CALCULATOR ALLOWED | Advanced level questions | 25 MIN

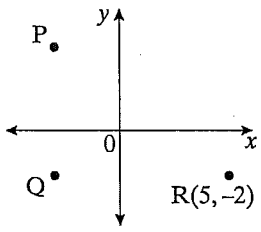
Mini Test 25: Algebra—Patterns, Expressions and Number Plane

1 Which expression is equivalent to $3(1 - p) - p$?

A $3 - 2p$ **B** $3 - p^2$ **C** $3 - 4p$ **D** $3 + 2p$

2 Ben used the rule 'multiply by 3 and then add 2' to get the next number in a pattern. The first three numbers are 1, 5 and 17. What is the seventh number in the pattern? ☐

3 R is the point $(5, -2)$. QR is parallel to the x-axis. PQ is parallel to the y-axis. PQ = 6 and QR = 8. What are the co-ordinates of P?



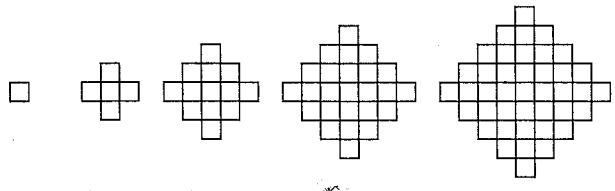
A $(-1, 8)$ **B** $(-3, 4)$ **C** $(-1, 4)$ **D** $(-3, 8)$

4 $3(5x + 4) + 2(3x - 7) =$

A $21x - 2$ **B** $14x - 2$
C $21x - 3$ **D** $14x - 3$

This pattern is used in questions 5 to 8.

Daniel made this pattern of squares with matches. He then drew up a table:



Shape 1 Shape 2 Shape 3 Shape 4 Shape 5

Shape	1	2	3	4	5
Number of squares	1	5	13	25	41
Number of matches	4	16	36	64	100

5 How many squares will there be in Shape 7? ☐

6 How many matches will Daniel need for Shape 7? ☐

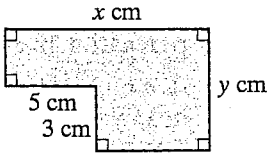
7 Which rule gives the number of matches needed for each shape?

A 4 × shape number
B 3 × number of squares + 1
C 2 × number of squares + 2
D (2 × shape number)^2

8 What is the largest number shape that can be made with 500 matches?

A 9 **B** 10 **C** 11 **D** 12

9 Which expression gives the area of the shaded shape in square centimetres?



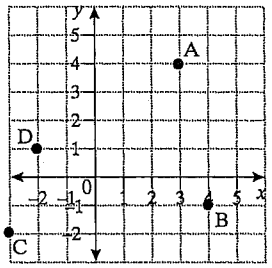
A $5x - 3y$ **B** $3x - 5y$
C $xy - 15$ **D** $xy - 8$

10 Which rule connects x and y?

x	1	2	3	4	5
y	1	13	33	61	97

A $y = x^2 + 1$ **B** $y = 2x^2 - 1$
C $y = 3x^2 - 2$ **D** $y = 4x^2 - 3$

11 Which point will the line $y = 7 - 2x$ pass through?



A A **B** B **C** C **D** D

12 What is the 27th number in this pattern?

1, 4, 7, 10, 13, ... ☐

13 $3(2x - 1) + 5 +$? $= 10x + 2$

What is the missing term? ☐

14 Which of these points lies on the straight line joining (1, 3) to (7, 15)?

A (2, 4) **B** (3, 8) **C** (4, 10) **D** (5, 11)

15 Which expression is equivalent to $4 + x - x^2$?

A $-x^2 + x + 4$ **B** $-x^2 - x + 4$
C $x^2 - x - 4$ **D** $x^2 + x - 4$

16 What is the next number in this pattern?

$\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \ldots$ ☐

Answers and explanations on pages 122–124

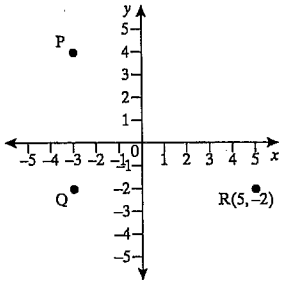
Mini Test 25: Algebra—Patterns, Expressions and Number Plane Page 28

1 C **2** 1457 **3** B **4** A **5** 85 **6** 196 **7** D **8** C **9** C **10** D **11** B **12** 79 **13** $4x$ **14** D **15** A **16** $\frac{11}{12}$

1 $3(1 - p) - p = 3 - 3p - p$
$= 3 - 4p$

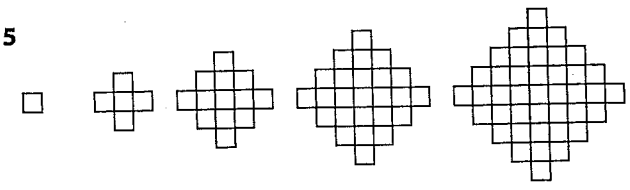
2 The third number = 17
The fourth number $= 3 \times 17 + 2$
$= 51 + 2$
$= 53$
The fifth number $= 3 \times 53 + 2$
$= 159 + 2$
$= 161$
The sixth number $= 3 \times 161 + 2$
$= 483 + 2$
$= 485$
The seventh number $= 3 \times 485 + 2$
$= 1457$

3 QR = 8 so Q is 8 units from R.
The co-ordinates of Q are $(-3, -2)$.
PQ = 6 so P is 6 units above Q.
The co-ordinates of P are $(-3, 4)$.



4 $3(5x + 4) + 2(3x - 7) = 15x + 12 + 6x - 14$
$= 21x - 2$

5



Shape 1 Shape 2 Shape 3 Shape 4 Shape 5

Shape	1	2	3	4	5
Number of squares	1	5	13	25	41
Number of matches	4	16	36	64	100

The number of squares increases by 4, then 8, then 12 and then 16.
The difference between the squares increases by 4 each time.
The next difference will be 20.
Shape 6 will have 41 + 20 or 61 squares.
The next difference will be 24.
Shape 7 will have 61 + 24 or 85 squares.

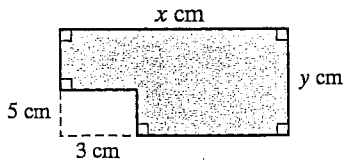
6 The number of matches increases by 12, then 20, then 28 and then 36.
The difference between the number of matches increases by 8 each time.
The next difference will be 44.
Shape 6 will need 100 + 44 or 144 matches.
The next difference will be 52.
Shape 7 will need 144 + 52 or 196 matches.

7 Try each option:
'4 × shape number'
There would be 4 matches for Shape 1.
There would be 8 matches for Shape 2.
This is not the rule.
'3 × number of squares + 1'
When there is 1 square there would be 4 matches.
When there are 5 squares there would be 16 matches.
When there are 13 squares there would be 40 matches.
This is not the rule.
'2 × number of squares + 2'
When there is 1 square there would be 4 matches.
When there are 5 squares there would be 12 matches.
This is not the rule.

'(2 × shape number)^2'
There would be $(2 \times 1)^2$ or 4 matches for Shape 1.
There would be $(2 \times 2)^2$ or 16 matches for Shape 2.
There would be $(2 \times 3)^2$ or 36 matches for Shape 3.
There would be $(2 \times 4)^2$ or 64 matches for Shape 4.
There would be $(2 \times 5)^2$ or 100 matches for Shape 5.
This is the rule.
The rule is (2 × shape number)^2.

8 [Use the rule found in the previous question and try each option.]
There would be $(2 \times 9)^2$ or 324 matches needed for Shape 9.
There would be $(2 \times 10)^2$ or 400 matches needed for Shape 10.
There would be $(2 \times 11)^2$ or 484 matches needed for Shape 11.
There would be $(2 \times 12)^2$ or 576 matches needed for Shape 12.
The largest number shape that can be made with 500 matches is Shape 11.

9 [The shaded area is the area of a large rectangle minus the area of a small rectangle.]



Area of large rectangle = length × width
$= x \times y$
$= xy$

Area of smaller rectangle = length × width
$= 5 \times 3$
$= 15$

Shaded area = $(xy - 15)$ cm^2

10

x	1	2	3	4	5
y	1	13	33	61	97

Try each option:

$y = x^2 + 1$
When $x = 1$,
$y = 1^2 + 1$
$= 2$
This is not the option.

$y = 2x^2 - 1$
When $x = 1$,
$y = 2 \times 1^2 - 1$
$= 1$
When $x = 2$,
$y = 2 \times 2^2 - 1$
$= 7$
This is not the option.

$y = 3x^2 - 2$
When $x = 1$,
$y = 3 \times 1^2 - 2$
$= 1$
When $x = 2$,
$y = 3 \times 2^2 - 2$
$= 10$
This is not the option.

$y = 4x^2 - 3$
When $x = 1$,
$y = 4 \times 1^2 - 3$
$= 1$
When $x = 2$,
$y = 4 \times 2^2 - 3$
$= 13$
When $x = 3$,
$y = 4 \times 3^2 - 3$
$= 33$
When $x = 4$,
$y = 4 \times 4^2 - 3$
$= 61$
When $x = 5$,
$y = 4 \times 5^2 - 3$
$= 97$
This is the option.

The rule is $y = 4x^2 - 3$.

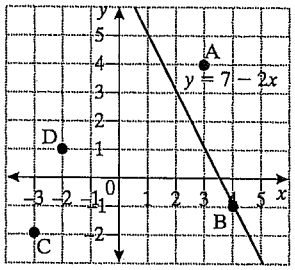
11 [If a line passes through a point the co-ordinates of that point must satisfy the equation of the line.]

The equation of the line is $y = 7 - 2x$.

Try each point:

At A, $x = 3$
When $x = 3$,
$y = 7 - 2 \times 3$
$= 7 - 6$
$= 1$ (not 4)
A does not lie on the line.

At B, $x = 4$
When $x = 4$,
$y = 7 - 2 \times 4$
$= 7 - 8$
$= -1$
B does lie on the line.



The point that lies on the line is B $(4, -1)$.

12 1, 4, 7, 10, 13, ...

The numbers increase by 3 each time.
The first number is $3 \times 1 - 2$.
The second number is $3 \times 2 - 2$.
The third number is $3 \times 3 - 2$.
So, following this pattern,
the 27th number $= 3 \times 27 - 2$
$= 81 - 2$
$= 79$

13 $3(2x - 1) + 5 + \boxed{?} = 10x + 2$

Now $3(2x - 1) + 5 = 6x - 3 + 5$
$= 6x + 2$

So the missing term is what must be added to $6x + 2$ to give $10x + 2$.

The missing term is $4x$.

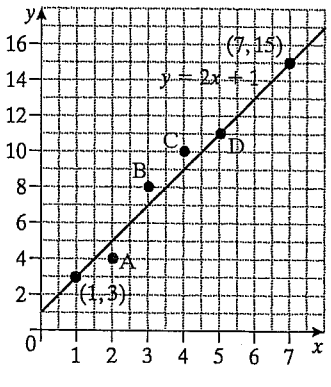
14 The line passes through $(1, 3)$ and $(7, 15)$.

The y-value in both points is 1 more than twice the x-value.

So that must be the equation of the line:

$y = 2x + 1$.

Considering the options, the only point with its y-value one more than twice its x-value is $(5, 11)$.



15 $4 + x - x^2 = -x^2 + x + 4$

[4 and x need to be added but x^2 needs to be subtracted.]

16 $\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \ldots$

All the fractions have a common denominator of 12.

[Write the pattern using fractions whose denominator is 12.]

$\frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \ldots$

The next number in the pattern will be $\frac{11}{12}$.