International Grammar School

Year 12
Mathematics
HSC Assessment Task 1
2010/2011

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 36

- Attempt Questions 1 – 3
- All questions are of equal value
- Start each question in a new writing answer booklet
- Write your name on each answer booklet
- If you do not attempt a question, submit a blank answer booklet marked with your name, question number and "N/A" on the page

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Total Marks – 36

Attempt Questions 1–3
All questions are of equal value

Answer each question on a SEPARATE answer booklet. Extra answer booklets are available.

QUESTION 1 (12 marks) — **Marks**

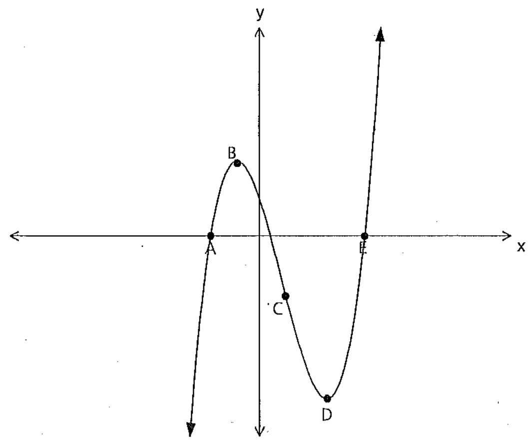
(a) Find the first and second derivatives of the following:

i) $y = \frac{4}{3}x^3$ — 2

ii) $f(x) = \frac{1}{x+1}$ — 2

iii) $f(x) = (x^2 + 1)^4$ — 3

(b) Given the graph of the function $f(x)$ below



Find the values of x for which

i) $f(x)$ is increasing — 2

ii) $f(x)$ is concave up — 1

iii) $f(x)$ is both concave up and increasing — 1

iv) $f'(x) = 0$ — 1

End of Question 1

QUESTION 2 (12 marks) — Marks

START A NEW ANSWER BOOKLET

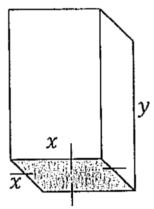
(a) A function $f(x)$ is defined by $f(x) = 6x^2 - 2x^3$.

i) Find the coordinates of the turning points of $y = f(x)$ and determine their nature. — 3

ii) Find the coordinates of any point(s) of inflexion. — 1

iii) Hence sketch the graph of $y = f(x)$. Label the turning points, any point(s) of inflexion and the points where the curve meets the x-axis. (Your graph should be one third of the page) — 3

(b) A box, **open** at the top is to be made from cardboard. The base of the box is a square of side of x cm and its height is y cm as shown in the diagram.



i) If the volume of the box is to be 32 cm^2, show that $y = \frac{32}{x^2}$. — 1

ii) Show that the area of cardboard needed will be $A = x^2 + \frac{128}{x}$ — 1

iii) Find the dimensions of the box which minimises the area — 3

End of Question 2

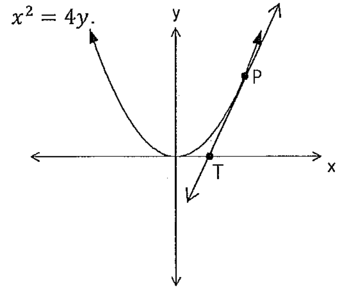
QUESTION 3 (12 marks) — Marks

START A NEW ANSWER BOOKLET

(a) A parabola has focus at (0,3) and directrix at $y = 1$. Draw a sketch of the parabola labelling the focus, directrix and vertex. — 2

(b) A parabola has focus at (2,0) and directrix at $x = -1$. Draw a sketch of the parabola labelling the focus, directrix and vertex. — 2

(c) P(8,16) is a point on the parabola $x^2 = 4y$ and F is the focus.



i) Show that the tangent at P has equation $4x - y - 16 = 0$ — 2

ii) The tangent at P meets the x axis at T. Find the coordinates of T and hence show that ∠PTF=90° — 2

(c) A, B are the points (-1,0) and (5,2) respectively. Show that the equation of the locus of the point P(x, y) which moves so that

i) P is equidistant from the points A and B is $3x + y = 7$. — 2

ii) PA is perpendicular to PB, is $x^2 + y^2 - 4x - 2y - 5 = 0$ — 2

End of Assessment Task

Question 1

a) i) $f'(x) = 4x^2$ ①

$f''(x) = 8x$ ①

ii) $f(x) = (x+1)^{-1}$

$f'(x) = -(x+1)^{-2} = \frac{-1}{(x+1)^2}$ ①

$f''(x) = 2(x+1)^{-3} = \frac{2}{(x+1)^3}$ ①

iii) $f'(x) = 4 \times 2x(x^2+1)^3$

$= 8x(x^2+1)^3$ ①

$f''(x) = v\,du + u\,dv$

$u = 8x$ ① $v = (x^2+1)^3$

$du = 8$ $dv = 6x(x^2+1)^2$

$f''(x) = 8(x^2+1)^3 + 48x^2(x^2+1)^2$ ①

$8(x^2+1)^2(x^2+1+6x^2)$

b i) $D < x < B$ ① ①

ii) $x > B$ ①

iii) $x > D$ ①

iv) $x = B$, $x = D$ ①

Question 2

$f(x) = 6x^2 - 2x^3$

$f'(x) = 12x - 6x^2$

$f''(x) = 12 - 12x$

i) St pts at $f'(x) = 0$

$12x - 6x^2 = 0$

$6x(2 - x) = 0$

$6x = 0$ $\quad 2 - x = 0$

$x = 0$ $\quad x = 2$

$y = 0$ $\quad y = 8$

$(0,0)$ $\quad (2,8)$

$f''(0) = 12 - 2(0) = 12 > 0$ min

$f''(2) = 12 - 2(2) = -12 < 0$ max

$(0,0)$ min and $(2,8)$ max

ii) POI at $f''(x) = 0$

$12 - 2x = 0$

$12 = 2x$

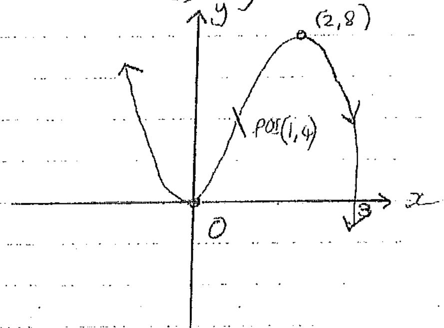
$x = 1$ a possible POI

	0	1	2
$f''(x)$	>0		<0

change in concavity

∴ POI at $(1, 4)$

iii)



b) i) $x^2 y = 32$

∴ $y = \frac{32}{x^2}$

ii) $A = x^2 + 4xy$

$= x^2 + 4x \times \frac{32}{x^2}$

∴ $A = x^2 + \frac{128}{x}$

$A' = 2x - \frac{128}{x^2}$

$2x^3 = 128$

$x^3 = 64$

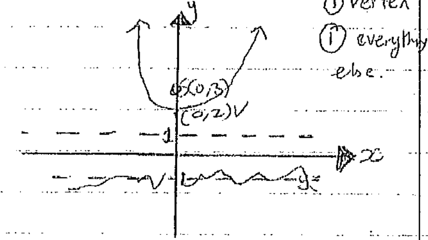
$x = 4$

When $x = 4$ $y = \frac{32}{16} = 2$

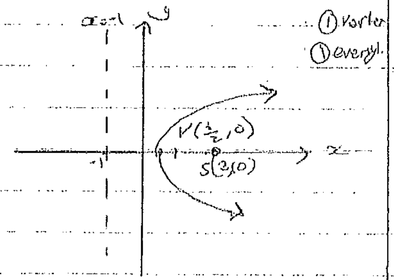
dimensions: $4 \times 4 \times 2$

Question 3

a) ① vertex ① everything else



b) ① vertex ① everything



c) i) $y = \frac{x^2}{4}$

$y' = \frac{x}{2}$ at $x = 8$ $y' = 4$

$y - 16 = 4(x - 8)$ ① gradient

$y - 16 = 4x - 32$

∴ $4x - y - 16 = 0$ ①

ii) $4x - 0 - 16 = 0$

$4x = 16$

$x = 4$

$T(4, 0)$ ← ① finding x int.

$F(0, 1)$

Perpendicular if $m_1 \times m_2 = -1$

ie. $m_{FT} \times m_{PT} = -1$

$m_{FT} = \frac{-1}{4}$ $\quad m_{PT} = \frac{16-0}{8-4} = \frac{16}{4} = 4$

$-\frac{1}{4} \times 4 = -1$

∴ $\angle PTF = 90°$ ①

d) i) $(x+1)^2 + (y-0)^2 = (x-5)^2 + (y-2)^2$ ①

$x^2 + 2x + 1 + y^2 = x^2 - 10x + 25 + y^2 - 4y + 4$

$12x + 4y - 28 = 0$

$3x + y - 7 = 0$

∴ $3x + y = 7$ ①

ii) $\frac{y-0}{x+1} \times \frac{y-2}{x-5} = -1$ ①

$\frac{y^2 - 2y}{x^2 - 4x - 5} = -1$

$y^2 - 2y = -x^2 + 4x + 5$

$x^2 + y^2 - 4x - 2y - 5 = 0$ ①