WESTERN REGION

2010
HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks **(120)**

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

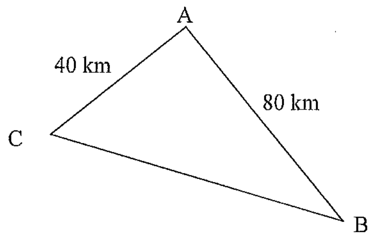
a) Evaluate $\int_1^5 (3x-7)\,dx$ **1**

b) For the arithmetic sequence

2, 7, 12, 17, ……….

i) Write a rule to describe the sequence in terms of $T_n = f(n)$ **1**
ii) Find the 23rd term **1**
iii) Find the sum of the first 47 terms **1**

c) Three towns form a triangle. Town A is 80 km from Town B and Town C is 40 km from Town A as shown below:



The bearing of Town B from Town A is $130°$. The bearing of Town C from Town A is $240°$.

i) Find the area enclosed by the 3 towns **2**
ii) Using the cosine rule, find the distance to the nearest kilometre between Town B and Town C **2**

d) Express the following as a single fraction

$$\frac{5}{2a+6}+\frac{a}{a^2-9}$$ **2**

e) Solve $|2x+5|<3$ **2**

End of Question 1

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) Differentiate with respects to x:

i) $\left(3x^2+7\right)^6$ **2**

ii) $4x^2e^{3x^3}$ **2**

iii) $\dfrac{\pi \cos x}{x^2}$ **2**

b) Find $\int \dfrac{dx}{3x+5}$ **2**

c) The line $2x+3y-13=0$ is translated 5 units up and parallel to the line. Find the equation of this transformation. **2**

d) Find the angle in degrees and minutes, that a line with gradient -2.5 makes with the positive x axis. **2**

End of Question 2

Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) Find $\lim_{x \to 3} \dfrac{x^2+8x+15}{x+3}$ **2**

b) Evaluate $\int_0^{\frac{\pi}{6}} (x^2+\sin 2x)\, dx$ **2**

c) Jake buys 5 tickets in a car raffle in which 350 tickets are sold. Three different tickets are to be drawn out. First prize is the car, second prize is a $1000 gift voucher and third prize is a Blue Ray player.

i) Draw a tree diagram to represent this information **2**

ii) Hence or otherwise find the probability that:

α) Jake wins all three prizes **1**

β) Jake wins at least 1 prize **1**

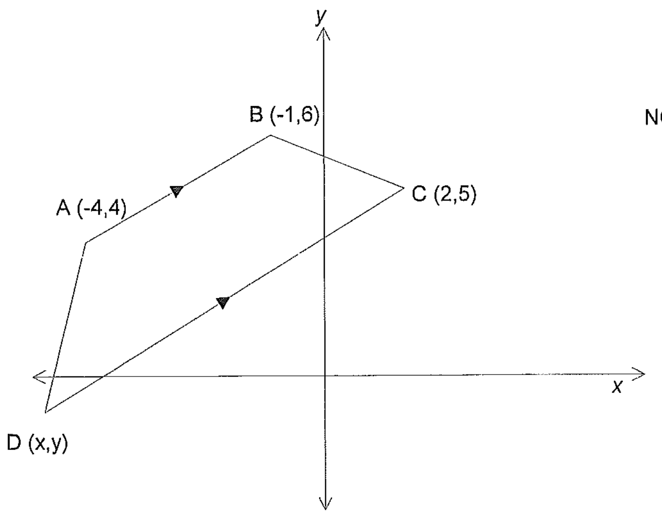
d) Find the equation of the parabola that passes through the points **2**

(0 , -8), (-2 , -10), (3 , 10)

e) For the curve $y=\sin \pi x$, state the period and amplitude **2**

End of Question 3

Question 4 (12 Marks) Use a Separate Sheet of paper **Marks**



a) Find the gradient of the line AB **1**

b) Show the distance AB is $\sqrt{13}$ units **2**

c) Show the equation of the line CD is $2x-3y+11=0$ **2**

d) Find the perpendicular distance from the line CD to the point A **2**

e) The distance between C & D is to be $\sqrt{117}$ units. Find the coordinates of point D, assuming $x<0$ **3**

f) Hence or otherwise find the area of the quadrilateral $ABCD$ **2**

End of Question 4

Question 5 (12 Marks) Use a Separate Sheet of paper **Marks**

a) Calculate the area of the region enclosed by the graph of $y=\cos 2x$ the x axis and the lines $x=0$ and $x=\dfrac{\pi}{4}$ **2**

b) The roots of the equation $2x^2-7x+12=0$ are α and β
Find:

i) $\alpha+\beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

iv) $\alpha^2+\beta^2$ **2**

c) For the curve $y=2x^3-12x^2-5x-3$ find:

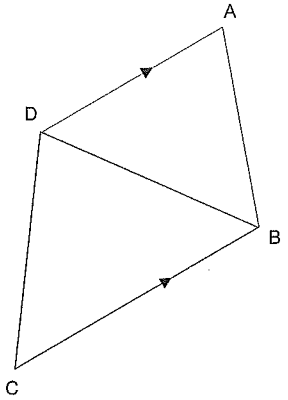
i) Any points of inflexion **2**

ii) The equation of the normal to this curve at the point of inflexion. **2**

End of Question 5

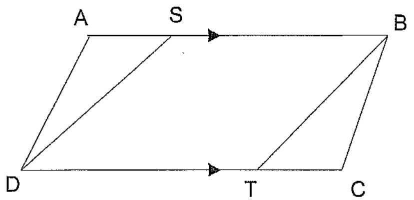
Question 6 (12 Marks) Use a Separate Sheet of paper **Marks**

a)



Given AD = AB, DB = DC, AD || BC and $\angle DAB = 100°$ **3**
Find $\angle BDC$ giving reasons for each step.

b)



ABCD is a parallelogram **3**
AS = CT
Prove DS = BT

c) The gradient function of a curve is $y' = \dfrac{4x}{x^2+1}$ and the curve passes through the point (0 , e). Find the equation of the curve. **2**

d) The number of bacteria N a person has after being infected with a virus after t hours is given by:

$$N = 10000e^{0.05t}$$

i) Find the number of bacteria after 10 hours **1**
ii) Find the time required for the number of bacteria to reach 100000 **2**
iii) At what rate is the bacteria increasing after 1 day **1**

End of Question 6

Question 7 (12 Marks) Use a Separate Sheet of paper **Marks**

a) i) Given $f(x) = \sqrt{4-x^2}$ complete the table of values to 3 decimal places. **1**

x	0	0.5	1	1.5	2
$f(x)$					

ii) Hence evaluate an approximation for $\int_0^2 \sqrt{4-x^2}\,dx$ using Simpson's rule with 5 function values. **2**

b) A pendulum on a grandfather clock is 50 cm long. When it swings the maximum length of the arc is makes is 40 cm.

i) In radians find the angle through which the pendulum swings. **1**

ii) Find the shortest distance between the maximum positions of the pendulum. **2**

c) i) Differentiate $y = 3^{4x-2}$ with respects to x **3**

ii) Hence find:

$$\int 3^{4x-2}\,dx$$ **1**

d) The area bounded by $y^2 = 3 - 2x - x^2$, $y \geq 0$ and between $x = -3$ and $x = 1$ is revolved about the x axis. Calculate the volume of the solid formed if this area is rotated about the x axis. **2**

End of Question 7

Question 8 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) Tiarn borrows $500 000 to buy a house. An interest rate of 9% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly instalements (R) over a 25 year period.

i) Show the amount owing after 3 months is: **2**

$$A_3 = 500000(1.0075)^3 - R\left[1 + 1.0075 + 1.0075^2\right]$$

ii) Assuming this pattern continues the monthly repayment can be calculated using: **2**

$$A_n = 500000(1.0075)^n - R\left[1 + 1.0075 + 1.0075^2 + \dots\dots\dots + 1.0075^{n-1}\right]$$

How much should Tiarn be paying each month?

iii) How much interest does Tiarn pay over the 25 years? **1**

iv) What is the equivalent simple interest rate of this loan? **1**

b) In Year 12 at Probability High School, every student must do Ancient History or Modern History. Students can also do both. In a group of 140 students surveyed, 73 do Ancient History and 82 students do Modern History. If one student is chosen at random from Year 12, find the probability that this student does:

i) both Ancient History and Modern History? **1**
ii) Ancient History only? **1**
iii) Modern History only? **1**

c) If $f(x) = 4 - 2^{-x}$ find:

i) $f(x^2)$ **1**
ii) $[f(x)]^2$ **1**
iii) Is $f(x)$ even, odd or neither **1**

End of Question 8

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) If $x = \dfrac{e^y - e^{-y}}{2}$ use the substitution $m = e^y$ to solve the equation for y in terms of x. **4**

b) The acceleration a ms^{-2} of a moving particle is given after t seconds by $a = -2$. Initially the particle is located at $x = -3$ and its velocity is 4 ms^{-1}

i) Find the velocity(v) and displacement (x) as functions of time (t) **2**

ii) Determine when the particle is at rest. **2**

iii) When will the particle first be at the origin? **2**

iv) Sketch displacement (x) as a function of time (t) **2**

End of Question 9

Question 10 (12 Marks) Use a Separate Sheet of paper **Marks**

a) A swimming pool is to be emptied for maintenance. The quantity of water Q litres, remaining in the pool at anytime, t minutes, after it starts to empty is given by:

$$Q(t) = 2000(25 - t)^2, \quad t \geq 0$$

i) At what rate is the pool being emptied at any time (t) **1**

ii) How long will it take to half empty the pool to the nearest minute? **2**

iii) At what time is the water flowing out at 20 kL / minute. **2**

iv) What is the average water flow in the first 10 minutes in litres? **2**

b) Adam is on a paddle board in the ocean 3 kilometres from the nearest point O on a straight beach. He needs to meet his friend Josh who is 6 kilometres along the beach from O. Adam is able to paddle at a rate of 4km/h and walk at a rate of 5km/h.

i) Draw a diagram to represent this information. **1**

ii) Show the total time T(x) hours, for Adam to reach Josh is given by: **2**

$$T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$$

iii) Find the minimum time for Adam to reach Josh on the beach. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x + \sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x + \sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

WESTERN REGION

2010
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

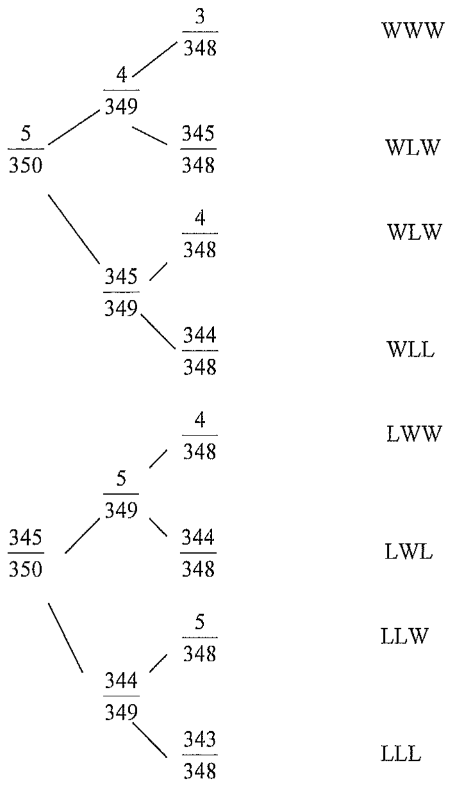
Question 1	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\int_1^5 (3x-7)dx$	1	
	$=\left[\frac{3x^2}{2}-7x\right]_1^5$		
	$=(\frac{75}{2}-35)(\frac{3}{2}-7)$		
	$=8$		
b)	i) $2,7,12,17,....$	1	
	$T_n = 5n-3$		
	ii) $T_{23} = 5\times 23-3$		
	$=112$	1	
	iii) $S_n = \frac{n}{2}[2a+(n-1)d]$		
	$=\frac{47}{2}[2\times 2+(47-1)5]$		
	$=5499$	1	
c)	i) $A=\frac{1}{2}ab\sin C$	2	
	$A=\frac{1}{2}\times 40\times 80\times \sin 110°$		1
	$=1503.5km^2$		1
	ii) $a^2 = b^2+c^2-2bc\cos A$	2	
	$a^2 = 40^2+80^2-2\times 40\times 80\times \cos 110°$		1
	$a^2 = 10188.93$		
	$a = 100.9$		
	$a = 101km$		1

Question 1	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
d)	$\frac{5}{2a+6}+\frac{a}{a^2-9}$	2	
	$=\frac{5}{2(a+3)}+\frac{a}{(a+3)(a-3)}$		1
	$=\frac{5(a-3)+2a}{2(a+3)(a-3)}$		
	$=\frac{5a-15+2a}{2(a+3)(a-3)}$		
	$=\frac{7a-15}{2(a^2-9)}$		1
e)	$\lvert 2x+5\rvert<3$		Only 1 if both cases not considered
	$2x+5<3$ or $2x+5>-3$	2	
	$2x<-2$ $\quad 2x>-8$		
	$x<-1$ $\quad x>-4$		
	check		
	$\therefore -4<x<-1$		
		/12	

Question 2	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i) $\frac{d}{dx}(3x^2+7)^6$	2	
	$=6\times 6x(3x^2+7)^5$		1
	$=36x(3x^2+7)^5$		1
	ii) $4x^2e^{3x^3}$	2	
	$u=4x^2 \quad v=e^{3x^3}$		
	$u'=8x \quad v'=9x^2e^{3x^3}$		
	$\frac{d}{dx}=8xe^{3x^3}+4x^2\times 9x^2e^{3x^3}$		1
	$=8xe^{3x^3}+36x^4e^{3x^3}$		1
	$=4xe^{3x^3}\left[2+9x^3\right]$		
	iii) $\frac{\pi\cos x}{x^2}$	2	
	$u=\pi\cos x \quad v=x^2$		
	$u'=-\pi\sin x \quad v'=2x$		
	$\frac{d}{dx}=\frac{-\pi x^2\sin x-2\pi x\cos x}{(x^2)^2}$		1
	$=\frac{\pi x(-x\sin x-2\cos x)}{x^4}$		1
	$=\frac{\pi(-x\sin x-2\cos x)}{x^3}$		
b)	$\int\frac{dx}{3x+5}$	2	
	$=\frac{1}{3}\int\frac{3}{3x+5}dx$		1
	$=\frac{1}{3}\ln(3x+5)+C$		1

Question 2	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	$2x+3y-13=0$ $y=\dfrac{-2x+13}{3}$ $m_1=-\dfrac{2}{3}$ $parallel\ m_1=m_2$ $b_1=\dfrac{13}{3}$ *transformation up 5 units* $b_2=\dfrac{13}{3}+5$ $=\dfrac{28}{3}$ $\therefore y=-\dfrac{2}{3}x+\dfrac{28}{3}$ $3y+2x-28=0$	2	1 1
d)	$m=\tan\theta$ $-2.5=\tan\theta$ $\theta=-68°12'$ $\therefore\theta=180°-68°12'$ $=111°48'$	2	1 1
		/12	

Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\lim_{x\to 3}\dfrac{x^2+8x+15}{x+3}$ $\dfrac{0}{0}$ DNE $\therefore$ factorise $\lim_{x\to 3}\dfrac{(x+5)(x\not+3)}{x\not+3}$ $=3+5$ $=8$	2	1 1
b)	$\int_0^{\frac{\pi}{6}} x^2+\sin 2x\,dx$ $=\left[\dfrac{x^3}{3}-\dfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$ $=\left[\dfrac{(\frac{\pi}{6})^3}{3}-\dfrac{1}{2}\cos(\dfrac{2\pi}{6})\right]-\left[-\dfrac{1}{2}\cos 0\right]$ $=\dfrac{\pi^3}{648}-\dfrac{1}{4}+\dfrac{1}{2}$ $=0.298$	2	1 1

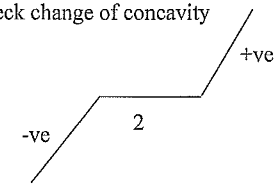
Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	i) 	2	1 for correct diagram 1 for correct fractions
	ii) α) $\frac{5}{350}\times\frac{4}{349}\times\frac{3}{348}$ $=\frac{1}{708470}$	1	
	β) $1-P(L)$ $=1-\left[\frac{345}{350}\times\frac{344}{349}\times\frac{343}{348}\right]$ $=0.042$	1	

Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
d)	$y=ax^2+bx+c$ $pt(0,-8)$ $8=0+0+c$ $\therefore c=-8$ $pt(3,10)$ $-10=4a-2b+c$ $-10=4a-2b-8$ $-2=4a-2b$ $-1=2a-b$.............(1) $pt(3,10)$ $10=9a+3b+c$ $10=9a+3b-8$ $18=9a+3b$ $6=3a+b$..............(2) solve simultaneously $(1)+(2)$ $-1=2a-b$.............(1) $6=3a+b$..............(2) $5a=5$ $a=1$ $6=3+b$ $b=3$ $\therefore y=x^2+3x-8$	2	1 1
e)	Amplitude = 1 Period = $\frac{2\pi}{\pi}=2$	2	1 1
		/12	

Question 4	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$m_{AB}=\dfrac{6-4}{-1--4}$ $=\dfrac{2}{3}$	1	1
b)	$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$ $=\sqrt{(6-4)^2+(-1--4)^2}$ $=\sqrt{2^2+3^2}$ $=\sqrt{13}units$	2	1 1
c)	$y-y_1=m(x-x_1)$ $y-5=\dfrac{2}{3}(x-2)$ $3y-15=2x+4$ $2x-3y+11=0$	2	1 1
d)	$d_{\perp}=\dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $=\dfrac{\lvert 2\times -4+-3\times 4+11\rvert}{\sqrt{2^2+(-3)^2}}$ $=\dfrac{\lvert -8-12+11\rvert}{\sqrt{13}}$ $=\dfrac{9}{\sqrt{13}}$ $=\dfrac{9\sqrt{13}}{13}units$	2	1 1

Question 4	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
e)	$d=\sqrt{(y-y_1)^2+(x-x_1)^2}$ $\sqrt{117}=\sqrt{(y-5)^2+(x-2)^2}$ $y^2-10y+25+x^2-4x+4=117$ $y^2-10y+x^2-4x=88$................(1) $2x-3y+11=0$ $y=\dfrac{2x+11}{3}$..............................(2) *sub* (2) into (1) $\left[\left(\dfrac{2x+11}{3}\right)^2-10\left(\dfrac{2x+11}{3}\right)+x^2-4x\right]=88$ $\dfrac{4x^2+44x+121}{9}-\left(\dfrac{20x+110}{3}\right)+x^2-4x-88=0$ $4x^2+44x+121-60x-330+9x^2-36x-792=0$ $13x^2-52x-1001=0$ $x^2-4x-77=0$ $(x-11)(x+7)=0$ $\therefore x=11$ or $x=-7$ $x<0$ $\therefore x=-7$ *when* $x=-7$ $y=\dfrac{2\times -7+11}{3}$ $\therefore D(-7-1)$	3	1 1 1
f)	$A=\dfrac{1}{2}h(a+b)$ $A=\dfrac{1}{2}\times\dfrac{9\sqrt{13}}{13}\times(\sqrt{13}+\sqrt{117})$ $=18units^2$	2	1 1
		/12	

Question 5	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\int_0^{\frac{\pi}{4}} \cos 2x$ $=\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{2}\sin\frac{\pi}{2}-\frac{1}{2}\sin 0$ $=\frac{1}{2}\times 1$ $=\frac{1}{2}\,unit^2$	2	1 1
b)	$2x^2-7x+12=0$ $\alpha+\beta=\frac{-b}{a}$ $\alpha\beta=\frac{c}{a}$ i) $\alpha+\beta=\frac{-b}{a}=\frac{--7}{2}$ $=\frac{7}{2}$ ii) $\alpha\beta=\frac{c}{a}=\frac{12}{2}$ $=6$ iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $=\frac{\left(\frac{7}{2}\right)}{6}$ $=\frac{7}{12}$	1 1 2	 1 1
	iv) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{7}{2}\right)^2-2\times 6$ $=\frac{1}{4}$	2	1 1

Question 5	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	i) $y=2x^3-12x^2-5x-3$ $y'=6x^2-24x-5$ $y''=12x-24$ for inflexions $y''=0$ $0=12x-24$ $0=12(x-2)$ $\therefore$ possible inflexion at $x=2$ check change of concavity  when $x=2$ $y=2\times(2^3)-12\times(2^2)-5\times 2-3$ $y=-45$ $\therefore$ pt of inflexion at $(2,-45)$	2	1 1
	ii) m when $x=2$ $y'=6x^2-24x-5$ $=6\times(2)^2-24\times 2-5$ $=24-48-5$ $=-29$ $\therefore m_1=-29$ for normal $\perp$ $m_1\times m_2=-1$ $\therefore m_2=\frac{1}{29}$ $at(2,-45)$ $y--45=\frac{1}{29}(x-2)$ $29y+1305=x-2$ $x-29y-1307=0$	2	 1 1
		/12	

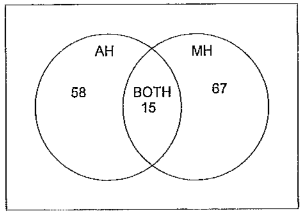
Question 6	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	Let $\angle ADB = x°$ ΔADB is isosceles $(AB = AD)$ $\angle ADB = \angle ABD$(*base angles of* $\Delta ADB =$) *then* $2x + 100 = 180°$(*angle sum* Δ) *so* $x = 40°$ $\therefore \angle ADB = 40°$ *then* $\angle DBC = 40°$(*alt* $\angle$'s = $AD \parallel BC$) $\because \Delta DBC$ is isosceles (DB=DC) $\therefore \angle DBC = 40°$(*base* $\angle$'s ΔDBC) $\therefore \angle BDC = 180° - 40° - 40°$(*angle* sum ΔDBC) $=100°$ $\therefore \angle BDC = 100°$	3	1 1 1
b)	i) Prove DS = BT $AS = TC$(*given*) $AD = BC$(*opp sides* parallelogram =) $\angle DAS = \angle BCT$(*opp* $\angle$'s parallelogram =) $\therefore \Delta DAS \equiv \Delta BCT(SAS)$ ii) $\because \Delta DAS \equiv \Delta BCT$ $DS = BT$(*corresponding sides* in $\equiv \Delta =$)	3	1 1 1
c)	$y' = \dfrac{4x}{x^2+1}$ $y = 2\int \dfrac{2x}{x^2+1}dx$ $= 2\ln\left[x^2+1\right] + c$ *when* $x = 0 \;\; y = e$ $e = 2\ln\left[0+1\right] + c$ $\therefore c = e$ $\therefore y = 2\ln\left[x^2+1\right] + e$	2	1 1

Question 6	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
d)	i) $N = 10000e^{0.05t}$ $N = 10000e^{0.05\times 10}$ $N = 16487$ ii) $100000 = 10000e^{0.05t}$ $10 = e^{0.05t}$ $\ln 10 = \ln(e^{0.05t})$ $\ln 10 = 0.05t(\ln e)$ $t = \dfrac{\ln 10}{0.05}$ $= 46$ *hours* iii) $N' = 500e^{0.05t}$ *when* $t = 24$ *hours* $N' = 500e^{0.05\times 24}$ $= 1660$ bacteria/hour	1 2 1	1 1 1 1
		/12	

Question 7	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\begin{array}{|c|c|c|c|c|c|}\hline x & 0 & 0.5 & 1 & 1.5 & 2 \\ \hline f(x) & 2 & 1.936 & 1.732 & 1.323 & 0 \\ \hline\end{array}$ $\int_a^b f(x)dx \approx \frac{h}{3}[(y_0+y_n)+(4\times odds)+(2\times evens)]$ $\approx \frac{0.5}{3}[(2+0)+4(1.936+1.323)+2(1.732)]$ ≈ 3.083 or $3\frac{1}{12}$	1 2	1 1 1
b)	i) $l = r\theta$ $40 = 50\theta$ $\theta = \frac{40}{50}$ $\theta = 0.8 radians$ ii) $a^2 = b^2 + c^2 - 2bc\cos A$ $a^2 = 50^2 + 50^2 - 2\times 50\times 50\times \cos\frac{4}{5}$ $a = 38.94cm$	1 2	1 1 1
c)	i) $y = 3^{4x-2}$ $4x - 2 = \log_3 y$ $4x - 2 = \frac{\ln y}{\ln 3}$ $x = \frac{\frac{\ln y}{\ln 3}+2}{4}$ $x = \frac{\ln y}{4\ln 3} + \frac{1}{2}$ $\frac{dx}{dy} = \frac{1}{4\ln 3}\times\frac{1}{y}$ $= \frac{1}{4\ln 3y}$ $\frac{dy}{dx} = 4\ln 3y$ $= 4\ln 3(3^{4x-2})$	3	1 1 1

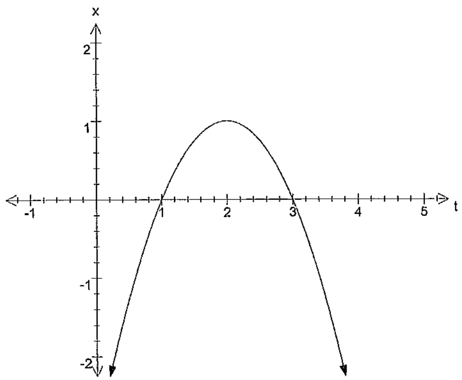
Question 7	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	ii) $\int 3^{4x-2}dx$ $= \frac{1}{4\ln 3}\int 4\ln 3(3^{4x-2})$ $= \frac{1}{4\ln 3}\times(3^{4x-2}) + c$ $= \frac{(3^{4x-2})}{4\ln 3} + c$	1	1
d)	$y^2 = 3 - 2x - x^2$ $V = \pi\int_a^b y^2 dx$ $V = \pi\int_{-3}^{1} 3 - 2x - x^2 dx$ $= \pi\left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}$ $= \pi\left[\left(3 - 1 - \frac{1}{3}\right) - (-9 - 9 + 9)\right]$ $= \frac{32\pi}{3}$	2	1 1
		/12	

Question 8	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i) $r = 9 \div 100 \div 12$ $r = 0.0075$ $A = P(1+r)^n$ $A_1 = 500000(1.0075)^1 - R$ $A_2 = A_1(1.0075)^1 - R$ $A_2 = 500000(1.0075)^2 - R(1.0075) - R$ $A_3 = A_2(1.0075)^1 - R$ $A_3 = 500000\left[(1.0075)^2 - R(1.0075) - R\right](1.0075) - R$ $= 500000(1.0075)^3 - R(1.0075)^2 - R(1.0075) - R$ $= 500000(1.0075)^3 - R\left[1 + 1.0075 + 1.0075^2\right]$ *as required*	2	1 1
	ii) $A_n = 0$ as all money is repaid $\therefore 0 = 500000(1.0075)^{300} -$ $R\left[1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{n-1}\right]$ $R = \dfrac{500000(1.0075)^{300}}{\left[1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{n-1}\right]}$ *geometric series* with $a = 1, r = 1.0075, n = 300$ $S_n = \dfrac{a\left(r^n - 1\right)}{r - 1}$ $S_{300} = \dfrac{1\left(1.0075^{300} - 1\right)}{1.0075 - 1}$ $S_{300} \approx 1121.121937$ $R = \dfrac{500000(1.0075)^{300}}{S_{300}}$ $R = \$4195.98$	2	1 1
	iii) Total repaid $= \$4195.98 \times 300$ $= \$1258794.00$ *Interest* $= \$1258794 - 500000$ $= \$758794.00$	1	1

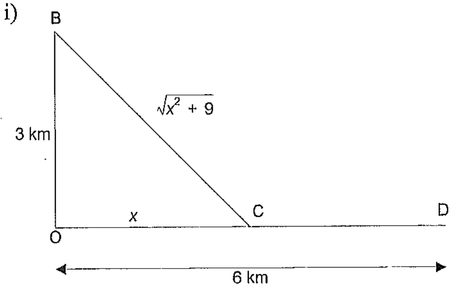
Question 8	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	iv) $SI = \Pr n$ $758794 = 500000 \times r \times 25$ $r = 6.07\%$	1	1
b)	 i) $\dfrac{15}{140} = \dfrac{3}{28}$ ii) $\dfrac{58}{140} = \dfrac{29}{70}$ iii) $\dfrac{67}{140}$	1 1 1	
c)	i) $f(x) = 4 - 2^{-x}$ $f(x^2) = 4 - 2^{-x^2}$ ii) $\left[f(x)\right]^2 = \left[4 - 2^{-x}\right] \times \left[4 - 2^{-x}\right]$ $= 16 - 2^3 \times 2^{-x} + (2^{-x})^2$ $= 16 - 2^{3-x} + 2^{-2x}$ iii) $f(-x) = 4 - 2^{-(-x)}$ $= 4 - 2^x$ $\neq f(x) or - f(x)$ $\therefore$ the function is neither odd nor even	1 1 1	
		/12	

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$x=\dfrac{e^y-e^{-y}}{2}$	4	
	$Let\ m=e^y$		
	$x=\dfrac{m-\frac{1}{m}}{2}$		
	$x=\dfrac{m^2-1}{m}\times\dfrac{1}{2}$		1
	$x=\dfrac{m^2-1}{2m}$		
	$2mx=m^2-1$		1
	$m^2-2xm-1=0$		
	using quadratic formula		
	$m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$		
	$m=\dfrac{2x\pm\sqrt{(4x^2-4\times1\times-1}}{2}$		
	$=\dfrac{2x\pm\sqrt{4x^2+4}}{2}$		
	$=\dfrac{2x\pm\sqrt{4(x^2+1)}}{2}$		
	$=\dfrac{2\left[x\pm\sqrt{(x^2+1)}\right]}{2}$		1
	$=x\pm\sqrt{(x^2+1)}$		
	$but\ m=e^y$		
	$e^y=x\pm\sqrt{(x^2+1)}$		
	$e^y>0$		
	$\therefore e^y=x+\sqrt{(x^2+1)}$		
	$\ln e^y=\ln(x+\sqrt{(x^2+1)})$		1
	$y=\ln(x+\sqrt{(x^2+1)})$		

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
b)	i)	2	
	$a=-2\quad x=-3\quad v=4ms^{-1}$		
	$a=-2$		
	$v=\int -2dt$		
	$=-2t+c$		
	$when\ t=0\ \ v=4$		
	$4=-2\times0+c$		
	$c=4$		1
	$\therefore v=-2t+4$		
	$x=\int -2t+4dt$		
	$=-t^2+4t+c$		
	$when\ t=0\ \ x=-3$		
	$-3=0+0=c$		
	$c=-3$		
	$\therefore x=-t^2+4t-3$		1
	ii)		
	Particle at rest when v=0	2	1
	$v=-2t+4$		
	$0=-2t+4$		
	$2t=4$		
	$t=2\sec onds$		1
	$\therefore\ particle$ at rest when $t=2\sec onds$		
	iii)		
	Particle at the origin when $x=0$	2	
	$x=-t^2+4t-3$		
	$0=-t^2+4t-3$		
	$0=-(t^2-4t+3)$		1
	$0=-(t-3)(t-1)$		
	$\therefore t=1\ or\ 3\sec onds$		1
	particle first at the origin when $t=1\sec ond$		

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
b)	iv) 	2	1 for correct shape 1 for correct intercepts
		/12	

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i)	1	
	$Q(t)=2000(25-t)^2, t\geq 0$		
	$Q'(t)=-4000(25-t)$		
	$\therefore$ it is emptying at a rate of		
	$4000(25-t)$ litres/minute		1
	ii)		
	Pool full at $t=0$	2	
	$Q(t)=2000(25-0)^2$		
	$=1250000 litres$		
	$\therefore$ half full = 625000 litres		1
	$625000=2000(25-t)^2$		
	$312.5=625-50t+t^2$		
	$t^2-50t+312.5=0$		
	$2t^2-100t+625=0$		
	$t=\frac{--100\pm\sqrt{100^2-4\times2\times625}}{2\times2}$		
	$t=\frac{100\pm\sqrt{5000}}{4}$		
	$t=\frac{100\pm50\sqrt{2}}{4}$		
	$t=\frac{2(50\pm25\sqrt{2})}{4}$		
	$t=\frac{50\pm25\sqrt{2}}{2}$		
	$t=7.322$ or 42.68		
	$\therefore t=7$ minutes		1
	$\therefore$ it will take $\approx$ 7 minutes to half empty the pool		
	iii)		
	$20kL=20000L/\min$	2	
	$20000=-4000(25-t)$		1
	$20000=-100000+4000t$		
	$4000t=120000$		
	$t=30\min s$		
	$\therefore$ the flow rate will be 20kL after 30 minutes		1

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	iv) when $t = 10$ $Q(t) = 2000(25-10)^2$ $= 2000 \times 225$ $= 450000L$ left in the pool when $t = 0$ $Q(t) = 1250000L$ $Average = \frac{(1250000-450000)}{10}$ $= 80000L/\min$	2	1 1
b)	i)  	1	1
	ii) Using Pythagoras and $S = \frac{D}{T}$ $\therefore$ he paddles a distance of $\sqrt{x^2+9}$ at 4km/h $\therefore Paddles - \frac{\sqrt{x^2+9}}{4} hours$ $\therefore$ he walks a distance of $6-x$ at 5km/h $\therefore Walks - \frac{6-x}{5} hours$ The total time $T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$	2	1 1

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
b)	iii) $T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$ $T'(x) = \frac{x}{4\sqrt{x^2+9}} - \frac{1}{5}$ $= \frac{5x-4\sqrt{x^2+9}}{20\sqrt{x^2+9}}$ Min when $T'(x) = 0$ $0 = \frac{5x-4\sqrt{x^2+9}}{20\sqrt{x^2+9}}$ $0 = 5x - 4\sqrt{x^2+9}$ $5x = 4\sqrt{x^2+9}$ (square both sides) $25x^2 = 16x^2 + 144$ $9x^2 = 144$ $x^2 = 16$ $x = \pm 4 \ (x \neq -4)$ $\therefore x = 4$ *check* min*imum* *when* $x < 4, T'(x) < 0$ *when* $x > 4, T'(x) > 0$ $\therefore$ min*imum* at $x = 4$ $\therefore$ Adam paddles to C - 4 kilometres from O $T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$ $T(4) = \frac{\sqrt{4^2+9}}{4} + \frac{6-4}{5}$ $= 1.65 hours$ $= 1 hour \ \& \ 39 \min s$	2	1 1
		/12	