35 Functions and mappings

Question 1   For each graph, state whether it is a function or a non-function:

(a)

$y = x + 1$

(b)
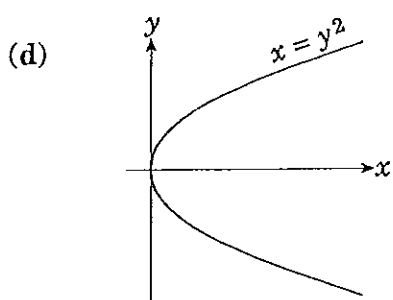
$y = x^2$

(c)

$x^2 + y^2 = 4$

(d)

$x = y^2$

(e)

$y = 2^x$

(f)
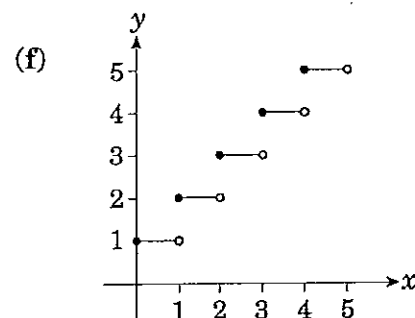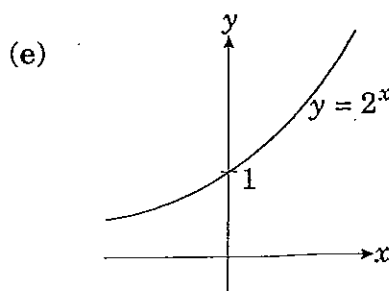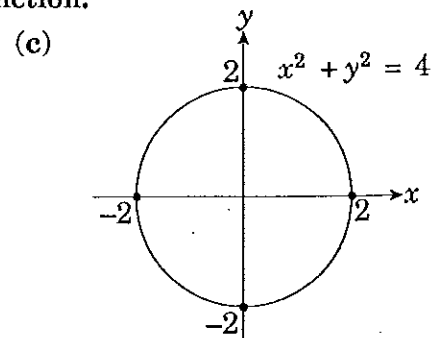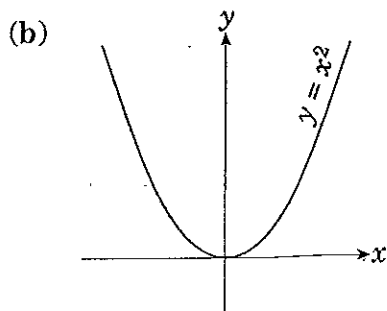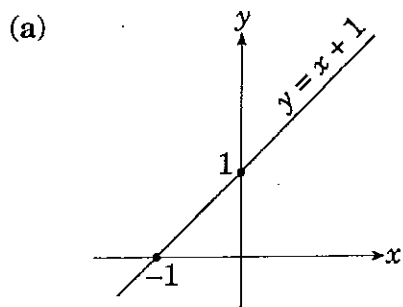
Question 2   If $f(x) = x^2 + 4$, find:

(a) $f(0)$

(b) $f(1)$

(c) $f(2)$

(d) $f(-3)$

Question 3   If $g(x) = \frac{1}{1+x}$, find:

(a) $g(0)$

(b) $g(1)$

(c) $g(a)$

(d) $g\left(\frac{1}{x}\right)$

Question 4   If $F(x) = x + \frac{1}{x}$, show that $F(2) = F\left(\frac{1}{2}\right)$:

Question 5 If $h(x) = 2^{x+1}$, find:

(a) $h(0)$

(b) $h(1)$

(c) $h(a)$

(d) $h(-3)$

Question 6 If $f(x) = 2x^2 - x + 1$, find:

(a) $f(2)$

(b) $f(x+h)$

(c) $f(x+h) - f(x)$

(d) $\dfrac{f(x+h) - f(x)}{h}$

Question 7 $f(x) = x^2 - 1$, $g(x) = 3x - 2$. Find:

(a) $f[g(x)]$

(b) $g[f(x)]$

Question 8 $f(x) = 3x + 4$ Find $f^{-1}(x)$ where $f^{-1}(x)$ is the inverse function of $f(x)$.

Question 9 $g(x) = \dfrac{7x+2}{3}$ If $g^{-1}(x)$ is the inverse function of $g(x)$, find:
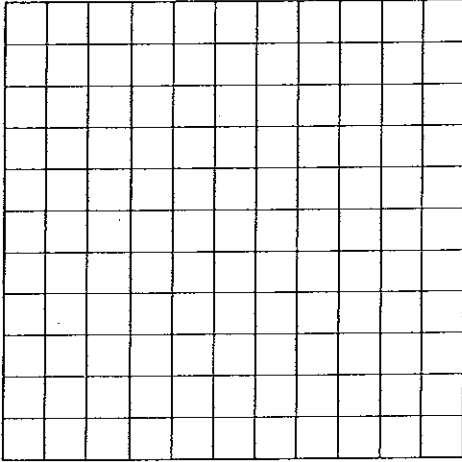
(a) $g^{-1}(x)$

(b) $g^{-1}(0)$

(c) $g^{-1}(2)$

(d) $g^{-1}(-2)$

Question 10 $\triangle PQR$ has coordinates $P(1,0)$, $Q(2,3)$, $R(3,1)$. In each case draw $\triangle PQR$ and its image using the given mapping.

(a) f: $(x, y) \rightarrow (-x, -y)$

(b) g: $(x, y) \rightarrow (y, x)$
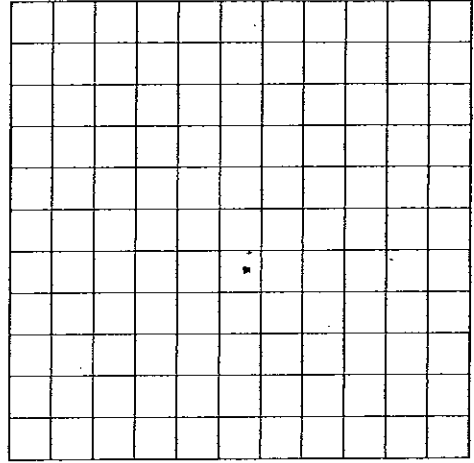
(c) h: $(x, y) \rightarrow (-y, x)$

(d) k: $(x, y) \rightarrow (-y, -x)$

Question 11 If h: $x \rightarrow 2x + 1$, g: $x \rightarrow x^2$, find:

(a) $hog(1)$

(b) $goh(1)$

(c) $hog(-2)$

(d) $goh(-2)$

(e) $h^2(-1)$

(f) $g^2(-1)$

Question 10

(a)

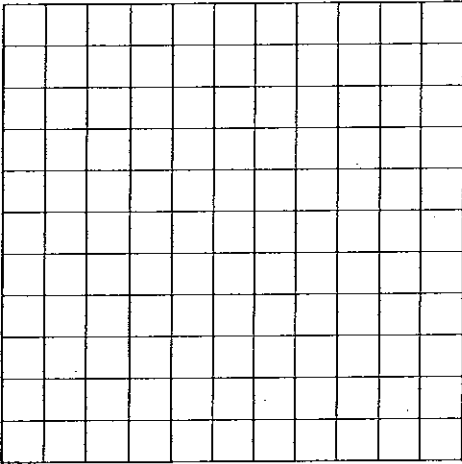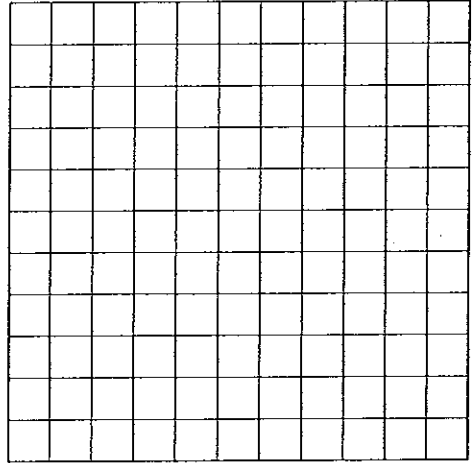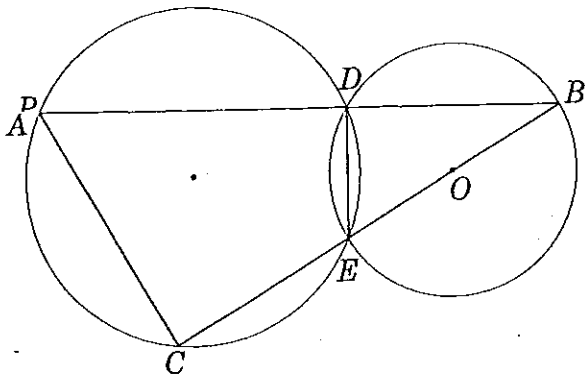*BE* is a diameter of the smaller circle.
Prove that $\angle ACB = 90°$.

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(b)
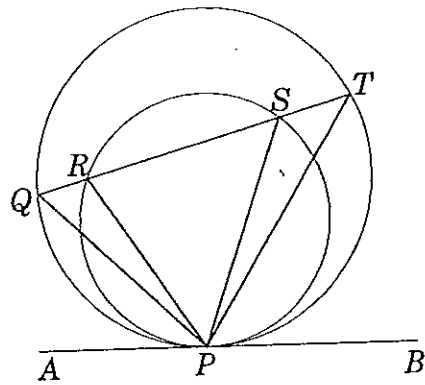
AB is a tangent.
Prove that $\angle QPR = \angle SPT$.

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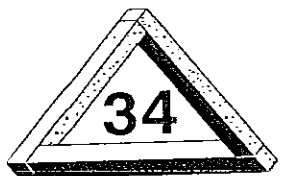
34 Logarithms

Question 1   Solve the following exponential equations:

(a) $3^{2x+1} = 81$

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(b) $8^x = 4$

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(c) $3^{x-1} = \dfrac{1}{27}$

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(d) $9^x = 243$

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(e) $4^{x-1} = \sqrt{2}$

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(f) $8^{3x-1} = 16$

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(g) $4^{2x-1} = \left(\dfrac{1}{32}\right)^x$

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(h) $2^{3-x} = 8^{x+1}$

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(i) $\left(3\sqrt{3}\right)^x = \left(9\sqrt{27}\right)^{2-x}$

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Question 2   Write each of the following in logarithmic form:

(a) $64 = 2^6$

(b) $100 = 10^2$

(c) $125 = 5^3$

(d) $3 = 2^x$

(e) $5 = 10^x$

(f) $15 = 4^x$

Question 3   Evaluate the following:

(a) $\log_2 16$

(b) $\log_9 81$

(c) $\log_5 625$

(d) $\log_9 27$

(e) $\log_{25} 125$

(f) $\log_8 32$

Question 4   Solve the following for the variable:

(a) $\log_2 32 = x$

(b) $\log_m 27 = 3$

(c) $\log_{16} 64 = y$

(d) $\log_p 125 = 3$

(e) $\log_n 6\frac{1}{4} = 2$

(f) $\log_x 8 = 3$

(g) $\log_y 8 = \frac{1}{4}$

(h) $\log_2 m = 7$

(i) $\log_{3\frac{1}{2}} q = 2$

Question 5   Evaluate the following:

(a) $\log_{10} 25 + \log_{10} 4$

(b) $\log_5 1000 - \log_5 8$

(c) $\log_6 12 + \log_6 3$

(d) $\log_4 32 + \log_4 2$

(e) $\log_{25} 300 - \log_{25} 12$

(f) $2\log_6 2 + 2\log_6 3$

Question 6 Solve the following equations for x:

(a) $\log_3 x = \log_3 7 + \log_3 2$

(b) $\log_a x = 2\log_a 3 - 3\log_a 2$

(c) $\log_a x + \log_a 5 = \log_a (x+1)$

(d) $2\log_3 x = \log_3 49$

(e) $\log_2 x + \log_2 (x+4) = 5$

(f) $\log_{12} 2x + \log_{12}(x-1) = 1$

Question 7 Solve the following exponential equations correct to 4 decimal places:

(a) $3^x = 7$

(b) $5^x = 15$

(c) $2^{x+1} = 9$

(d) $3^x = 6^{x-1}$

(e) $6^{x+2} = 10^{x-7}$

(f) $3^{x+1} = 5^{x-1}$

(d) $x = 95°$, $y = 93°$

(e) $a = 100°$, $b = 160°$

(f) $m = 50°$, $n = 65°$

(a) 10 cm   (b) 8 cm   (c) 24 cm

and 4 Proofs

(a) 42 cm   (b) $x = 15$ cm, $y = 68°$

Proofs

(a) 36°   (b) $a = 58°$, $b = 58°$ $c = 58°$, $d = 39°$

Proofs

(a) 3 cm   (b) 6 cm

Proofs

34 Logarithms

(a) $x = \frac{3}{2}$ (b) $x = \frac{2}{3}$ (c) $x = -2$

(d) $x = \frac{5}{2}$ (e) $x = \frac{5}{4}$ (f) $x = \frac{7}{9}$

(g) $x = \frac{2}{9}$ (h) $x = 0$ (i) $x = \frac{7}{5}$

(a) $\log_2 64 = 6$ (b) $\log_{10} 100 = 2$

(c) $\log_5 125 = 3$ (d) $\log_2 3 = x$

(e) $\log_{10} 5 = x$ (f) $\log_4 15 = x$

(a) 4   (b) 2   (c) 4   (d) $\frac{3}{2}$

(e) $\frac{3}{2}$ (f) $\frac{5}{3}$ 4 (a) 5   (b) 3

(c) $\frac{3}{2}$ (d) 5   (e) $\frac{5}{2}$ (f) 2

(g) 4096   (h) 128   (i) $12\frac{1}{4}$

(a) 2   (b) 3   (c) 2   (d) 3

(e) 1   (f) 2

(a) $x = 14$ (b) $x = \frac{9}{8}$ (c) $x = \frac{1}{4}$

(d) $x = 7$ (e) $x = 4$ (f) $x = 3$

(a) 1·7712   (b) 1·6826   (c) 2·1699

(d) 2·5850   (e) 38·5682   (f) 5·3013

35 Functions and mappings

(a) function   (b) function   (c) non-function
(d) non-function   (e) function   (f) function

(a) 4   (b) 5   (c) 8   (d) 13

(a) 1   (b) $\frac{1}{2}$ (c) $\frac{1}{1+a}$ (d) $\frac{x}{x+1}$

Proof

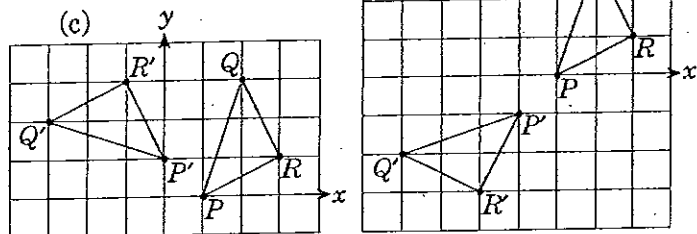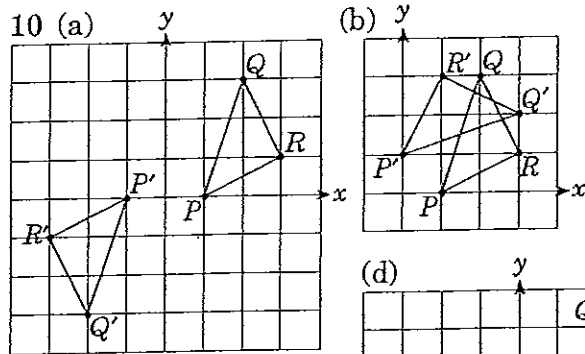(a) 2   (b) 4   (c) 2^{a+1} (d) $\frac{1}{4}$

(a) 7   (b) $2x^2 + 4xh + 2h^2 - x - h + 1$

(c) $2h^2 + 4xh - h$ (b) $2h + 4x - 1$

(a) $9x^2 - 12x + 3$ (b) $3x^2 - 5$

(a) $\frac{1}{3}x - \frac{4}{3}$ 9 (a) $\frac{3x-2}{7}$

(b) $-\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $-\frac{8}{7}$

10 (a)

(b)

(c)

(d)

11 (a) 3   (b) 9   (c) 9   (d) 9   (e) −1   (f) 1

36 Polynomials

1 (a) deg. = 3, l.c. = 2   (b) deg. = 4, l.c. = 5

   (c) deg. = 5, l.c. = −6   (d) deg. = 2, l.c. = $\frac{1}{3}$

   (e) deg. = 3, l.c. = 1   (f) deg. = 6, l.c. = $-\frac{1}{10}$

2 (a) $7x^3 - 4x^2 + 3x - 2$

   (b) $5x^3 - 5x^2 + 5x - 8$

   (c) $14x^4 + 3x^3 + 8x^2 + x + 9$

   (d) $6x^4 - 4x^3 + x^2 + 3x - 5$

   (e) $x^3 + x^2 - 12$

   (f) $6x^4 - x^3 - 9x^2 - x + 1$

3 (a) $2x^4 - 9x^3 - x^2 + 2x$

   (b) $2x^4 + 17x^3 - 6x^2 - 3x + 2$

   (c) $12 - 25x - 11x^2 + 2x^3 - 7x^4 - x^5$

   (d) $2x^6 + 19x^5 + 30x^4 + 8x^3 - 4x^2 + 29x + 12$

4 (a) $x^2 + 3x + 8 + \frac{7}{x-1}$

   (b) $3x^3 - 13x^2 + 34x - 70 + \frac{135}{x+2}$

   (c) $5x^2 + 8x + 25 + \frac{82}{x-3}$

   (d) $2x^2 + 2x - 5 + \frac{7}{2x+1}$

   (e) $7x^2 - 3x - 29 + \frac{22x + 143}{x^2 + 5}$

   (f) $x + 15 + \frac{101x - 13}{x^2 - 7x + 1}$

   (g) $5x^2 + 17x + 36 + \frac{69}{x-2}$

   (h) $7x^4 - 6x^3 + 4x^2 - 4x + 5 - \frac{2}{x+1}$

5 (a) $P(1) = 7$ (b) $P(-2) = 34$

   (c) $P(3) = 463$ (d) $P(-1) = 8$

   (e) $P(2) = 11$ (f) $P(2) = 31$

6 (a) $P(-2) = 0$ (b) $P(2) = 0$