Newington College

2009

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is proved at the back of this paper
- All necessary working should be shown in every question

Total marks: 120

- Attempt Questions 1–10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Total Marks - 120
Attempt Questions 1 – 10
All questions are equal value.
Answer each question in a SEPARATE writing booklet.

Question 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of $\sqrt{\dfrac{50}{\pi}}$ correct to three significant figures. 2

(b) Factorise $3x^2+5x-12$. 2

(c) Simplify $\dfrac{3}{5}-\dfrac{x-1}{4}$. 2

(d) Find the primitive of $4-\dfrac{1}{x^2}$. 2

(e) Find the values of x for which $|x-4|>3$. 2

(f) If $f(x)=\begin{cases} 7-3x & \text{for } x\le 1 \\ x^3+1 & \text{for } x>1 \end{cases}$

evaluate $f(0)+f(2)$. 2

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Differentiate the following:

(i) $(2x^3-1)^5$ 2

(ii) $e^{5x}\tan x$ 2

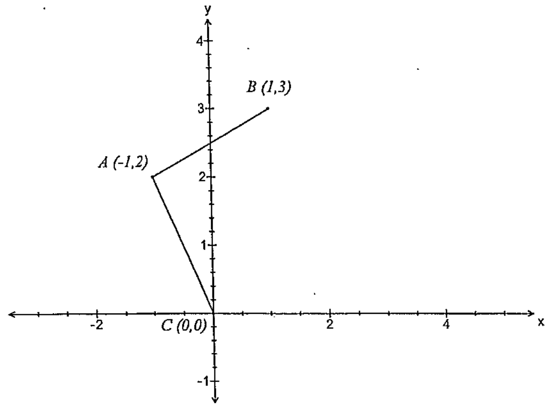
(iii) $\dfrac{\ln x}{x^2}$ 2

(b) Find the equation of the tangent to the curve $y=2x(1-x^2)$ at the point $(1,0)$. 3

(c) Evaluate $\displaystyle\int_2^3 \frac{3x}{x^2-1}\,dx$, give answer in exact form. 3

Question 3 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram above $A(-1,2)$, $B(2,3)$ and $C(0,0)$ are as shown.

(i) Prove that $AB \perp AC$. **2**

(ii) Find the equation of CD such that $CD \parallel AB$. **2**

(iii) If BD parallel to the x-axis find the co-ordinates of D. **2**

(iv) Describe the special quadrilateral $ABDC$ geometrically. **1**

(v) Find the area of $ABDC$. **3**

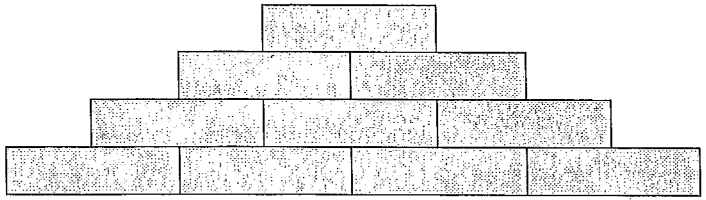
(b) The table shows values of a function $f(t)$ for the five values of t.

t	0	3	6	9	12
$f(t)$	0	2	5	8	4

Use the Trapezoidal Rule with these 5 values to estimate $\int_0^{12} f(t)\ dx$. **3**

Question 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)



Top four layers.

A bricklayer builds a triangular wall. The top of the wall is shown above. If the bricklayer uses 171 bricks, how many layers did he build? **3**

(b) Sally has a bag of sweets which are all identical in shape. The bag contains 6 orange drops and 4 lemon drops. She selects one sweet at random, eats it, and then takes another at random. Determine the probability that:

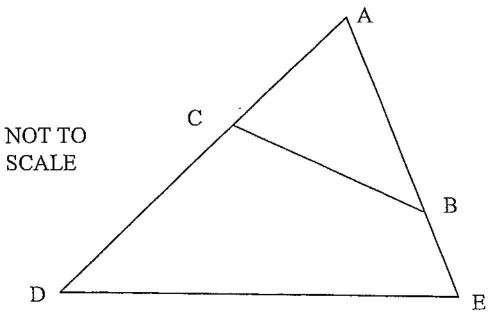
(i) both sweets were lemon drops **2**

(ii) at least one of the drops was an orange drop. **2**

(c) Evaluate $\sum_{n=1}^{10} 2^n$. **3**

(d) In the diagram below, ΔABC is similar to ΔADE. $\angle ADE = \angle ABC$, $AC = 8\,cm$, $AB = 12\,cm$ and $BE = 4\,cm$ Find the length of CD. **2**

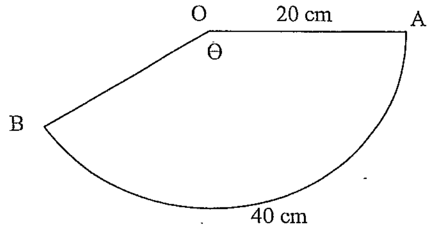
NOT TO SCALE



Question 5 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\log_3(5x+1)=2$. 2

(b)

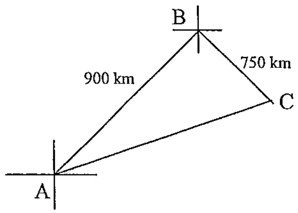


In the diagram AB is an arc of a circle with centre O and radius 20 cm. The arc has a length of 40 cm. Find:

(i) the exact size of $\angle AOB$ in radians, 1

(ii) the exact area of the sector AOB. 1

(c) A plane travels 900 km from A to B on a bearing of 50^0T. The plane then travels 750 km on a bearing of 135^0T to a point C.



Copy the diagram into your answer booklet.

(i) Show that ∠ABC is 95°, giving reasons. 2

(ii) Find the distance of CA, correct to one decimal place. 2

(iii) Find ∠ACB. 2

(iv) Find the bearing of A from C, to the nearest minute. 2

Question 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)

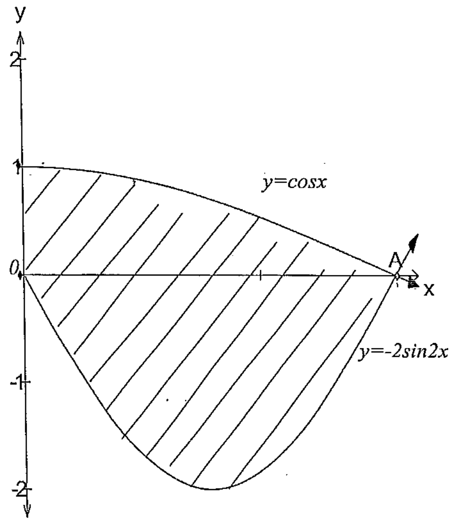


DIAGRAM NOT DRAWN TO SCALE

The diagram shows the graph of $y=-2\sin 2x$ and $y=\cos x$, for $x\geq 0$. The graphs intersect at point A.

(i) Show that point A has co-ordinates $\left(\frac{\pi}{2}, 0\right)$. 2

(ii) Find the shaded area enclosed by the two graphs for $0\leq x\leq\frac{\pi}{2}$. 3

(b) Given the equation $2x^2+7x-3=0$ has roots α and β, evaluate the following:

(i) $\alpha+\beta$ 1

(ii) $\alpha\beta$ 1

(iii) $2\alpha^2+2\beta^2$. 2

(c) Find the values of A, B and C, such that:

$$4x^2+5x-3\equiv A(x-1)^2+B(x-1)+C$$ 3

Question 7 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Given the parabolic equation:

$$8y = x^2 - 2x - 15$$

Find:

(i) the coordinates of the vertex **2**

(ii) the coordinates of the focus **1**

(iii) the equation of the directrix. **1**

(b) For the curve $f(x) = 2x^3 + 3x^2 - 12x - 4$

(i) Find the coordinates of the stationary points and determine their nature. **3**

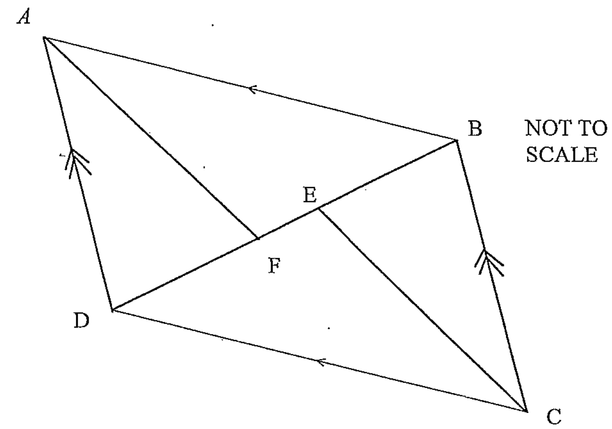
(ii) Find the coordinates of the point of inflexion. **2**

(iii) Hence sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and where the curve meets the y-axis. **2**

(iv) For what values of x is the graph of $f(x)$ concave up? **1**

Question 8 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Copy or trace the diagram into your writing booklet.



$ABCD$ is a parallelogram. FA bisects $\angle BAD$ and EC bisects $\angle BCD$.

(i) Prove that ΔADF and ΔCBE. **3**

(ii) Hence find the length of EF if $BD = 40\,cm$ and $DF = 16\,cm$. **1**

(b) The position x *cm* of a particle P moving along the x-axis after t seconds is given by $x = 25t - 10\ln t$ cm, where $t \geq 1$.

(i) Find expressions for the particle's velocity and acceleration. **2**

(ii) Find the velocity and acceleration when $t = e$ minutes. **2**

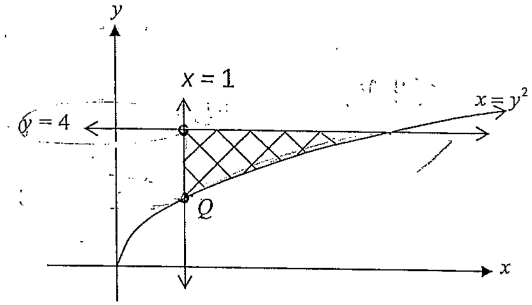
(iii) Discuss the velocity as $t \to \infty$ **2**

(iv) Sketch the graph of the velocity function? **2**

Question 9 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $4\cos^2\theta - 3 = 0$, if $0 \le \theta \le 2\pi$. 3

(b)



(i) Find the coordinates of Q, the intersection of $x = y^2$ and $x = 1$. 1

(ii) The shaded region in the diagram is the area bounded by the lines $y = 4$, $x = 1$ and the parabola $x = y^2$. This region is rotated about the y-axis. Find the volume of the solid formed. 3

(c) The amount Q grams of a carbon isotope in a dead tree trunk is given by $Q = Q_0e^{-kt}$ where Q_0 and k are positive constants and time t is measured in years from the death of the tree.

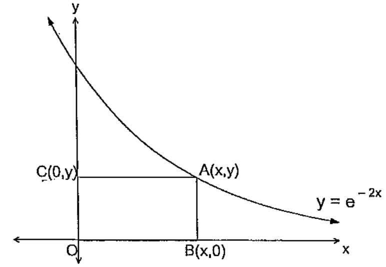
(i) Show that Q satisfies the equation $\frac{dQ}{dt} = -kQ$ 1

(ii) Show that if the half-life of the isotope is 4500 years, then $k = \frac{1}{4500}\ln 2$ 2

(iii) For a particular dead tree trunk, the amount of isotope is only 15% of the original amount in the living tree. How long ago did the tree die? Give your answer to the nearest 1000 years. 2

Question 10 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) A rectangle is placed under the curve $y = e^{-2x}$ as shown. If $A(x)$ gives the area of $OBAC$,



(i) find an expression for $A(x)$ in terms of x. 2

(ii) determine the coordinates of $A(x)$ such that the rectangle has a maximum area. 3

(b) A couple wish to purchase a home. They need to calculate the maximum amount they can borrow given that the current interest rate is 9% pa, compounded monthly and the term of the loan is 30 years. They can afford to repay \$2700 per month, paid at the end of every month. Let A_n be the amount owing at the end of n months and \$$P$ be the amount borrowed.

(i) Show that after three months the amount of money owing is
$$A_3 = P(1.0075)^3 - 2700(1 + 1.0075 + 1.0075^2).$$ 2

(ii) Hence show that if the loan is to be paid off after n months then
$$P = \frac{360000(1.0075^n - 1)}{(1.0075)^n}$$ 2

(iii) Calculate, correct to the nearest cent, the amount that they can borrow if the loan is to be paid off in thirty years. 2

(iv) How much interest, in dollars, will they pay for loan? 1

End of Paper.

Q1

(a) $\sqrt{\frac{50}{\pi}} = 3.98942280\ldots$ ✓

$= \underline{3.99}$ [3 sig fig] ✓

2 for correct answer only

(b) $3x^2+5x-12$ ✓

$=(3x-4)(x+3)$ ✓

(c) $\frac{3}{5} - \frac{x-1}{4}$

$= \frac{12-5(x-1)}{20}$ ✓

$= \frac{12-5x+5}{20}$

$= \frac{17-5x}{20}$ ✓

(d) $\int 4 - \frac{1}{x^2}\,dx = \int 4 - x^{-2}\,dx$ ✓

$= 4x + x^{-1} + c$

$= 4x + \frac{1}{x} + c$ ✓

(e) $|x-4| > 3$

$\therefore x-4 > 3$ or $x-4 < -3$

$\therefore x > 7$ or $x < 1$

✓ ✓

(f) $f(0) = 7-3(0)$

$f(2) = 2^3+1$

$\therefore f(0)+f(2) = 7+9$ ✓

$= 16$ ✓

Q2

(12)

(a) (i) $y = (2x^3-1)^5$

$\frac{dy}{dx} = 5(2x^3-1)^4 \times 6x^2$

$= 30x^2(2x^3-1)^4$

✓ ✓

(ii) $y = e^{5x}\tan x$

$\frac{dy}{dx} = e^{5x}\cdot\sec^2 x + \tan x \cdot 5e^{5x}$ ✓✓ Product Rule.

$= e^{5x}(\sec^2 x + 5\tan x)$

(iii) $y = \frac{\ln x}{x^2}$ Quotient Rule

$\frac{dy}{dx} = \frac{x^2 \times \frac{1}{x} - 2x\ln x}{x^4}$ ✓

$\left.\begin{aligned} &= \frac{x - 2x\ln x}{x^4} \\ &= \frac{1-2\ln x}{x^3} \end{aligned}\right\}$ ✓

(b) $y = 2x(1-x^2)$

$\therefore y = 2x - 2x^3$

$\frac{dy}{dx} = 2 - 6x^2$

at $x=1$ $\frac{dy}{dx} = 2-6$

$= -4$ ✓

$\therefore$ Equation of tangent at (1,

$y - y_1 = m(x-x_1)$ ✓

$y - 0 = -4(x-1)$

$y = -4x + 4$ ✓

$\therefore 4x + y - 4 = 0$ is the equation of tangent. Note: Accepted form

Q2 (c) $\int_2^3 \frac{3x}{x^2-1}\,dx$

$= \frac{3}{2}\int_2^3 \frac{2x}{x^2-1}\,dx$ ✓

$= \frac{3}{2}\left[\ln(x^2-1)\right]_2^3$ ✓

$= \frac{3}{2}\left[\ln 8 - \ln 3\right]$ ← Accept either.

$= \frac{3}{2}\ln\frac{8}{3}$ ✓

Question 3

a i) $M_{AB} = \frac{3-2}{1--1}$ $= \frac{1}{2}$ $M_{AC} = \frac{2}{-1}$ $= -2$

Since $M_{AB} \times M_{AC} = -1$
$AB \perp AC$ (2)

ii) $y = mx$ as $b = 0$
$\therefore$ Equation of CD is $y = \frac{1}{2}x$ (1)

iii) at D $y = 3$
$\therefore 3 = \frac{1}{2} \times x$
$x = 6$ (2)
$\therefore D = (6, 3)$

iv) ABCD is a Trapezium
(Right $\angle$) (1)

v) $A = \frac{1}{2} \times 6 \times 3 - \frac{1}{2} \times \frac{1}{2} \times 1 + \frac{1}{2} \times 2.5 \times 1$
$= 10$ units^2

(2)

b) $\int_a^b f(x) \doteq \frac{h}{2}\left[f(a) + f(b)\right]$

$= \frac{3}{2}\left[f(0) + f(3) + f(3) + f(6) + f(6) + f(9) + f(9) + f(12)\right]$

$= \frac{3}{2}\left[0 + 2 + 2 + 5 + 5 + 8 + 8 + 4\right]$

$= 51$ (3)

Question 4

a) $S_n = 171$

$d = 1 \qquad a = 1$

$\frac{n}{2}\left[2a + (n-1)d\right] = 171$

$\frac{n}{2}\left[2(1) + (n-1)1\right] = 171$

$n(n+1) = 342$

$n^2 + n = 342$

$n^2 + n - 342 = 0$

$(n+19)(n-18) = 0$

$n = -19 \quad n = 18$

$\therefore n = 18$ as $n > 0$

$\therefore$ 18 layers built (3)

b) i) $P(LL) = \frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{15}$ (2)

ii) $P(\text{At least 1 Orange}) = 1 - P(LL)$

$= 1 - \frac{2}{15}$

$= \frac{13}{15}$ (2)

c) $\sum_{n=1}^{10} 2^n = 2^1 + 2^2 + 2^3 + 2^4 \dots 2^{10}$

$= 2046$ (3)

OR $= S_n$ GP

$= \frac{a(1-r^n)}{1-r}$

$= \frac{2(1-2^{10})}{1-2}$

$= 2046$

d) $\frac{AD}{12} = \frac{16}{8}$

$\therefore AD = 12 \times \frac{16}{8}$

$AD = 24$ (2)

$\therefore CD = 24 - 8$

$= 16\text{cm}$

OR

$\frac{x+8}{12} = \frac{16}{8}$

$x + 8 = 12 \times \frac{16}{8}$

$x + 8 = 24$

$x = 16$

$\therefore DC = 16\text{cm}$

Q 5 (a) $\log_3(5x+1) = 2$

$5x + 1 = 9$ ✓

$5x = 8$

$x = \frac{8}{5}$ ✓

(b) (i) Arc length $= r\theta$

$\therefore 20\theta = 40$

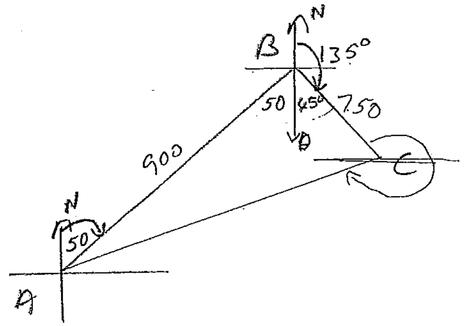
$\theta = 2$ radians ✓

(ii) Area of Sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(20)^2 \times 2$

$= 400\ \text{cm}^2$ ✓

(c)



✓

(i) From diagram $\angle ABD = 50°$ [Alternate angles]

$\angle DBC = 45°$ [Supplementary angles] ✓

$\therefore \angle ABC = 95°$ Note: Working can be shown on diagram.

(ii) $CA^2 = 750^2 + 900^2 - 2 \times 750 \times 900 \times \cos 95°$ ✓ (BC^2, AB^2)

$CA = 1220.7$ Km. [correct to one decimal place] ✓ [Note: DO … DEDUCT MA… FOR ROUNDIN…]

(iii) $\frac{\sin\angle ACB}{900} = \frac{\sin 95}{1220.7}$

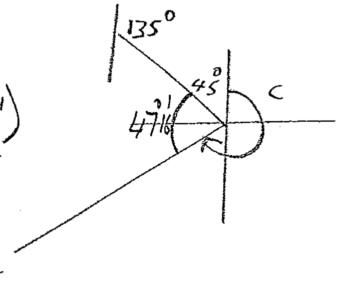
$\sin\angle ACB = \frac{900 \times \sin 95}{1220.7}$

$\angle ACB = 47°16'$ [Note: Accept 47° … 47°15']

5 c) (iv) Bearing of A from C

$= 360^\circ - (45^\circ + 47^\circ 16')$

$= 267^\circ 44'$



Q6 (a) (i) If $y = -2\sin 2x$, $x = \frac{\pi}{2}$

$\therefore y = -2\sin\pi$

$= 0$ $\therefore$ curves intersect at $A(\frac{\pi}{2}, 0)$

If $y = \cos x$, $x = \frac{\pi}{2}$

$y = \cos\frac{\pi}{2}$

$= 0$

(ii) Enclosed Area $= \int_0^{\frac{\pi}{2}} \left[\cos x - (-2\sin 2x)\right] dx$

$= \int_0^{\frac{\pi}{2}} \left[\cos x + 2\sin 2x\right] dx$

$= \left[\sin x - \cos 2x\right]_0^{\frac{\pi}{2}}$

$= 1 - (-1) - (0 - 1)$

$= 3\ u^2$

(b) $2x^2 + 7x - 3 = 0$

(i) $\alpha + \beta = -\frac{7}{2}$

(ii) $\alpha\beta = -\frac{3}{2}$

(iii) $2\alpha^2 + 2\beta^2$

$= 2(\alpha^2 + \beta^2)$

$= 2\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

$= 2\left[\frac{49}{4} + 3\right]$

$= 30.5$

Q6(a)

(i) $y = -2\sin$ [illegible]

when $x = \frac{\pi}{2}$

Q6(c) $4x^2 + 5x - 3 \equiv A(x-1)^2 + B(x-1) + C$

Now $A = 4$ [coefficient of x^2]

Let $x = 1$ $\therefore 6 = C$

Let $x = 0$

$\therefore -3 = A - B + C$

$-3 = 4 - B + 6$

$B = 13$

$\therefore A = 4,\ B = 13,\ C = 6.$

$4x^2 + 5x - 3 \equiv A(x-1)^2 + B(x-1) + C$

$= A(x^2 - 2x + 1) + Bx - B + C$

$= Ax^2 + (B - 2A)x + (A - B + C)$

$A = 4$

$B - 2A = 5$

$B - 2(4) = 3$

$B = 13$

$A - B + C = -3$

$4 - 13 + C = -3$

$-9 + C = -3$

$C = 6$

Q6 (c) $4x^2+5x-3 \equiv A(x-1)^2+B(x-1)+C$

let $x=1$ $\therefore 6 = C$

let x:

Q7

a) i) $8y = x^2-2x-15$

$x^2-2x+1 = 8y+16$ ✓

$(x-1)^2 = 8(y+2)$ ✓

Vertex $(1,-2)$ ✓

(ii) $4a = 8$

$a = 2$

focus $(1,0)$ ✓ ✓

iii) $\cancel{y=-4}$

$y = m(x-1)$

b) i) $f'(x) = 6x^2+6x-12$

Stationary pts $f'(x)=0$

$6x^2+6x-12=0$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x = -2$ or 1 ✓

$f''(x) = 12x+6$

$f''(-2) = -24+6<0$

$\therefore$ max at $(-2,16)$ ✓

$f''(1) = 12+6>0$

$\therefore$ min at $(1,-11)$ ✓

ii) $f''(x)=0$ ✓

$12x+6=0$

$x = (-\frac{1}{2}, 2\frac{1}{2})$ ✓

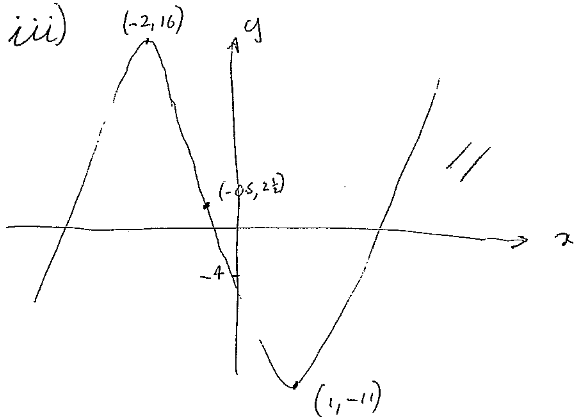
Check

x	-2	$-\frac{1}{2}$	1
y''	<0	0	>0

change of concavity

$\therefore$ point of inflection at $(-0.5, 2\frac{1}{2})$

iii)



✓✓

IV) $x > -0.5$ ✓

Q8

Q1(a)(i) Consider $\triangle ADF$ and $\triangle CBE$.

$AD = CB$ (opposite sides of parallelogram equal)
$A\hat{D}F = C\hat{B}E$ (alternate angles $AD \parallel BC$) ✓

Now $D\hat{A}B = B\hat{C}E$ (opposite angles of parallelogram equal)
and since AF bisects $A\hat{D}B$ and EC bisects $B\hat{C}D$.
$D\hat{A}F = B\hat{C}E$ ✓

$\therefore \triangle ADF \equiv \triangle CBE$ (AAS) ✓

(ii) $DF = EB = 16$ (corresponding sides in congruent triangles)
$\therefore EF = 40 - 32$
$= 8$ cm. ✓

(b) (i) $v = \dfrac{dx}{dt} = 25 - \dfrac{10}{t}$ ✓

$a = \dfrac{dv}{dt} = \dfrac{10}{t^2}$ ✓

(ii) since t in seconds find $t = 60e$ seconds.

$\therefore v = 25 - \dfrac{10}{60e}$

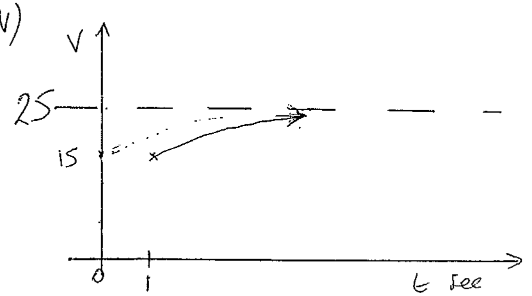
$= 25 - \dfrac{1}{6e}$ ✓ (24.94)

$a = \dfrac{10}{(60e)^2}$

$= \dfrac{1}{360e^2}$ ✓ (0.00038)

(iii) As $t \to \infty$ $v \to 25$ (as $\frac{10}{t} \to 0$) ✓✓

(iv)



✓✓

Q8

Problems with Q1.

(a)(i) On the whole this was done quite poorly.
- some proved similarity using equiangular.
- many tried to show $AF = EC$ by nonsense reasoning
- language used was incomplete (eg alternate angles on parallel lines).
- many used given for reasons when inappropriate.

(ii) the majority got this correct, regardless of (i).

(b) (i) Most got these, but a few differentiation errors (e.g with negative sign in second term)

(ii) few candidates realised they had to substitute $t = 60e$ secs.

(iii) Many just wrote $v \to 25$ with no explanation – two marks usually requires more. Here they needed to show $\frac{10}{t} \to 0$.

(iv) - Many did not attempt.
- Those that did made a number of errors such as
- starting graph at $t = 0$
- not putting in asymptote

Q9

(a) $4\cos^2\theta - 3 = 0 \quad , 0 \le \theta \le 2\pi$

$\cos\theta = \pm \frac{\sqrt{3}}{2}$ ✓ (3)

$\therefore \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓✓ [3 for all, 2 for $\frac{\pi}{6}, \frac{11\pi}{6}$]

(b) (i) if $x = y^2$, let $x = 1$

$\therefore y = 1 \quad \therefore Q(1,1)$ ✓ (1)

(ii) Volume of Solid $= \pi\int_1^4 x^2\,dy$ − Volume of cylinder

$= \pi\int_1^4 y^4\,dy - \pi \times 1^2 \times 3$ ✓ ✓

$= \pi\left[\frac{y^5}{5}\right]_1^4 - 3\pi$ (3)

$= \frac{\pi}{5} \times 1023 - 3\pi$

$= \frac{1008\pi}{5}\ u^3$ ✓ ACCEPT [633.365]

(c)(i) If $Q = Q_0 e^{-kt}$

$\frac{dQ}{dt} = -kQ_0 e^{-kt}$ (1)

$= -kQ$ ✓

(ii) $\therefore \frac{Q_0}{2} = Q_0 e^{-4500k}$

$\therefore e^{-4500k} = \frac{1}{2}$ (2)

$-4500k = \ln\frac{1}{2}$ ✓

$k = -\frac{1}{4500}\ln\frac{1}{2}$

$= \frac{1}{4500}\ln 2$ ✓

(iii) $0.15Q_0 = Q_0 e^{-kt}$ ✓

$e^{-kt} = 0.15$

$-kt = \ln 0.15$ (2)

$t = \frac{\ln 0.15}{-k}$

$= 12316.35$ years

$= 12000$ years ✓

Q10 (a) (i) $A(x) = x \times y$ ✓ (2)

$= x e^{-2x}$ ✓

(ii) $A'(x) = -2xe^{-2x} + e^{-2x}$

$= e^{-2x}(1-2x)$ ✓ (3)

let $A'(x) = 0 \therefore x = \frac{1}{2}$

Test for MAX/MIN using first derivative

x:	0.4	$\frac{1}{2}$	0.6
$A'(x)$:	>0	0	<0
	/	—	\

✓ $\therefore$ MAX

$\therefore$ as $x = \frac{1}{2}$

$y = e^{-1}$ ✓

$\therefore A\left(\frac{1}{2}, \frac{1}{e}\right)$

(b)(i) If rate = 9% pa (2)

rate/month = 0.75%

$\therefore$ Amount owing after 1 month $= P(1.0075) - 2700$

✓ after 2 months $= P(1.0075)^2 - 2700(1.0075) - $

after 3 months $= P(1.0075)^3 - 2700(1.0075)^2 - 2700$
-2

✓ $= P(1.0075)^3 - 2700(1 + 1.0075 + 1.00$

(ii) $\therefore A_n = P(1.0075)^n - \frac{2700\left(1.0075^n - 1\right)}{1.0075 - 1}$

let $A_n = 0 \therefore P = \frac{360\,000(1.0075^n - 1)}{1.0075^n}$ (2)

Q10

(b) (iii) Let $n = 360$

$$\therefore P = \frac{360000(1.0075^{360} - 1)}{1.0075^{360}}$$

(2)

$= \$335\,561.04$

(iv) Interest paid $= 360 \times 2700 - 335561.04$

$= \$636438.96$

(1)