NSW INDEPENDENT SCHOOLS

2014

Higher School Certificate

Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in Question 11 – 16
- Write your student number and/or name at the top of every page

Total marks – 100

Section I - Pages 3 – 6

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 7 – 13

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

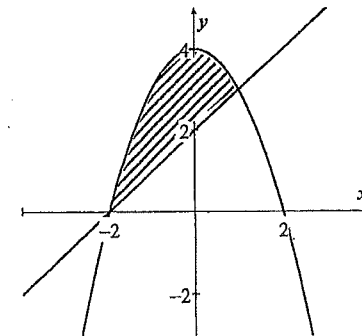|	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Section I

10 marks
Attempt Question 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

1. What is 5.9974932 correct to 3 significant figures?

    A.   5.99

    B.   6.00

    C.   5.997

    D.   5.998

2. Which pair of inequalities represents the shaded region.

    A. $\begin{cases} y \le x+2 \\ y \le 4-x^2 \end{cases}$

    B. $\begin{cases} y \le x+2 \\ y \ge 4-x^2 \end{cases}$

    C. $\begin{cases} y \ge x+2 \\ y \ge 4-x^2 \end{cases}$

    D. $\begin{cases} y \ge x+2 \\ y \le 4-x^2 \end{cases}$
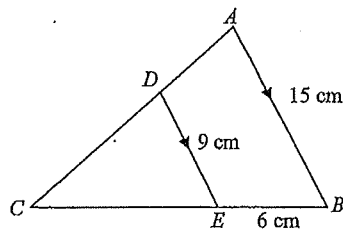
3. If $\dfrac{2\sqrt{3}}{\sqrt{3}+3}=a\sqrt{3}+b$, what are the values of a and b?

   A. $a=1,\ b=1$

   B. $a=1,\ b=-1$

   C. $a=-1,\ b=1$

   D. $a=-1,\ b=-1$

4. What is the perpendicular distance of the point $(-3,\ 1)$ from the line $3x-2y=4$?

   A. $\dfrac{7}{\sqrt{13}}$

   B. $\dfrac{7}{\sqrt{5}}$

   C. $\dfrac{15}{\sqrt{13}}$

   D. $\dfrac{15}{\sqrt{5}}$

5. In the diagram below ABC is a triangle and $AB \parallel DE$

   Given that $AB=15$ cm, $DE=9$ cm and $BE=6$ cm, what is the value of BC?

   A. 3.6 cm

   B. 6 cm
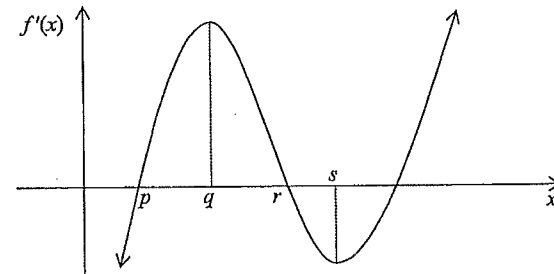
   C. 9 cm

   D. 15 cm

6. $\displaystyle\int \dfrac{1}{2x}\,dx=?$

   A. $\dfrac{1}{2}\ln x+c$

   B. $\ln 2x+c$

   C. $2\ln x+c$

   D. $2\ln 2x+c$

7. The midpoint of (a,b) and $(5,-3)$ is $(-1,4)$. What are the values of a and b?

   A. $a=2,\ b=\dfrac{1}{2}$

   B. $a=-7,\ b=11$

   C. $a=11,\ b=-10$

   D. $a=3,\ b=3\dfrac{1}{2}$

8. The diagram shows the graph $y=f'(x)$.

   What is the x value of the maximum turning point?

   A. $x=p$

   B. $x=q$

   C. $x=r$

   D. $x=s$

9. Brooke owns 4 red jackets and 5 blue jackets. She selects, at random, one jacket for herself and another for each of her 2 friends.
What is the probability that all 3 are blue?

A. $\dfrac{5}{42}$

B. $\dfrac{125}{729}$

C. $\dfrac{5}{9}$

D. $\dfrac{39}{42}$

10. An infinite geometric series has a first term of 10 and a limiting sum 30.
What is the common ratio?

A. $\dfrac{1}{3}$

B. $\dfrac{1}{2}$

C. $\dfrac{2}{3}$

D. $\dfrac{3}{4}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Simplify $\dfrac{4}{x^2-x}-\dfrac{x-1}{x^2}$.   **2**

(b) Solve $|2x-3|=3-x$.   **2**

(c) Differentiate $\dfrac{5}{\sqrt{2-3x^2}}$ with respect to x.   **2**

(d) Find the equation of the tangent to the curve $y=3e^x-1$ at the point where $x=0$.   **3**

(e) Use Simpson's Rule and all the values in the table to find an approximate value for $\displaystyle\int_0^3 f(x)\,dx$.   **3**

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0.3	0	−1.3	−2.1	0	1.2	5

(f) The quadratic equation $x^2-7x+8=0$ has roots α and β. Find:-

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **1**
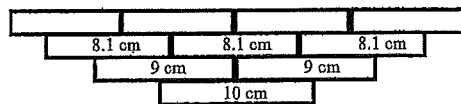
Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x.

    (i) $\left(x^2-1\right)\tan\left(3x-1\right)$ **2**

    (ii) $\dfrac{e^{3x}}{\ln x}$ **2**

(b) Evaluate $\displaystyle\int_2^3 \dfrac{6x^2}{x^3-2}\,dx$. Give your answer in exact form.   **3**

(c) Jenny is using blocks to build a tower. The bottom brick is 10 cm long. She then places two 9cm blocks on top of this. The third layer consists of three 8.1 cm blocks, as shown in the diagram below. The length of the blocks in each row is 90% of the length of the blocks in the previous row.

Her tower contains 10 rows of blocks. How long is the top layer?   **3**

(d)
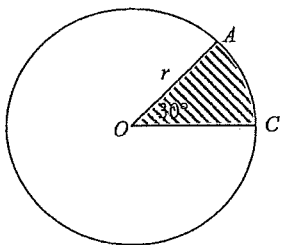
    (i) Find the radius of the circle if the area of the shaded sector is $12\pi\,\text{cm}^2$.   **3**

    (ii) Hence find the exact length of major arc AC.   **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch, on the same set of axes, the curves $y=\sin 2x$ and $y=\cos x$ for $0\le x\le\dfrac{\pi}{2}$.   **2**

    (ii) Verify that the curves intersect at $x=\dfrac{\pi}{6}$ and $x=\dfrac{\pi}{2}$.   **1**

    (iii) Hence, find the area between the two curves from $x=\dfrac{\pi}{6}$ to $x=\dfrac{\pi}{2}$.   **3**

(b) Rachel borrowed \$35 000 from a credit union to purchase a new car. Interest on the loan is calculated monthly at the rate of 0.7% per month and is charged immediately before each monthly repayment of \$R is made.
Let A_n be the amount in dollars owing on the loan after the nth repayment has been made.

    (i) Show that $A_3=35000\times1.007^3-R\left(1+1.007+1.007^2\right)$.   **1**

    (ii) Show that $A_n=35000\times1.007^n-\dfrac{1000R\left(1.007^n-1\right)}{7}$.   **2**

    (iii) If the loan is to be paid out after 5 years what would the value of R be?   **2**

    (iv) If Rachel decides to pay \$800 per month in repayments, how long would it take to pay out her loan?   **2**

(c) Find the coordinates of the focus of the parabola $\left(y+1\right)^2=12\left(3-x\right)$.   **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The acceleration of a particle is given by

$$a = 1 - 2t$$

where a is measured in centimetres per second per second and t is measured in seconds.

Initially the particle is at rest 2 cm to the right of the origin.

    (i) At what time is the particle next at rest?   2

    (ii) Where is the particle at this time?   2

(b) Felicity receives a money box on the day she's born.
Her parents decide that each month, on the 1st of the month, they will deposit money into her money box and give her this money on her 21st birthday.

The 1st month they deposit \$10 into the money box.
The 2nd month they deposit \$20, the 3rd month they deposit \$30.
Each month they deposit \$10 more into the money box than they did the month before.

    (i) How many times over the 21 years will Felicity's parents deposit money into her money box?   1

    (ii) How much will be deposited into the money box in the month of her 21st birthday?   2

    (iii) How much will Felicity receive from her parents on her 21st birthday?   2

(c) Peter and Paul catch a train to work each day but they get on at different stations.   3

The probability that Peter gets a seat each morning is $\dfrac{1}{8}$.

The probability that Paul gets a seat each morning is $\dfrac{1}{21}$.

What is the probability that, on the next three mornings, at least one of them gets a seat?

(d) Solve $2\sin x \cos x = \cos x$ for $0 \le x \le 2\pi$.   3

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) The population, P, of Booby Birds on Christmas Island is decreasing at a rate proportional to P.
That is $\dfrac{dP}{dt} = -kP$, where k is a positive integer and t is measured in years.

In January 2000 there were 3000 Booby Birds and by January 2010 the population had decreased to 2750.

    (i) Show that the value of k is 0.0087, correct to 4 decimal places.   2

    (ii) If the population continues to decrease at this rate what will be the expected population in 2020?   2

(b) The graphs $y = x^2 - 3x - 5$ and $y = x + k$ have only one point of intersection, P.

    (i) Show that the x–coordinate of P satisfies $x^2 - 4x - 5 - k = 0$.   1

    (ii) Find the value of k.   2

    (iii) Find the coordinates of P.   2

(c) Consider the curve $y = xe^x$.

    (i) Find the stationary point and determine its nature.   2

    (ii) Find the point of inflexion.   2

    (iii) Sketch the curve showing all important features.   2

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a)
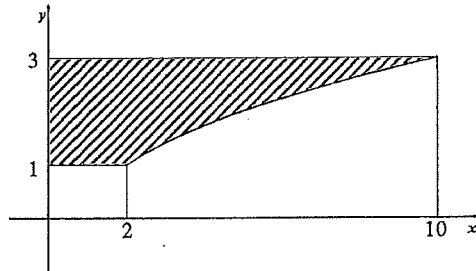
The diagram shows the shaded region enclosed by the curve $y = \sqrt{x-1}$, the y – axis and the lines $y = 1$ and $y = 3$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the y – axis.   3

(b)   After a week of rain the local dam starts to fill until, at 10am Sunday the dam overflows. At this point the height (H) of the river starts to change at the rate of

$$\left(1 - \frac{t}{20}\right) \text{ metres per hour.}$$

Initially the height of the river is 5 metres.

(i)   Show that the height of the river is given by the formula   1
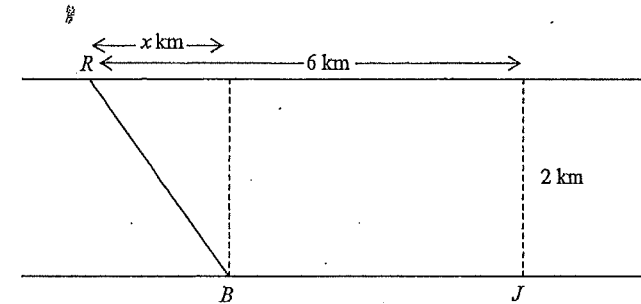
$$H = -\frac{t^2}{40} + t + 5$$

(ii)   Find the maximum height of the river during this flood.   2

(iii)   A bridge crossing this river will be blocked once the height of the river reaches 12.5 metres.
At what times and days will the bridge be blocked and then re-opened.   3

Question 16 (continued).

(c)

Romeo (R) and Juliet (J) live on 2 parallel streets which are 2 km apart and run east–west as shown in the diagram.

When Juliet calls Romeo to let him know her parents are out he needs to get there as soon as possible.
Romeo has hidden a bike, at point B, on Juliet's street.

To get to Juliet's house, Romeo runs from his house, R, through the bush to his bike, B at 8 km/h. He then rides his bike to Juliet's house, J at 16 km/h.

Let x km represent the distance the bike is east of Romeo's house.

(i)   Show that the time (T hours) taken for Romeo to get to Juliet's house is given by   2

$$T = \frac{\sqrt{x^2 + 4}}{8} + \frac{6 - x}{16}$$

(ii)   Find the distance of the bike from Juliet's house in order to minimize the time taken for Romeo to get to Juliet.   3

(iii)   Find the minimum time taken.   1

End of Paper

NSW INDEPENDENT TRIAL EXAMS – 2014
MATHEMATICS (YR12 TRIAL EXAM)
MARKING GUIDELINES

Section I

Question	Answer	Solution	Outcome
1.	B	6.0	P3
2.	D	$\begin{cases} y \geq x+2 \\ y \leq 4-x^2 \end{cases}$	P4
3.	B	$\dfrac{2\sqrt{3}}{\sqrt{3}+3} = \dfrac{2\sqrt{3}}{\sqrt{3}+3} \times \dfrac{\sqrt{3}-3}{\sqrt{3}-3}$ $= \dfrac{6-6\sqrt{3}}{3-9}$ $= \dfrac{6-6\sqrt{3}}{-6}$ $= \sqrt{3}-1 \qquad a=1, \quad b=-1$	P3
4.	C	$d = \left\| \dfrac{3(-3)-2(1)-4}{\sqrt{9+4}} \right\|$ $= \left\| \dfrac{-9-2-4}{\sqrt{13}} \right\|$ $= \dfrac{15}{\sqrt{13}}$	P3
5.	D	$\dfrac{CE}{CE+6} = \dfrac{9}{15}$ $\dfrac{CE}{CE+6} = \dfrac{3}{5}$ $5CE = 3CE+18$ $2CE = 18$ $CE = 9$ $BC = 9+6$ $= 15$	P4
6.	A	$\int \dfrac{1}{2x}\,dx = \dfrac{1}{2}\int \dfrac{1}{x}\,dx$ $= \dfrac{1}{2}\ln x + c$	H3
7.	B	$\dfrac{a+5}{2} = -1 \qquad \dfrac{b-3}{2} = 4$ $a+5 = -2 \qquad b-3 = 8$ $a = -7 \qquad b = 11$	P4
8.	C	$x = r$	H7
9.	A	$\dfrac{5}{9} \times \dfrac{4}{8} \times \dfrac{3}{7} = \dfrac{5}{42}$	H5
10.	C	$30 = \dfrac{10}{1-r}$ $30-30r = 10$ $30r = 20$ $r = \dfrac{2}{3}$	H5

Section II.

Question 11.
11a)   Outcome Assessed: P3, P4

Marking Guidelines

Criteria	Marks
• Factorises and expresses with common denominator.	1
• Correct answer	1

Answer:

$$\dfrac{4}{x^2-x} - \dfrac{x-1}{x^2} = \dfrac{4}{x(x-1)} - \dfrac{x-1}{x^2}$$

$$= \dfrac{4x-(x-1)(x-1)}{x^2(x-1)}$$

$$= \dfrac{4x-(x^2-2x+1)}{x^2(x-1)}$$

$$= \dfrac{6x-x^2-1}{x^2(x-1)}$$

11b)   Outcome Assessed: P3, P4

Marking Guidelines

Criteria	Marks
• 1 correct answer tested.	1
• Two Correct answers tested	1

Answer:

$|2x-3| = 3-x$

$2x-3 = 3-x \qquad\qquad 2x-3 = -(3-x)$
$3x = 6 \qquad\qquad\qquad\quad = x-3$
$x = 2 \qquad\qquad\qquad\quad x = 0$
$\quad$ Test $\qquad\qquad\qquad\qquad$ Test
$|2(2)-3| = 3-2 \qquad\quad |2(0)-3| = 3-0$
$|1| = 1 \quad \therefore$ True $\qquad |-3| = 3 \quad \therefore$ True
$x = 0, 2$

11c) Outcome Assessed: P7, P8

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Expressing function with correct negative index.	1
• Correct differentiation.	1

Answer:

$$\frac{d}{dx}\frac{5}{\sqrt{2-3x^2}}=\frac{d}{dx}5\left(2-3x^2\right)^{-\frac{1}{2}}$$

$$=-\frac{5}{2}\left(2-3x^2\right)^{-\frac{3}{2}}\times -6x$$

$$=15x\left(2-3x^2\right)^{-\frac{3}{2}}$$

$$=\frac{15x}{\left(2-3x^2\right)^{\frac{3}{2}}}$$

11d) Outcome Assessed: H3, H6

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Correct y value.	1
• Correct gradient.	1
• Correct equation of tangent.	1

Answer:

$x=0,\qquad y=3e^0-1$

$\qquad\qquad =2$

$y=3e^x-1$

$y'=3e^x$

$x=0$

$y'=3$

$y-2=3(x-0)$

$\qquad y=3x+2$

11e) Outcome Assessed: H8

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Correct h value (0.5).	1
• Correct substitution into correct formula.	1
• Correct solution.	1

Answer:

$$\int_0^3 f(x)dx \doteqdot \frac{0.5}{3}\left(0.3+5+4\left(0+-2.1+1.2\right)+2\left(-1.3+0\right)\right)$$

$$=\frac{0.5}{3}(-0.9)$$

$$=-0.15$$

11f) Outcome Assessed: P4

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Correct value for $\alpha+\beta$	1
• Correct value for $\alpha\beta$.	1
• Correct solution for $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$.	1

Answer:

(i) $\qquad \alpha+\beta=7$

(ii) $\qquad \alpha\beta=8$

(iii) $\qquad \dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\beta+\alpha}{\alpha\beta}$

$$\qquad\qquad =\frac{7}{8}$$

Question 12.

12a) (i) Outcome Assessed: P7, H5

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Correct product rule.	1
• Correct differentiation of tan function.	1

Answer:

$$\frac{d}{dx}\left(x^2-1\right)\tan(3x-1)=2x\tan(3x-1)+3\left(x^2-1\right)\sec^2(3x-1)$$

12a) (ii) Outcome Assessed: P7, H3, H5

<div align="center">Marking Guidelines</div>

Criteria	Marks
• Correct quotient rule.	1
• Correct differentiation of exponential and logarithmic functions.	1

Answer:

$$\frac{d}{dx}\frac{e^{3x}}{\ln x}=\frac{3e^{3x}\ln x-e^{3x}\frac{1}{x}}{\left(\ln x\right)^2}$$

$$=\frac{e^{3x}\left(3x\ln x-1\right)}{x\left(\ln x\right)^2}$$

12b)   Outcome Assessed: H3,H5

Marking Guidelines

Criteria	Marks
• Identifying logarithmic function.	1
• Substitution into correct integral	1
• Correct solution	1

Answer:

$$\int_2^3 \frac{6x^2}{x^3-2}\,dx = \left[2\ln\left(x^3-2\right)\right]_2^3$$
$$= 2\left(\ln 25 - \ln 6\right)$$
$$= 2\ln\frac{25}{6}$$

12c)   Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Recognises the term of a GP and correct common ratio.	1
• Correct expression for GP.	1
• Correct solution.	1

Answer:

$r = 0.9$

$$l = 10\times\left(10\times 0.9^9\right)$$
$$= 38.742 \text{ cm}$$

12d) (i)   Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Converts 30° to radians.	1
• Correct expression area of sector.	1
• Correct solution.	1

Answer:

$$30° = \frac{\pi}{6}$$
$$12\pi = \frac{1}{2}\times\frac{\pi}{6}\times r^2$$
$$r^2 = 144$$
$$r = 12$$

12d) (ii)   Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Circumference of circle	1
• Correct solution.	1

Answer:
$$C = 24\pi$$
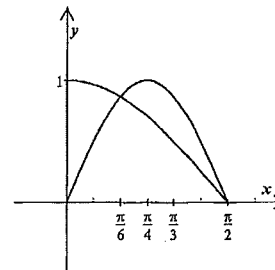$$\text{Arc } AC = \frac{\pi}{6}\times 12$$
$$= 2\pi$$
$$\text{Arc } ABC = 24\pi - 2\pi$$
$$= 22\pi$$

Question 13.
13a) (i) Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Correct sin graph.	1
• Correct cos graph.	1

Answer:

13a) (ii) Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Correctly verified.	1

Answer:

At $x = \dfrac{\pi}{6}$, $y = \sin 2\left(\dfrac{\pi}{6}\right)$, $y = \cos\left(\dfrac{\pi}{6}\right)$

$\qquad\qquad y = \dfrac{\sqrt{3}}{2}$ $\qquad y = \dfrac{\sqrt{3}}{2}$

At $x = \dfrac{\pi}{2}$, $y = \sin 2\left(\dfrac{\pi}{2}\right)$, $y = \cos\left(\dfrac{\pi}{2}\right)$

$\qquad\qquad y = 0$ $\qquad\qquad y = 0$

13a) (iii)   Outcome Assessed: H5, H8

Marking Guidelines

Criteria	Marks
• Correct integral.	1
• Correct integration and substitution.	1
• Correct solution	1

Answer:

$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\sin 2x - \cos x\right)dx$$

$$= \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[-\frac{1}{2}\cos\pi - \sin\frac{\pi}{2}\right] - \left[-\frac{1}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\right]$$

$$= \left(\frac{1}{2}-1\right) - \left(-\frac{1}{4}-\frac{1}{2}\right)$$

$$= \left(-\frac{1}{2}\right) - \left(-\frac{3}{4}\right)$$

$$= \frac{1}{4}$$

13b) (i) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correctly shown	1

Answer:

$A_1 = 35000 + 35000 \times 0.007 - R$

$\quad = 35000(1.007) - R$

$A_2 = A_1(1.007) - R$

$\quad = \left(35000(1.007) - R\right)(1.007) - R$

$\quad = 35000(1.007)^2 - R(1.007) - R$

$\quad = 35000(1.007)^2 - R(1 + 1.007)$

$A_3 = A_2(1.007) - R$

$\quad = \left(35000(1.007)^2 - R(1.007) - R\right)(1.007) - R$

$\quad = 35000(1.007)^3 - R(1.007)^2 - R(1.007) - R$

$\quad = 35000(1.007)^3 - R\left(1 + 1.007 + 1.007^2\right)$

13b) (ii) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correct expanded expression .	1
• Correct sum of GP.	1

Answer:

$A_n = 35000(1.007)^n - R\left(1 + 1.007 + 1.007^2 + 1.007^3 + \ldots + 1.007^{n-1}\right)$

$\quad = 35000(1.007)^n - R\left(\dfrac{1(1.007^n - 1)}{1.007 - 1}\right)$

$\quad = 35000(1.007)^n - R\left(\dfrac{1(1.007^n - 1)}{0.007}\right)$

$\quad = 35000(1.007)^n - \dfrac{1000R(1.007^n - 1)}{7}$

13b) (iii) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correct term and equating to 0.	1
• Correct solution.	1

Answer:

$A_n = 35000(1.007)^n - \dfrac{1000R(1.007^n - 1)}{7}$

$A_{60} = 35000(1.007)^{60} - \dfrac{1000R(1.007^{60} - 1)}{7}$

$A_{60} = 0$

$0 = 35000(1.007)^{60} - \dfrac{1000R(1.007^{60} - 1)}{7}$

$\dfrac{1000R(1.007^{60} - 1)}{7} = 35000(1.007)^{60}$

$R = \dfrac{35000(1.007)^{60} \times 7}{1000(1.007^{60} - 1)}$

$\quad = \$716.39$

13b) (iv) Outcome Assessed: H3, H5

Marking Guidelines

Criteria	Marks
• Correct equation involving 800.	1
• Correct solution.	1

Answer:

$A_n = 35000(1.007)^n - \dfrac{1000R(1.007^n - 1)}{7}$

$0 = 35000(1.007)^n - \dfrac{1000 \times 800(1.007^n - 1)}{7}$

$35000(1.007)^n = \dfrac{1000 \times 800(1.007^n - 1)}{7}$

$245(1.007)^n = 800(1.007)^n - 800$

$555(1.007)^n = 800$

$(1.007)^n = \dfrac{160}{111}$

$n\ln(1.007) = \ln\left(\dfrac{160}{111}\right)$

$n = 52.4174 \text{ months}$

13c) Outcome Assessed: P4

Marking Guidelines

Criteria	Marks
• Correct vertex.	1
• Correct focus.	1

Answer:

Vertex $= (3, -1)$

Focus $= (0, -1)$

Question 14.

14a) (i) Outcome Assessed: H5, H8

Marking Guidelines

Criteria	Marks
• Correct velocity equation.	1
• Correct time.	1

Answer:

$a = 1 - 2t$

$v = t - t^2 + c$

$t = 0, \ v = 0 \ \therefore c = 0$

$v = t - t^2$

$v = 0$

$0 = t - t^2$

$\quad = t(1 - t)$

$t = 0, \ 1$

Particle at rest after 1 second.

14a) (ii) Outcome Assessed: H5, H8

Marking Guidelines

Criteria	Marks
• Correct distance equation.	1
• Correct position.	1

Answer:

$x = \frac{1}{2}t^2 - \frac{1}{3}t^3 + k$

$t = 0, \ x = 2 \ \therefore k = 2$

$x = \frac{1}{2}t^2 - \frac{1}{3}t^3 + 2$

$t = 1$

$x = \frac{1}{2} - \frac{1}{3} + 2$

$\quad = 2\frac{1}{6}$

14b) (i) Outcome Assessed: P4

Marking Guidelines

Criteria	Marks
• Correct solution.	1

Answer:

$21 \times 12 = 252$

14b) (ii) Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Correct identification of a term of an AP.	1
• Correct solution.	1

Answer:

$T_{252} = 10 + 251 \times 10$

$\quad = \$2520$

14b) (iii) Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Correct identification of a sum of an AP.	1
• Correct solution.	1

Answer:

$S_{252} = \frac{252}{2}(20 + 251 \times 10)$ $\qquad$ $S_{252} = \frac{252}{2}(10 + 2520)$

$\quad = \$318780$ $\qquad\qquad\qquad\quad = \$318780$

14c) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correct probability for neither get a seat on 1 day.	1
• Correct probability for neither get a seat on 3 day.	1
• Correct solution	1

Answer:

Peter $- P(\bar{S}) = \frac{7}{8},$ $\qquad$ Paul $- P(\bar{s}) = \frac{20}{21}$

$P(S\bar{s}) = \frac{7}{8} \times \frac{20}{21}$

$\qquad = \frac{5}{6}$

3 Days $- P(S\bar{s})^3 = \left(\frac{5}{6}\right)^3$

$\qquad\qquad = \frac{125}{216}$

$P(\text{at least 1 gets seat}) = 1 - \frac{125}{216}$

$\qquad\qquad\qquad = \frac{91}{216}$

14d) Outcome Assessed: H5

Marking Guidelines

Criteria	Marks
• Correct factorization.	1
• Correct solutions for $\cos x$.	1
• Correct solutions for sin function.	1

Answer:

$2\sin x \cos x = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad\qquad$ $2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \ \frac{3\pi}{2}$ $\qquad\qquad$ $\sin x = \frac{1}{2}$

$\qquad\qquad\qquad\qquad x = \frac{\pi}{6}, \ \frac{5\pi}{6}$

Question 15.

15a) (i) Outcome Assessed: H3, H5

Marking Guidelines

Criteria	Marks
• Substituting and forming correct exponential equation.	1
• Correct expression for k.	1

Answer:

$$P = 3000e^{-kt}$$

$$t = 10, \quad P = 2750$$

$$2750 = 3000e^{-10k}$$

$$\frac{2750}{3000} = e^{-10k}$$

$$\ln\left(\frac{11}{12}\right) = -10k$$

$$k = \ln\left(\frac{11}{12}\right) \div -10$$

$$k = 0.0087$$

15a) (ii) Outcome Assessed: H3, H5

Marking Guidelines

Criteria	Marks
• Substituting correct time	1
• Correct solution.	1

Answer:

$$P = 3000e^{-0.0087t}$$

$$t = 20$$

$$P = 3000e^{-0.0087 \times 20}$$

$$= 2520.89..$$

$$= 2521$$

15b) (i) Outcome Assessed: P4

Marking Guidelines

Criteria	Marks
• Correctly shown	1

Answer:

$$\begin{cases} y = x^2 - 3x - 5 \\ y = x + k \end{cases}$$

$$x^2 - 3x - 5 = x + k$$

$$x^2 - 4x - 5 - k = 0$$

15b) (ii) Outcome Assessed: P4

Marking Guidelines

Criteria	Marks
• Recognising the need for the discriminant and equating to zero.	1
• Correct solution	1

Answer:

As only 1 point of intersection the discriminant = 0.

$$\Delta = 16 - 4(-5 - k)$$

$$0 = 16 + 20 + 4k$$

$$4k = -36$$

$$k = -9$$

15b) (iii) Outcome Assessed: P4

Marking Guidelines

Criteria	Marks
• Correct value of x.	1
• Correct value of y.	1

Answer:

$$x^2 - 4x - 5 + 9 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x = 2$$

$$y = (2)^2 - 3(2) - 5 \quad \text{or} \quad y = 2 - 9$$

$$y = -7 \qquad\qquad\qquad y = -7$$

$$P(2, -7)$$

15c) (i) Outcome Assessed: H3, H6

Marking Guidelines

Criteria	Marks
• Correct x value of stationary point	1
• Correct minimum	1

Answer:

$$y = xe^x$$

$$y' = e^x + xe^x$$

$$= e^x(1 + x)$$

Stationary points occur when $y' = 0$

$$0 = e^x(1 + x)$$

$$x = -1, y = -e^{-1}$$

$$y'' = e^x + e^x + xe^x$$

$$= 2e^x + xe^x$$

$$= e^x(2 + x)$$

At $x = -1$

$$y'' = e^{-1} > 0$$

$$\therefore \text{ Minimum at } \left(-1, -e^{-1}\right)$$

15c) (ii) Outcome Assessed: H3, H6

Marking Guidelines

Criteria	Marks
• Correct Point of Inflection	1
• Correct test	1

Answer:

$y'' = 2e^x + xe^x$

$\quad = e^x(2+x)$

$0 = e^x(2+x)$

$x = -2, \quad y = -2e^{-2}$

x	<-2	-2	>-2
y''	$y'' < 0$	0	$y'' > 0$

15c) (iii)   Outcome Assessed: H6

Marking Guidelines

Criteria	Marks
• Correct shape & graph through origin	1
• Minimum & POI labelled	1

Answer:

$\left(-2,-2e^{-2}\right) \left(-1,-e^{-1}\right)$
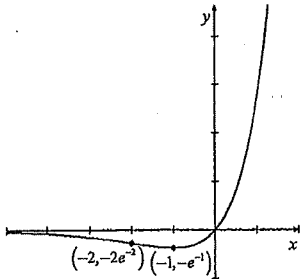
Question 16.

16a)   Outcome Assessed: H8

Marking Guidelines

Criteria	Marks
• Correct statement for volume	1
• Correct integration	1
• Correct answer	1

Answer:

$y = \sqrt{x-1}$

$y^2 = x-1$

$x = y^2 + 1$

$V = \pi \int_1^3 \left(y^2+1\right)^2 dy$

$\quad = \pi \int_1^3 \left(y^4 + 2y^2 + 1\right) dy$

$\quad = \pi \left[\dfrac{y^5}{5} + \dfrac{2y^3}{3} + y\right]_1^3$

$\quad = \pi \left(\left(\dfrac{3^5}{5} + \dfrac{2(3^3)}{3} + 3\right) - \left(\dfrac{1}{5} + \dfrac{2}{3} + 1\right)\right)$

$\quad = \pi \left(69\dfrac{3}{5} - 1\dfrac{13}{15}\right)$

$\quad = \dfrac{1016\pi}{15} u^3$

16b)(i) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correctly shown	1

Answer:

$\dfrac{dH}{dt} = 1 - \dfrac{t}{20}$

$H = t - \dfrac{t^2}{40} + c$

$\quad t = 0, \quad H = 5 \quad \therefore c = 5$

$H = t - \dfrac{t^2}{40} + 5$

16b)(ii)   Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Time when maximum occurs	1
• Maximum Height	1

Answer:

$$\frac{dH}{dt} = 1 - \frac{t}{20}$$

$$0 = 1 - \frac{t}{20}$$

$$\frac{t}{20} = 1$$

$$t = 20$$

$$H = -\left(\frac{20^2}{40}\right) + 20 + 5$$

$$= 15 \text{ metres}$$

16b)(iii)   Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Equating Height equation to 12.5	1
• Solving equation for the 2 times.	1
• Mark for opening and closing of bridge	1

Answer:

$$12.5 = -\frac{t^2}{40} + t + 5$$

$$\frac{t^2}{40} - t + 7.5 = 0$$

$$t^2 - 40t + 300 = 0$$

$$(t - 10)(t - 30) = 0$$

$$t = 10, \ 30 \text{ hours.}$$

Bridge blocked – 10am Sunday + 10 hours = 8 pm Sunday
Bridge opened – 8 pm Sunday + 20 hours = 4 pm Monday

16c)(i)   Outcome Assessed: H2, H4, H5

Marking Guidelines

Criteria	Marks
• Time to BR	1
• Correct shown.	1

Answer:

$$\text{Distance } BR = \sqrt{x^2 + 2^2}$$

$$= \sqrt{x^2 + 4}$$

$$\text{Time to } BR = \frac{\sqrt{x^2 + 4}}{8}$$

$$\text{Distance } BJ = 6 - x$$

$$\text{Time to } BJ = \frac{6 - x}{16}$$

$$\text{Total time } T = \frac{\sqrt{x^2 + 4}}{8} + \frac{6 - x}{16}$$

16c)(ii) Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Differentiate and equate to zero.	1
• Value of x.	1
• Distance BJ	1

Answer:

$$T = \frac{\sqrt{x^2 + 4}}{8} + \frac{6 - x}{16}$$

$$T' = \frac{1}{2}\left(x^2 + 4\right)^{-\frac{1}{2}} \times 2x \times \frac{1}{8} - \frac{1}{16}$$

$$= \frac{x}{8\sqrt{x^2 + 4}} - \frac{1}{16}$$

$$0 = \frac{x}{8\sqrt{x^2 + 4}} - \frac{1}{16}$$

$$\frac{1}{16} = \frac{x}{8\sqrt{x^2 + 4}}$$

$$\sqrt{x^2 + 4} = 2x$$

$$x^2 + 4 = 4x^2$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}}$$

$$BJ = 6 - \frac{2}{\sqrt{3}}$$

16c)(iii)   Outcome Assessed: H4, H5

Marking Guidelines

Criteria	Marks
• Correct time	1

Answer:

$$T = \frac{\sqrt{x^2+4}}{8} + \frac{6-x}{16}$$

$$x = \frac{2}{\sqrt{3}}$$

$$T = \frac{\sqrt{\left(\frac{2}{\sqrt{3}}\right)^2+4}}{8} + \frac{6-\frac{2}{\sqrt{3}}}{16}$$

$$= \frac{\sqrt{\frac{4}{3}+4}}{8} + \frac{6-\frac{2\sqrt{3}}{3}}{16}$$

$$= \frac{4}{8\sqrt{3}} + \frac{18-2\sqrt{3}}{48}$$

$$= \frac{4\sqrt{3}}{24} + \frac{9-\sqrt{3}}{24}$$

$$= \frac{9+3\sqrt{3}}{24}$$

$$= \frac{3+\sqrt{3}}{8}$$

$$= 0.591506 \text{ hour}$$

$$= 35 \text{ minutes } 29 \text{ seconds}$$