Outcome 1 – Graphs (30 Marks)

1. Sketch each of the following and indicate on your sketch: **6**

   (i) the coordinates of the vertex

   (ii) the y-intercept.

   (a) $y = x^2$ (b) $y = (x+1)^2 - 2$

2. For each parabola, find: **6**

   (i) the equation of the axis of symmetry

   (ii) the vertex

   (iii) the minimum or maximum value of the function

   (a) $y = -x^2 + 4x + 5$ (b) $y = \frac{1}{2}x^2 + 6x - 3$

3. In each of the following circles, find: **6**

   (i) the coordinates of the centre

   (ii) the length of the radius

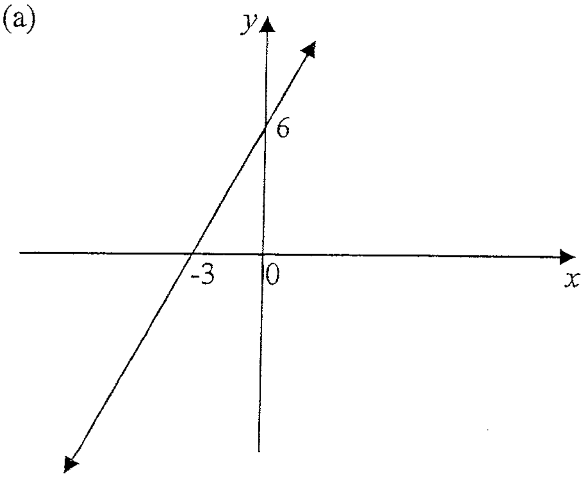
   (iii) the x-intercepts and y-intercepts (if they exist).

   (a) $x^2 + y^2 = 9$ (b) $(x-4)^2 + (y-3)^2 = 25$

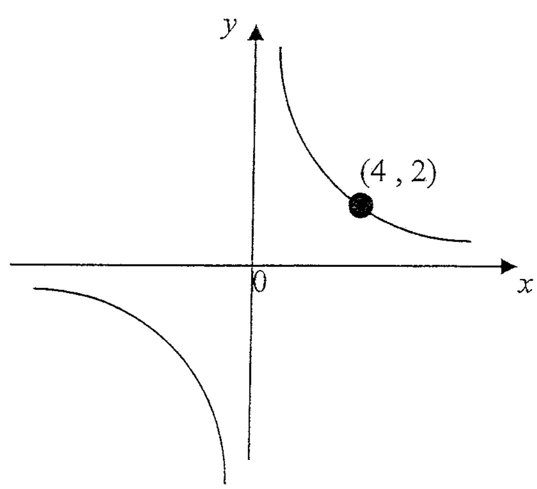
4. Find the coordinates of the point(s) of intersection between the curves $y = x^3$ and $y = \frac{1}{x}$. **2**

5. Find the equation of each graph **2**

(a)



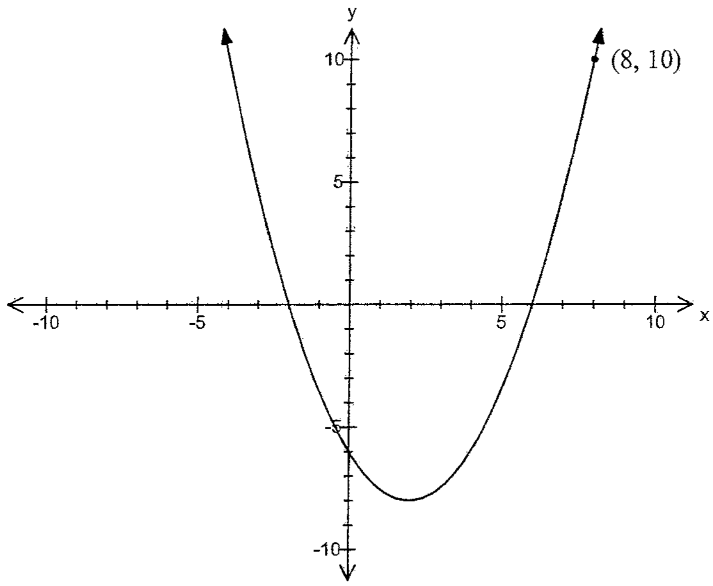
(b)



6. Sketch the following curves showing all essential features. **6**

(a) $3x+2y=6$ (b) $y=2^x$ (c) $y=\frac{-2}{x}$

7. Find the equation of the following parabola in the form $y=k(x-a)(x-b)$ **2**



Outcome 2 – Polynomials (25 Marks)

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1. For each of these polynomials state: **6**

   (i) the degree

   (ii) the leading coefficient

   (iii) the constant term.

   (a) $P(x)=8x^5+9x^3-2x-1$ (b) $P(x)=3\sqrt{5}x^3-4x+6$

2. Perform the following division and express your answer in the form: **3**

   Dividend = divisor x quotient + remainder.

$$\left(2x^3-x^2+5x+1\right)\div\left(x+6\right)$$

3. Expand and simplify $(3x-4)(5x^3+9x^2-7)$. **2**

4. Given $P(x)=2x^3+6x^2+4x+7$ and $Q(x)=x^3+3x-6$ find $P(x)-Q(x)$. **1**

3. Find the value of k given: **4**

(a) The remainder is 6 when $P(x)=2x^3+6x^2-3x+k$ is divided by $(x-1)$.

(b) $P(x)=x^3-10x^2+kx-8$ is exactly divisible by $(x-2)$.

4. $P(x)=x^3+ax^2+bx-30$ is divisible by $(x-3)$ but leaves a remainder of -8 when divided by $(x-1)$. Find a and b. **3**

5. Sketch the following polynomial functions show the x and y intercepts. **6**

(a) $y=(x+1)(x-3)(x-7)$

(b) $y=-x(x-4)^3$

(c) $y=x^3+8x^2-20x$

Outcome 3 – Functions and Logarithms (25 Marks)

START A NEW PAGE

1. Given $f(x) = x^2 - x - 3$: **2**

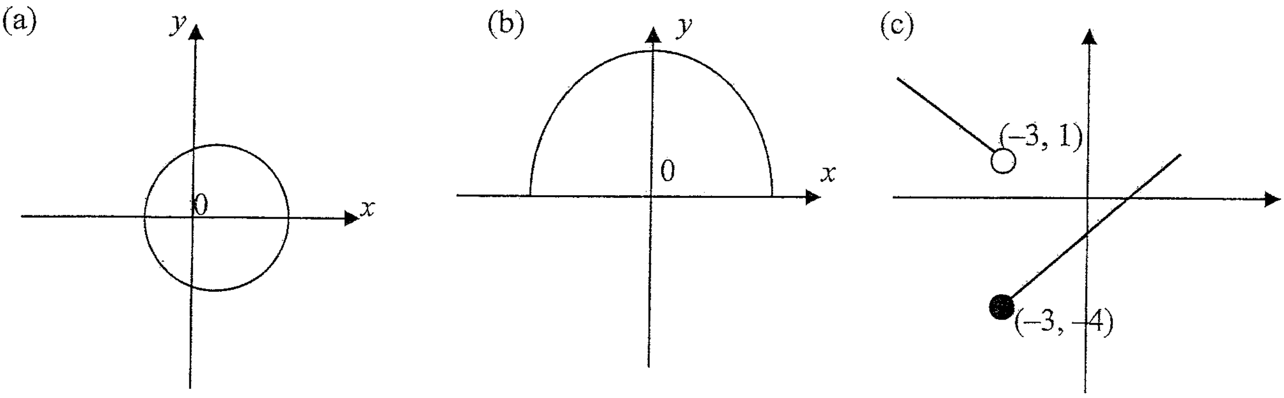
   (a) $f(2)$ (b) $f(y-3)$

2. Sketch $f(x) = x^2 + 1$ for $x \geq 0$. On the same number plane diagram, sketch $y = f^{-1}(x)$, the inverse function of $y = f(x)$. **2**

3. Find the inverse of $y = \dfrac{3x-2}{3}$. **2**

4. State whether or not the following diagrams represent the sketch of a function. **3**

(a) (b) (c)



5. Write the statement $3^4 = 81$ in logarithm form: **1**

6. Write the statement $\log_2 \dfrac{1}{32} = -5$ in index form: **1**

7. Solve each of the following equations: **6**

   (a) $\log_x 9 = 2$ (b) $\log_{\sqrt{2}} 8 = x + 2$ (c) $3^{2x} = 26$

8. Simplify, fully, each expression. **6**

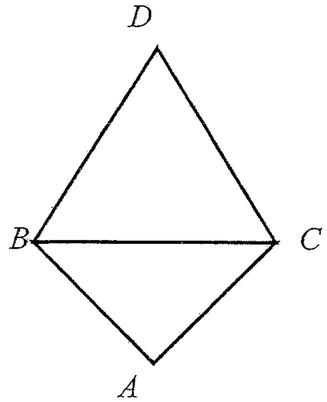
   (a) $\log_6 9 + \log_6 4$ (b) $\log_4 144 - \log_4 9$ (c) $\log_{100} 20 - \dfrac{1}{2}\log_{100} 4$

9. At the beginning of 2007, David deposited \$150 000 in an account which will pay an interest rate of 6% per annum, compounding monthly. During which year will David's investment be worth twice the original deposit? **2**

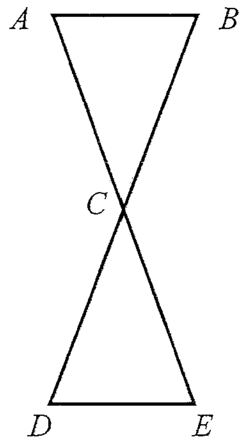
Outcome 4 – Geometry and Similarity (25 Marks)

1. **5**

(a) ΔABC is a right-angled isosceles triangle. ΔDBC is equilateral. Find the size of $\angle ABD$.



(b) $AC = BC = DC = EC$. Prove that AB and DE are parallel.

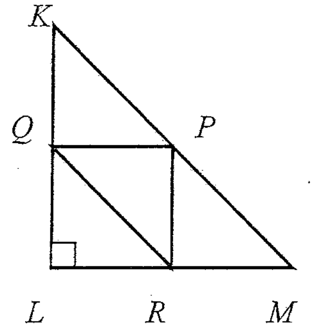


2. ΔKLM is right-angled at L. Q, P and R are the midpoints of the sides of ΔKLM. $KL = 15$ cm, $LM = 10$ cm. Find the: **4**

(i) area of ΔKLM

(ii) area of ΔPQR

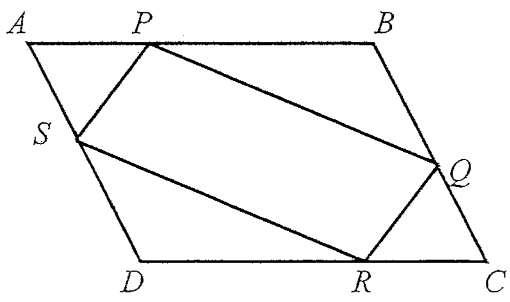
(iii) ratio of area ΔLRQ to area trapezium $KQRM$. Show all reasons.



3. $ABCD$ is a parallelogram. $AP = AS = CQ = CR$. By using congruent triangles, or otherwise: **5**

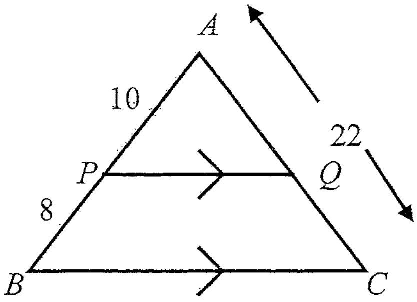
(i) Prove $QR = PS$ and $PQ = SR$.

(ii) What shape is $PQRS$? Justify your answer.



4. (i) Prove that $\triangle APQ$ and $\triangle ABC$ are similar. **4**

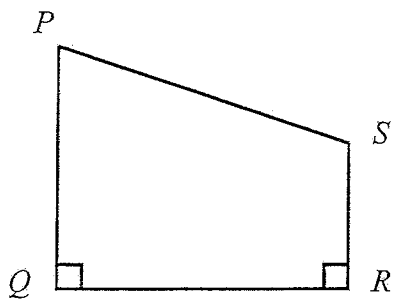
(ii) Hence find the length of PQ.



5. Two similar cones have surface areas in the ratio 256:81. What is the ratio of the corresponding: **2**

(i) slant heights (ii) volumes.

6. In the quadrilateral $PQRS$, $PQ \perp QR$ and $SR \perp QR$ **5**



(i) Prove that $PR^2 - QS^2 = PQ^2 - RS^2$

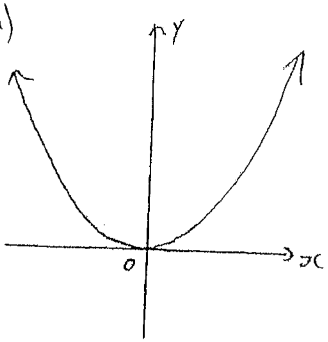
(ii) Hence, prove that $PS^2 - QR^2 = (PQ - RS)^2$

END OF EXAM.

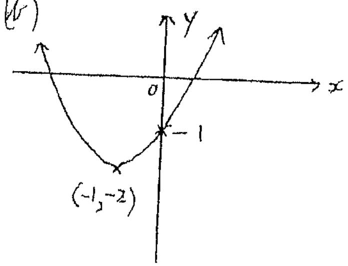
YEAR 10 PATHWAY A
YEARLY 2007

Outcome ①

(a)



(b)



② (a) $y = -x^2 + 4x + 5$

(i) $x = 2$

(ii) $(2, 9)$

(iii) 9

(b) $y = \frac{1}{2}x^2 + 6x - 3$

(i) $x = -6$

(ii) $(-6, -21)$

(iii) -21

③ (a) $x^2 + y^2 = 9$

(i) $(0,0)$

(ii) 3

(iii) $x = \pm 3,\ y = \pm 3$

(b) $(x-4)^2 + (y-3)^2 = 25$

(i) $(4,3)$

(ii) 5

(iii) $x = 8,\ x = 0,\ y = 6,\ y = 0$

④ $x^3 = \frac{1}{x}$

$x^4 = 1,\ x^4 - 1 = 0$

$(x-1)(x+1)(x^2+1) = 0$

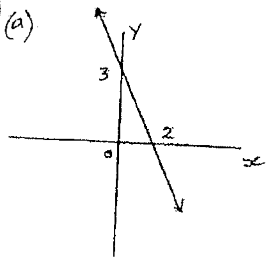
$x = \pm 1$

∴ curves intersect at $(1,1)$ and $(-1,-1)$

⑤ (a) $y = 2x + 6$

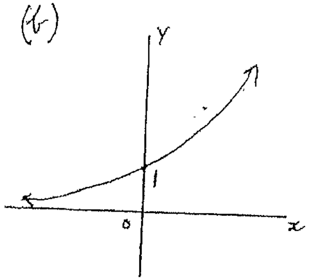
(b) $xy = 8$

⑥ (a)



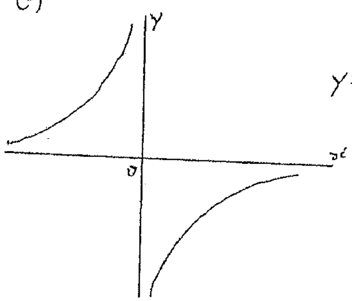
$3x + 2y = 6$

(b)



$y = 2^x$

(c)



$y = -\frac{2}{x}$

⑦ $a = 6,\ b = -2$

$y = k(x-6)(x+2)$

$-6 = k(0-6)(0+2)$

$k = \frac{1}{2}$

∴ $y = \frac{1}{2}(x-6)(x+2)$

Outcome ②

① (a) (i) 5

(ii) 8

(iii) −1

(b) (i) 3

(ii) $3\sqrt{5}$

(iii) 6

②

$$\begin{array}{r} 2x^2 - 13x + 83 \\ x+6 \overline{)\,2x^3 - x^2 + 5x + 1} \\ 2x^3 + 12x^2 \\ \hline -13x^2 \\ -13x^2 - 78x \\ \hline 83x \\ 83x + 498 \\ \hline -497 \end{array}$$

$2x^3 - x^2 + 5x + 1 = (x+6)(2x^3 - x^2 + 5x + 1) - 497$

③ $15x^4 + 27x^3 - 21x - 20x^3 - 36x^2 + 28$

$= 15x^4 + 7x^3 - 36x^2 - 21x + 28$

④ $x^3 + 6x^2 + x + 13$

⑤ (a) $2 \times 1^3 + 6 \times 1^2 - 3 \times 1 + k = 6$

$2 + 6 - 3 + k = 6$

$k = 1$

(b) $2^3 - 10 \times 2^2 + 2k - 8 = 0$

$8 - 40 + 2k - 8 = 0$

$k = 20$

⑥ $3^3 + 9a + 3b - 30 = 0$

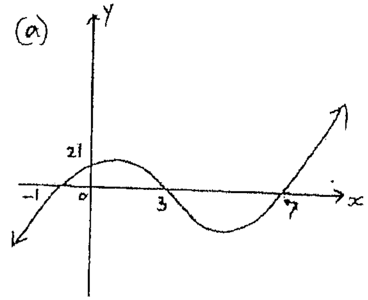
$3a + b = 1$ —— ①

$1 + a + b - 30 = -8$

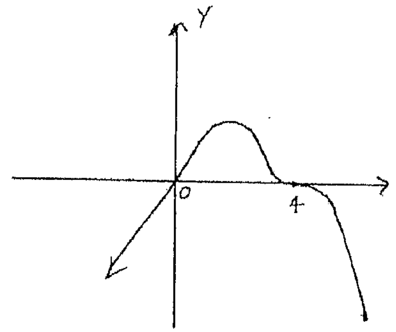
$a + b = 21$ —— ②

$a = -10,\ b = 31$

⑦ (a)

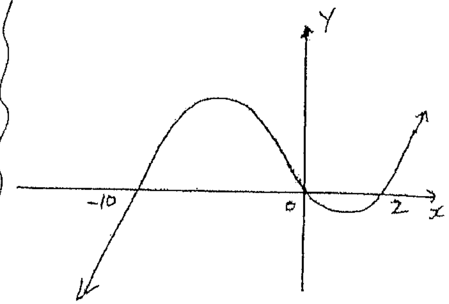


(b)



(c) $y = x(x^2 + 8x - 20)$

$= x(x+10)(x-2)$



Outcome ③

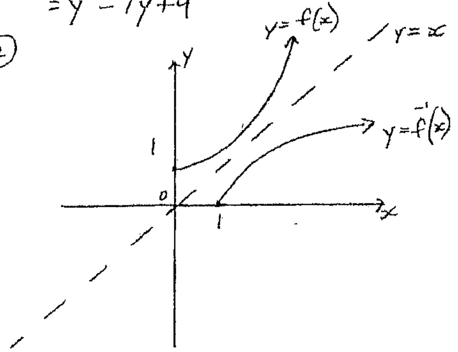
① (a) -1

(b) $(y-3)^2-(y-3)-3$

$=y^2-6y+9-y+3-3$

$=y^2-7y+9$

②



③ $x=\frac{3y-2}{3}$

$3x=3y-2$

$3y=3x+2$

$y=\frac{1}{3}(3x+2)$

or $f^{-1}(x)=\frac{3x+2}{3}$

④ (a) No (b) Yes (c) Yes

⑤ $\log_3 81=4$

⑥ $2^{-5}=\frac{1}{32}$

⑦ (a) $x^2=9$

$x=\pm 3$

but $x\neq -3$

$\therefore x=3$

(b) $(\sqrt{2})^{x+2}=8$

$2^{\frac{x+2}{2}}=2^3$

$\frac{x+2}{2}=3$

$x=4$

(c) $\log_3 26=2x$

$x=\frac{1}{2}\log_3 26$

⑧ (a) $\log_6 36=2$

(b) $\log_4\left(\frac{144}{9}\right)=2$

(c) $\log_{100}\left(\frac{20}{\sqrt{4}}\right)$

$=\log_{100}10$

$=\frac{1}{2}$

⑨ $A=P\left(1+\frac{0.5}{100}\right)^n$

$150000(1.005)^n=300000$

$1.005^n=2$

$n\log_{10}1.005=\log_{10}2$

$n=\frac{\log_{10}2}{\log_{10}1.005}$

≈ 139

After approx. 139 months, the amount doubles

i.e during 2020.

Outcome ④

① (a) $\angle ABC=\frac{1}{2}(180°-90°)$ (equal base ∠ of right isos △)

$=45°$

$\angle DBC=60°$ (△DBC equilateral)

$\therefore \angle ABD=105°$

(b) $AC=EC$ (Given)

$BC=DC$ (Given)

$\angle ACB=\angle DCE$ (vert. opp. ∠s)

$\therefore \triangle ABC\equiv\triangle DCE$ (SAS)

$\therefore \angle BAC=\angle DEC$ (base ∠s of congruent isosceles △s)

$\therefore AB\parallel DE$ (alternat ∠s equa)

② (a) $75\ cm^2$

(b) $\frac{75}{4}\ cm^2$

(c) $\triangle LRQ\equiv\triangle PRQ\equiv\triangle QPK\equiv\triangle RMP$

Area Trapezium = Area △PRQ + Area △QPK + Area △RMP

= 3 × Area △LRQ

∴ Ratio = 1 : 3

③ (i) $\angle PAS=\angle QCR$ (opp. ∠s of ∥gram)

$AP=QC$ (Given)

$AS=RC$ (Given)

$\therefore \triangle APS\equiv\triangle QCR$ (SAS)

$\therefore QR=PS$

$AB=DC$ (Opp sides of ∥gram)

$\therefore BP=AB-AP$

$=DC-RC$ ($AP=RC$ → given)

$=DR$

Similarly $BQ=DS$

$\angle SDR=\angle QBP$ (" of ∥gram)

$\therefore \triangle SDR\equiv\triangle QBP$ (SAS)

(ii) Parallelogram (Opposite sides are equal)

④ (i) $\angle APQ=\angle ABC$ (corresp. ∠s in ∥ lines)

$\angle AQP=\angle ACB$ (")

$\therefore \triangle APQ\parallel\triangle ABC$

(ii) $\frac{AQ}{22}=\frac{10}{18}$ (sides of sim △s)

$AQ=12.\dot{2}$

⑤ (a) $16:9$

(b) $16^3:9^3$

$=4096:729$

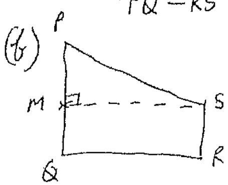
⑥ (a) $PR^2=PQ^2+QR^2$

$PR^2-QS^2=PQ^2+QR^2-QS^2$

$=PQ^2-(QS^2-QR^2)$

$=PQ^2-RS^2$

(b)



$PS^2=QR^2+PM^2$

$PS^2-QR^2=PM^2$

$=(PQ-QM)^2$

$=(PQ-RS)^2$