0^1\frac{2\,dt}{2+2t}$ $=[\ln(1+t)]_0^1$ $=\ln 2-\ln 1$ $=\ln 2$	1	
		/15	

Question 2	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. $AB=(3+4i)(1-i)$ $=3-3i+4i+4$ $=7+i$	1	
	ii. $\frac{A}{iB}=\frac{3+4i}{i(1-i)}$		
	$=\frac{3+4i}{1+i}\times\frac{1-i}{1-i}$	1	
	$=\frac{3-3i+4i+4}{2}$		
	$=\frac{7+i}{2}=\frac{7}{2}+\frac{1}{2}i$	1	
	iii. Let $\sqrt{A}=a+ib$ (a and b real)		
	$\therefore A=a^2-b^2+2abi$ $\therefore a^2-b^2=3$, $2ab=4$ $ab=2$ $\therefore b=\frac{2}{a}$	1	
	$\therefore a^2-\frac{4}{a^2}=3$ $a^4-3a^2-4=0$ $(a^2-4)(a^2+1)=0$		Any fair method
	$\therefore a=\pm 2$ only real solution $\therefore b=\pm 1$	1	
	$\therefore\sqrt{A}=\pm(2+i)$	1	

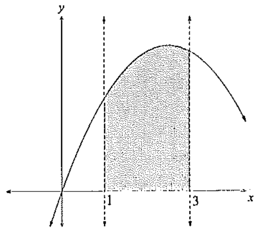
Question 2	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)	i. $\|w\| = \sqrt{3+1} = 2$	1	
	$w = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ If $Arg\ w = \theta$ $\quad cos\ \theta = \frac{\sqrt{3}}{2},\ sin\ \theta = -\frac{1}{2}$ $\therefore \theta = -\frac{\pi}{6}$ $\therefore Arg\ \ w = -\frac{\pi}{6}$	1	
	ii. $w = \sqrt{3} - i = 2\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$ $\therefore w^5 = 32\left[\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right]$	1	
	$= 32\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ $= -16\sqrt{3} - 16i$	1	
c)	i. $OB = iOA \times 2$ $= i\,(x + iy) \times 2$ $= -2y + 2ix$	1	
	ii. $OC = OB + OA$ $= (-2y + 2ix) + (x + iy)$ $= (x - 2y) + (2x + y)i$	1	
	iii. $BA = BO + OA$ $= -OB + OA$ $= -(-2y + 2ix) + (x + iy)$ $= (x + 2y) + (y - 2x)\,i$	1	0 if signs incorrect

d)	 	2	1 for region inside circle 1 for between lines
		/15	

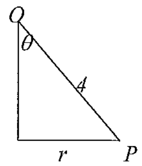
Question 3	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. $y=\frac{1}{f(x)}$ 	2	2 marks each, deduct a mark for a major feature missing or incorrect, e.g. asymptotes not correct in (i)
	ii. $y=[f(x)]^2$ 	2	
	iii. $y=f'(x)$ 	2	

Question 3	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)	i. $y=\frac{c^2}{x}$ $\frac{dy}{dx}=-\frac{c^2}{x^2}$, $at\ \ x=ca\quad \frac{dy}{dx}=-\frac{1}{a^2}$ $\therefore$ Gradient of normal $=a^2$	1	
	$\therefore$ Equation of normal is $y-\frac{c}{a}=a^2(x-ca)$ $=a^2x-ca^3$ $y=a^2x+\frac{c}{a}-ca^3$ $y=a^2x+\frac{c}{a}(1-a^4)$	1	
	ii. Solving $y=\frac{c^2}{x}$ with equation in (i) $\frac{c^2}{x}=a^2x+\frac{c}{a}(1-a^4)$ $\therefore a^2x^2+\frac{c}{a}(1-a^4)x-c^2=0$	1	
	Product of roots $=-\frac{c^2}{a^2}$	1	
	The roots are $x=cb\ and\ x=ca$ $\therefore c^2ab=-\frac{c^2}{a^2}$ $\therefore b=-\frac{1}{a^3}$	1	
	iii. Asymptotes become $y=\pm x$ Original Vertices $(\pm c,\pm c)$	1	Any logical reasoning
	Distance from O is $\sqrt{2}\ c$ $\therefore$ New vertices $=(\pm\sqrt{2},0)$	1	
	$\therefore a=\sqrt{2}\ c$ As it is rectangular $b=\sqrt{2}\ c$	1	
	$\therefore \frac{x^2}{2c^2}-\frac{y^2}{2c^2}=1$ $\therefore x^2-y^2=2c^2$	1	

Question 4	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. $a = 2$, $b = 1$		
	$\therefore$ Equation is $\frac{x^2}{4} + y^2 = 1$	1	
	ii. At $x = k$, $\quad y^2 = 1 - \frac{k^2}{4}$		
	$\therefore y = \pm\sqrt{\frac{4-k^2}{4}}$		
	$\therefore$ Length of side of triangle = $\sqrt{4-k^2}$	1	
	$\therefore$ Area = $\frac{1}{2}\sqrt{4-k^2}\,.\sqrt{4-k^2}\ \sin 60°$		
	$= \frac{1}{2}(4-k^2)\cdot\frac{\sqrt{3}}{2}$		
	$= \frac{\sqrt{3}}{4}(4-k^2)$	1	
	iii. Let slice thickness = δk		
	$\therefore$ Volume of slice $\delta V = \frac{\sqrt{3}}{4}(4-k^2).\,\delta k$		
	$\therefore V = \int_{-2}^{2}\frac{\sqrt{3}}{4}(4-k^2)\,dk$	1	
	$= \frac{\sqrt{3}}{4}\left[4k - \frac{k^3}{3}\right]_{-2}^{2}$		
	$= \frac{\sqrt{3}}{4}\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$		
	$= \frac{\sqrt{3}}{4}\cdot\frac{32}{3}$		
	Volume = $\frac{8\sqrt{3}}{3}$ units ^{3}	1	

Question 4	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)			
	At $x = k$		
	$y = 5k - k^2$		
	Let thickness of shell be δk		
	$\therefore$ Volume of shell		
	$\delta V = \pi\left(k^2 - (k-\delta k)^2\right)y$		
	$= \pi\left(2k\delta k - \delta k^2\right)y$	1	Can use formula
	As $\delta k \to 0$		
	$V = \int_1^3 2\pi.\,k.\,y\,dk$	1	
	$= 2\pi\int_1^3 k\left(5k-k^2\right)dk$	1	
	$= 2\pi\left[\frac{5}{3}k^3 - \frac{k^4}{4}\right]_1^3$		
	$= 2\pi\left[\left(45-\frac{81}{4}\right)-\left(\frac{5}{3}-\frac{1}{4}\right)\right]$		
	Volume = $\frac{140\pi}{3}$ units ^{3}	1	

Question 4	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
c)	i. $X = x^2$ $\therefore x = \sqrt{X}$ $\therefore X\sqrt{X} - 3X + 9 = 0$ $X\left(\sqrt{X} - 3\right) = -9$ $\sqrt{X} - 3 = \frac{-9}{X}$ $\sqrt{X} = \frac{-9}{X} + 3$ $X = \frac{81}{X^2} - \frac{54}{X} + 9$ $X^3 = 81 - 54X + 9X^2$ Required equation is $x^3 - 9x^2 + 54x - 81 = 0$	1 1	Any method
	ii. from equation in (i) sum of roots is given by $\alpha^2 + \beta^2 + \chi^2 = \frac{-b}{a} = 9$ Now, in original equation $\left.\begin{array}{l}\alpha^3 - 3\alpha^2 + 9 = 0\\ \beta^3 - 3\beta^2 + 9 = 0\\ \chi^3 - 3\chi^2 + 9 = 0\end{array}\right\\}$ as $x = \alpha, \beta, \chi$ are roots Adding, $\alpha^3 + \beta^3 + \chi^3 - 3\left(\alpha^2 + \beta^2 + \chi^2\right) + 27 = 0$ $\alpha^3 + \beta^3 + \chi^3 - 3(9) + 27 = 0$ $\therefore \alpha^3 + \beta^3 + \chi^3 = 0$	1 1	Any method
d)	Let $P(x) = (x - \alpha)^2\, Q(x)$ $\therefore P'(x) = 2(x - \alpha)\, Q(x) + (x - \alpha)^2\, Q'(x)$ $= (x - \alpha)\left[2\,Q(x) + (x - \alpha)\, Q'(x)\right]$ $\therefore P'(x)$ has a single root at $x = \alpha$	1 1	
		/15	

Question 5	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. Let $\angle POQ = \theta$ and $\angle OQP = \alpha$ Note when $OQ = 5m$, $\angle OPQ = 90°$ Vertically at Q $T_1 \cos\alpha = 2g$ $T_1 \cdot \frac{3}{5} = 2g$ $\therefore T_1 = \frac{10g}{3}\ N$ Vertically at P $T_2 \cos\theta = T_1 \cos\alpha + 3g$ $T_2 \cdot \frac{4}{5} = 2g + 3g$ $T_2 = \frac{25g}{4}\ N$	1 2	1 for correct resolving
	ii. Horizontally at P $T_1 \sin\alpha + T_2 \sin\theta = 3rw^2$ *Finding r*  $\frac{r}{4} = \sin\theta = \frac{3}{5}$ $r = \frac{12}{5}$ $\therefore \frac{10g}{3} \cdot \frac{4}{5} + \frac{25g}{4} \cdot \frac{3}{5} = 3 \cdot \frac{12}{5} \cdot w^2$ $\frac{77g}{12} = \frac{36}{5} w^2$ $g = 10$ $\therefore w^2 = \frac{1925}{216}$ $\therefore w = 3 \cdot 0$ *Angular Velocity 3·0 rad s^{-1}*	1 1 1	

Question 5	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)	Let $y=\dfrac{u(x)}{v(x)}$		
	$\therefore \ln y=\ln[u(x)]-\ln[v(x)]$		
	$\dfrac{1}{y}\cdot\dfrac{dy}{dx}=\dfrac{u'(x)}{u(x)}-\dfrac{v'(x)}{v(x)}$	1	
	$\dfrac{v(x)}{u(x)}\cdot\dfrac{dy}{dx}=\dfrac{u'(x)}{u(x)}-\dfrac{v'(x)}{v(x)}$		
	$\dfrac{dy}{dx}=\dfrac{u'(x)}{v(x)}-\dfrac{v'(x).u(x)}{[v(x)]^2}$		
	$\therefore \dfrac{dy}{dx}=\dfrac{v(x).u'(x)-u(x).v'(x)}{[v(x)]^2}$	1	
c)	i. $I_n=\int \tan^n x\,dx$		
	$=\int \tan^2 x.\tan^{n-2}x\,dx$	1	
	$=\int(\sec^2 x-1)\tan^{n-2}x\,dx$	1	
	$=\int \sec^2 x\tan^{n-2}x\,dx-\int\tan^{n-2}x\,dx$	1	
	$=\dfrac{1}{n-1}\tan^{n-1}x-I_{n-2}$	1	
	ii. $\int_0^{\frac{\pi}{4}}\tan^3 x\,dx=\left[\dfrac{1}{2}\tan^2 x\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\tan x\,dx$	1	
	$=\dfrac{1}{2}[1-0]+[\ln(\cos x)]_0^{\frac{\pi}{4}}$	1	
	$=\dfrac{1}{2}+\left[\ln\left(\dfrac{1}{\sqrt{2}}\right)-\ln 1\right]$		
	$=\dfrac{1}{2}+\ln\dfrac{1}{\sqrt{2}}$	1	
		/15	

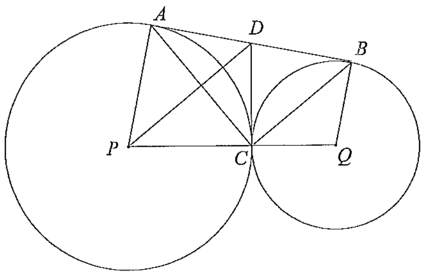
Question 6	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. $\ddot{x}=g-kv$	1	
	ii. $\ddot{x}=\dfrac{dv}{dt}=g-kv$		
	$\dfrac{dt}{dv}=\dfrac{1}{g-kv}$		
	$t=-\dfrac{1}{k}\ln(g-kv)+c$	1	
	When $t=0$, $v=0$		
	$\therefore c=\dfrac{1}{k}\ln g$		
	$\therefore t=\dfrac{1}{k}\ln g-\dfrac{1}{k}\ln(g-kv)$		
	$kt=\ln\left(\dfrac{g}{g-kv}\right)$	1	
	$e^{kt}=\dfrac{g}{g-kv}$		
	$ge^{kt}-kve^{kt}=g$		
	$ge^{kt}-g=kve^{kt}$		
	$g(e^{kt}-1)=kve^{kt}$		
	$\therefore v=\dfrac{g}{k}(1-e^{-kv})$	1	

Question 6	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	iii. $\ddot{x} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$		
	$= \dfrac{d}{dv}\left(\dfrac{1}{2}v^2\right)\cdot\dfrac{dv}{dx}$		
	$= v\dfrac{dv}{dx}$		
	$\therefore v\dfrac{dv}{dx} = g - kv$	1	
	$\dfrac{dv}{dx} = \dfrac{g-kv}{v}$		
	$\dfrac{dx}{dv} = \dfrac{v}{g-kv}$	1	
	$\dfrac{dx}{dv} = \dfrac{1}{k}\left[\dfrac{v}{\frac{g}{k}-v}\right] = \dfrac{1}{k}\left[\dfrac{v-\frac{g}{k}}{\frac{g}{k}-v} + \dfrac{\frac{g}{k}}{\frac{g}{k}-v}\right]$		
	$\dfrac{dx}{dv} = \dfrac{1}{k}\left[-1 + \dfrac{\frac{g}{k}}{\frac{g}{k}-v}\right]$		
	$\therefore x = \dfrac{1}{k}\left[-v - \dfrac{g}{k}\ln\left(\dfrac{g}{k}-v\right)\right] + c$	1	
	When $x = 0$, $v = 0$		
	$\therefore c = \dfrac{1}{k}\cdot\dfrac{g}{k}\ln\dfrac{g}{k}$		
	$\therefore x = \dfrac{1}{k}\left[\dfrac{g}{k}\ln\dfrac{g}{k} - v - \dfrac{g}{k}\ln\left(\dfrac{g}{k}-v\right)\right]$		
	$= \dfrac{1}{k}\left[\dfrac{g}{k}\ln\left(\dfrac{\frac{g}{k}}{\frac{g}{k}-v}\right) - v\right]$		
	$x = \dfrac{g}{k^2}\left[\ln\left(\dfrac{g}{g-kv}\right) - \dfrac{kv}{g}\right]$	1	

Question 6	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)	i. $z^4 = cos\,4\theta + i sin\,4\theta$		
	also by expansion		
	$= c^4 + 4ic^3s - 6c^2s^2 - 4ics^3 + s^4$	1	
	Where $c = cos\,\theta$ and $s = sin\,\theta$		
	$= c^4 + s^4 - 6c^2s^2 + 4i\left(c^3s - cs^3\right)$		
	Equating real parts		
	$cos\,4\theta = cos^4\theta + sin^4\theta - 6cos^2\theta\, sin^2\theta$	1	
	$= cos^4\theta + (1 - cos^2\theta)^2$		
	$- 6\,cos^2\theta(1 - cos^2\theta)$		
	$= cos^4\theta + 1 - 2\,cos^2\theta + cos^4\theta$		
	$- 6\,cos^2\theta + 6\,cos^4\theta$		
	$= 8\,cos^4\theta - 8\,cos^2\theta + 1$	1	
	ii. $cos\,4\theta = -1$ (Equating real parts)		
	$\therefore 4\theta = \pi, -\pi, 3\pi, -3\pi$	1	
	$\theta = \dfrac{\pi}{4}, -\dfrac{\pi}{4}, \dfrac{3\pi}{4}, -\dfrac{3\pi}{4}$		
	$\therefore$Roots are $cis\,\dfrac{\pi}{4}, cis\left(-\dfrac{\pi}{4}\right), cis\,\dfrac{3\pi}{4}, cis\left(-\dfrac{3\pi}{4}\right)$	1	
	iii. $z + \dfrac{1}{z} = 2\cos\theta$		
	$\left(z + \dfrac{1}{z}\right)^4 = 16\cos^4\theta$		
	LHS $= z^4 + 4z^3.\dfrac{1}{z} + 6z^2.\dfrac{1}{z^4} + 4z.\dfrac{1}{z^3} + \dfrac{1}{z^4}$		
	$= z^4 + \dfrac{1}{z^4} + 4\left(z^2 + \dfrac{1}{z^2}\right) + 6$	1	
	$\therefore 16cos^4\theta = 2cos\,4\theta + 8cos\,2\theta + 6$		
	$\therefore \cos^4\theta = \dfrac{1}{8}\cos 4\theta + \dfrac{1}{2}\cos 2\theta + \dfrac{3}{8}$	1	
		/15	

Question 7	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
a)	i. $\cos[(k-1)\theta] - 2\cos\theta\cos k\theta$ $= \cos k\theta\cos\theta + \sin k\theta\sin\theta - 2\cos\theta\cos k\theta$ $= -(\cos k\theta\cos\theta - \sin k\theta\sin\theta)$ $= -\cos(k\theta+\theta)$ $= -\cos[(k+1)\theta]$	1	
	ii. When $n = 1$ LHS = 1 RHS = $\frac{1-\cos\theta-\cos\theta+\cos 0}{2-2\cos\theta}$ $= \frac{2-2\cos\theta}{2-2\cos\theta} = 1 = LHS$	1	
	$\therefore$ True for $n = 1$ Assume true for $n = k$. i.e. $1+\cos\theta+\ldots\ldots+\cos[(k-1)\theta] = \frac{1-\cos\theta-\cos k\theta+\cos[(k-1)\theta]}{2-2\cos\theta}$		
	When $n = k+1$ $1+\cos\theta+\ldots\ldots\cos[(k-1)\theta]+\cos k\theta$ $= \frac{1-\cos\theta-\cos k\theta+\cos[(k-1)\theta]}{2-2\cos\theta}+\cos k\theta$	1	
	$= \frac{1-\cos\theta-\cos k\theta+\cos[(k-1)\theta]+2\cos k\theta-2\cos\theta\cos k\theta}{2-2\cos\theta}$ $= \frac{1-\cos\theta+\cos k\theta-\cos[(k+1)\theta]}{2-2\cos\theta}$ $= \frac{1-\cos\theta-\cos[(k+1)\theta]+\cos[((k+1)+1)\theta]}{2-2\cos\theta}$	1	
	If true for $n = 1$ then true for $n = 1+1 = 2$ etc $\therefore$ By induction true for all n positive integers.	1	

Question 7	Trial HSC Examination - Mathematics Extension 2		2010
Part	**Solution**	**Marks**	**Comment**
b)	Let Pulling force = F and distance pulled up be x metres. $\frac{dF}{dx} = -10$ $F = -10x + c$ When $x = 0$, $F = 250$ $\therefore F = 250 - 10x$	1	
	At the mass $20\ddot{x} = F - 20g$ $= 250 - 10x - 200$ $= 50 - 10x$ $\therefore \ddot{x} = \frac{5}{2} - \frac{x}{2}$	1	
	$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{5}{2} - \frac{x}{2}$ $\frac{1}{2}v^2 = \frac{5x}{2} - \frac{x^2}{4} + c$ When $x = 0$, $v = 0$, $\therefore c = 0$ $\frac{1}{2}v^2 = \frac{5x}{2} - \frac{x^2}{4}$ $v^2 = 5x - \frac{x^2}{2}$ When $x = 10$, $v^2 = 0$ $\therefore$ The mass is stationary.	1	
c)	When dividing by $(x-1)(x-2)$ the remainder is in the form $ax+b$. $P(x) = (x-1)(x-2)\,Q(x) + ax + b$	1	
	$P(1) = a + b = 3$ $P(2) = 2a + b = 5$ $\therefore a = 2, b = 1$	1	
	$\therefore$ Remainder is $2x + 1$	1	
d)	$(x^2-1)^2 \ge 0$ $x^4 - 2x^2 + 1 \ge 0$ $x^4 + x^2 + 1 \ge 3x^2$ $\frac{x^4+x^2+1}{x^2} \ge 3$	2	
e)	abc represents $100a + 10b + c$ If $b = a + c$	1	
	Then $100a + 10(a+c) + c$ $= 110a + 11c$ $= 11(10a + c)$	1	
		/15	

Question 8	Trial HSC Examination - Mathematics Extension 2	2010	
Part	**Solution**	**Marks**	**Comment**
a)			
	i. $\angle PAD = \angle DCP = 90°$ (Radius is perpendicular to tangent at point of contact) $\therefore PADC$ is cyclic (Opposite angles supplementary) Similar for $CDBQ$	2	Any fair proof
	ii. Let $\angle ADC = \theta$ $\therefore \angle BQC = \theta$ (Ext. angle of cyclic quadrilateral) $DA = DC$ (Equal Tangents) $\therefore \Delta ADC$ is isosceles $\therefore \angle DAC = \angle DCA = \left(90 - \frac{\theta}{2}\right)^\circ$ $BQ = CQ$ (Equal radii) $\therefore \Delta BQC$ is isosceles $\therefore \angle BCQ = \angle CBQ = \left(90 - \frac{\theta}{2}\right)^\circ$ $\therefore \Delta ADC \vert\vert\vert \Delta BQC$ (AAA)	3	
	iii. From above $\angle APC = 180 - \theta$ (opposite $\angle$ of $PADC$) $\angle PDC = \left(90 - \frac{\theta}{2}\right)^\circ$ (PD bisects $\angle APC$) $= \angle BCQ$ (from (ii)) $\therefore PD \parallel CB$ (corresponding $\angle$ equal)	2	

Question 8	Trial HSC Examination - Mathematics Extension 2	2010	
Part	**Solution**	**Marks**	**Comment**
b)	i. $P \sin \frac{\theta}{2}$ $= (1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta)\sin\frac{\theta}{2}$ $= \sin\frac{\theta}{2} + 2\cos\theta\sin\frac{\theta}{2} + 2\cos 2\theta\sin\frac{\theta}{2} + 2\cos 3\theta\sin\frac{\theta}{2}$	1	
	$\therefore = \sin\frac{\theta}{2} + \sin\frac{3\theta}{2} - \sin\frac{\theta}{2} + \sin\frac{5\theta}{2} - \sin\frac{3\theta}{2} + \sin\frac{7\theta}{2} - \sin\frac{5\theta}{2}$	1	
	$= \sin\frac{7\theta}{2}$	1	
	ii. From (i) $P\sin\frac{\theta}{2} = \sin\frac{7\theta}{2}$ When $\theta = \frac{2\pi}{7}$ $P\sin\frac{\pi}{7} = \sin\pi$ $= 0$	1	
	As $\sin\frac{\pi}{7} \neq 0$ then $P = 0$ i.e. $1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta = 0$	1	
	iii. $P = 1 + 2\cos\theta + 2(2\cos^2\theta - 1) + 2(4\cos^3\theta - 3\cos\theta)$ $= 8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1$	1	
	$P = 8x^3 + 4x^2 - 4x - 1$ when $x = \cos\theta$ From (ii) $P = 0$ when $\theta = \frac{2\pi}{7}$	1	
	$\therefore x = \cos\frac{2\pi}{7}$ is a solution.	1	
		/15	