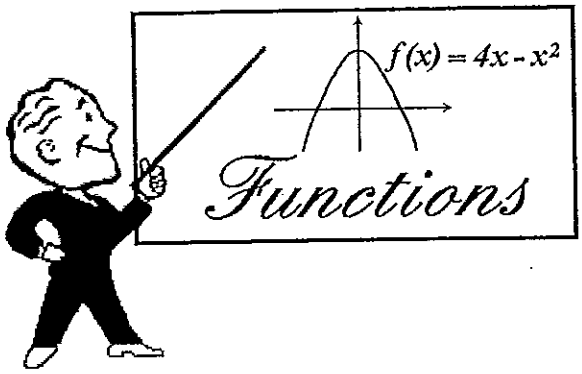


We saw in the last chapter that equations with 2 variables have an infinite set of solutions that are ordered pairs. These equations where called linear because the variables had no powers and their graphs were straight lines.

In this chapter we will study the graphs of non-linear equations (whose graphs are *curves*), including a special type called a function.

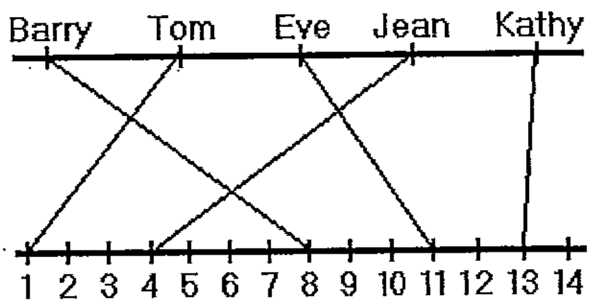
<u>RELATIONS</u>: - Any set of ordered pairs is called a relation.
Where elements x_i from one set, X, are associated with (paired with) elements y_i from another set Y - expressed in a consistent order (x_i, y_i)

Eg 1: { (Kathy,13) , (Eve,11) , (Barry,8) , (Jean,4) , (Tom,1) }

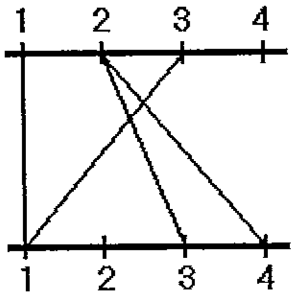
Eg 2: { (1,1) , (2,4) , (3,1) , (2,3) }

Eg 3: { $(x,y) : x + y = 7$ } , this set has an infinite number of such ordered pairs.

<u>Eg 1.</u>



<u>Eg 2.</u>



FUNCTIONS: - A set of ordered pairs in which all the first elements are different, is called a function. (Eg1. on the previous page is an example of this - Eg2. is not.)

or

A *function* is a relation in which NO two ordered pairs have the same first element. Not all relations are functions.

Domain - The set of all first values (x-values) in the set of ordered pairs is called the *Domain*, we use the letter $\mathcal{D}$ for this set.

Range - The set of all second values (y-values) in the set of ordered pairs is called the *Range*, we use the letter $\mathcal{R}$ for this set.

In the general case, the elements are all members of the set of real numbers.

Independent - the *independent variable*, x, has values from the domain.

Dependent - the *dependent variable*, y, has values from the range. (the y-value depends on the x-value chosen.)

We write $y = f(x)$ - (y is a function of x)

EXAMPLES:

Eg 1. For the function given by { (0,3) , (1,2) , (2,5) , (3,-1) , (4,2) , (5,4) }

The Domain, $\mathcal{D}$, is the set of 1st values . . . $\mathcal{D} = \{0, 1, 2, 3, 4, 5\}$

The Range, $\mathcal{R}$, is the set of 2nd values . . . $\mathcal{R} = \{-1, 2, 3, 4, 5\}$

Eg 2. For the function given by: $\{(x,y) : y = x^2\}$

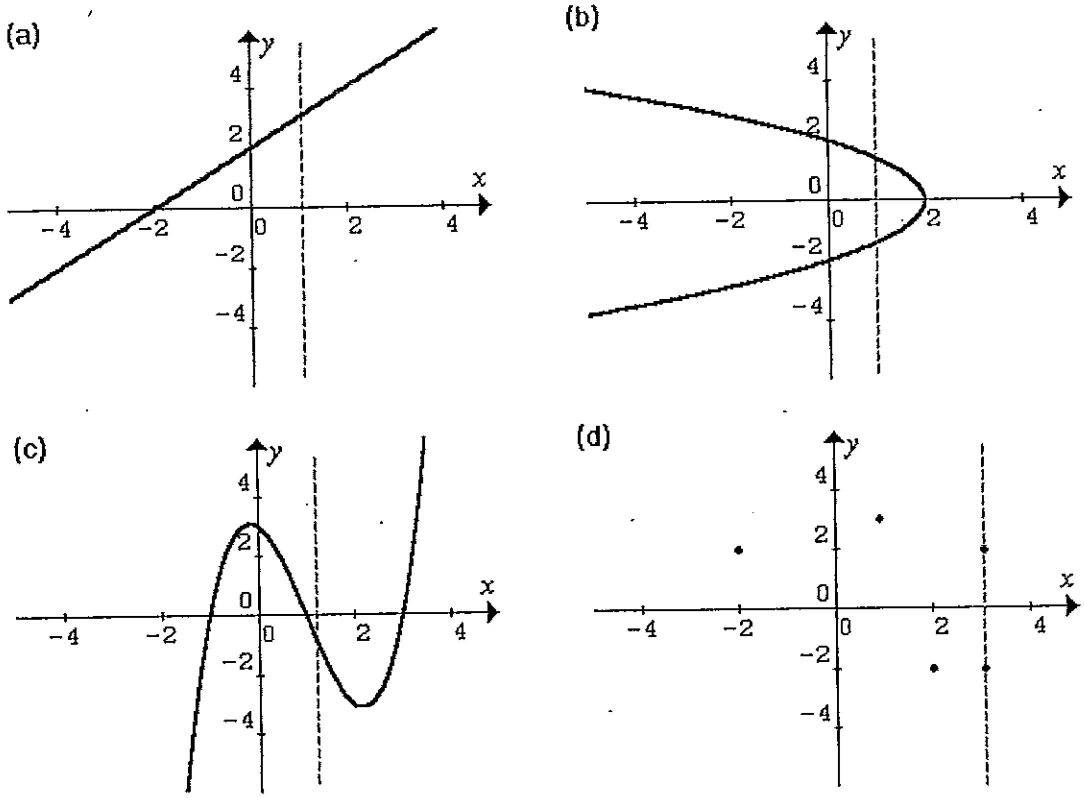
The Domain is the set of all Real numbers. $\mathcal{D} = \{x: -\infty < x < +\infty\}$

The Range is the set of all non-negative real numbers. $\mathcal{R} = \{y: y \geq 0\}$

EXERCISE 35 – Testing and Evaluating a Function:

If a vertical line cuts a graph only ONCE anywhere along the graph, it is the graph of a function.

1. Which of the graphs below are a set of points which form a function?



Answer:...

FUNCTION NOTATION

When an equation is written with y as the subject (explicitly) eg. $y = x^2 - x + 6$, then the expression on the right-hand side of the equation is the "function of x". For this example the function of x is: $f(x) = x^2 - x + 6$.

The *value* of a function for a particular x-value is found by substituting this x-value into the function $f(x) = x^2 - x + 6$.

eg. The value of the above function at $x = 3$ is written and evaluated as follows:
$f(3) = (3)^2 - (3) + 6 = 12$

2. Evaluate: (a) $f(0) =$
(b) $f(5) =$
(c) $f(-4) =$

3. Given $f(x) = 3x - x^2$, evaluate $f(-4) =$

EXERCISE 36 – Domain and Range:

1. List the Domain & Range of the following relations:

(a) { (2,5) , (3,-1) , (3,1) , (1,0)

$\mathcal{D}$ = { } $\mathcal{R}$ = { }

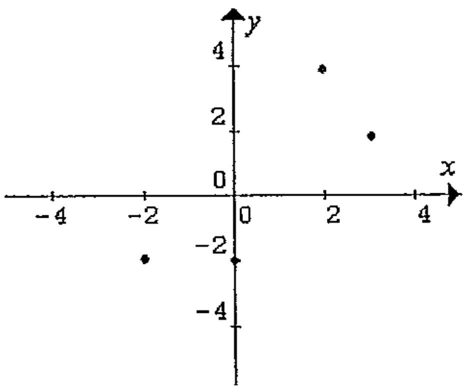
(b) { (-2,3) , (-1,2) , (0,1) , (1,0) , (2,-1)

$\mathcal{D}$ = { } $\mathcal{R}$ = { }

(c) { (1,1) , (2,1) , (0,1) }

$\mathcal{D}$ = { } $\mathcal{R}$ = { }

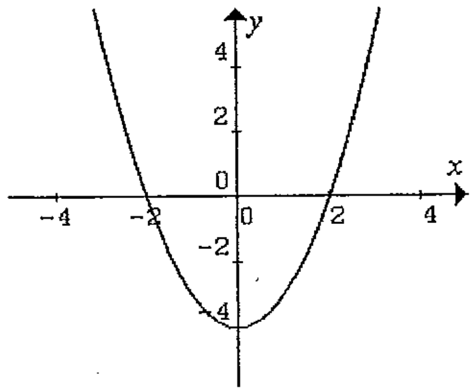
(d)



$\mathcal{D}$ = { }

$\mathcal{R}$ = { }

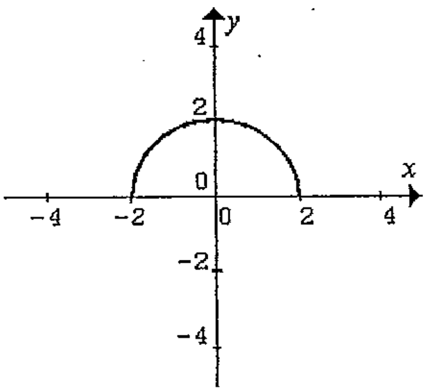
(e)



$\mathcal{D}$ = { }

$\mathcal{R}$ = { }

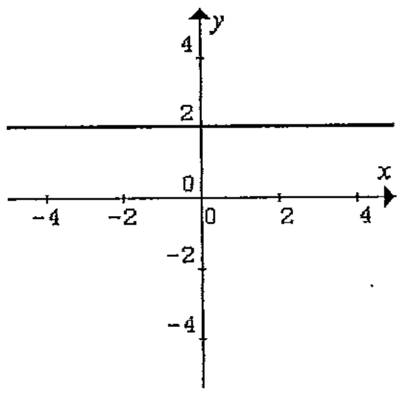
(f)



$\mathcal{D}$ = { }

$\mathcal{R}$ = { }

(g)



$\mathcal{D}$ = { }

$\mathcal{R}$ = { }

2. Which of the above relations are functions (Yes / No)?

(a) (b) (c) (d)

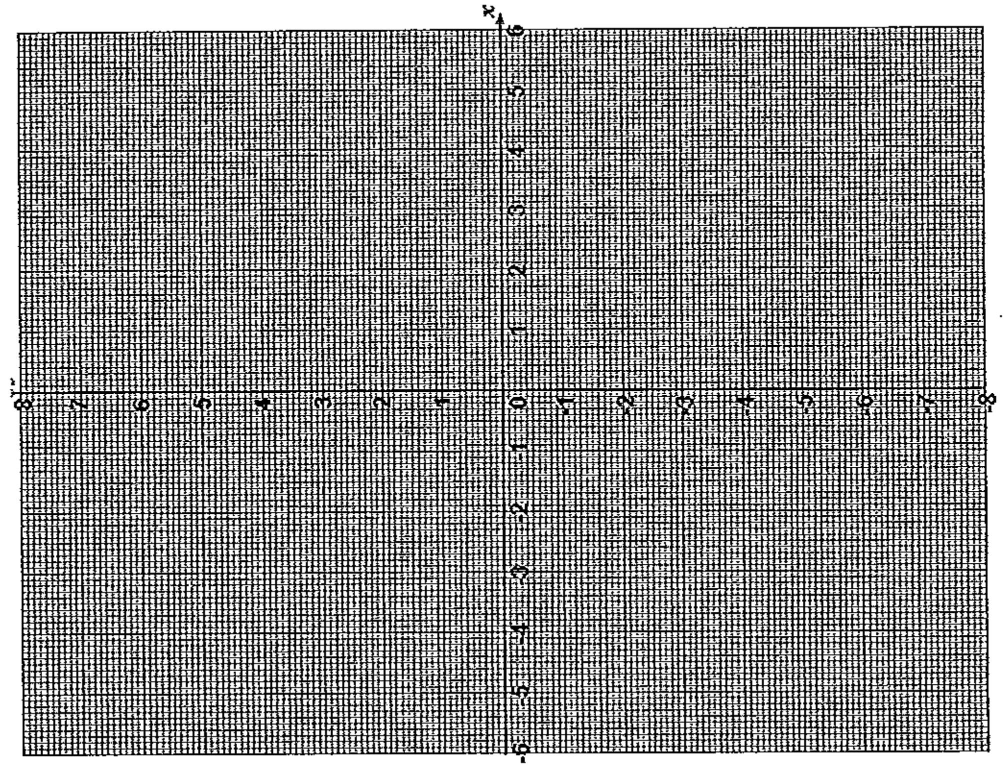
(e) (f) (g)

EXERCISE 37 – Graphing Non-Linear Functions

(1)

$y = 2^x$

x	-3	-2	-1	-.5	0	.5	1	2	3
y									

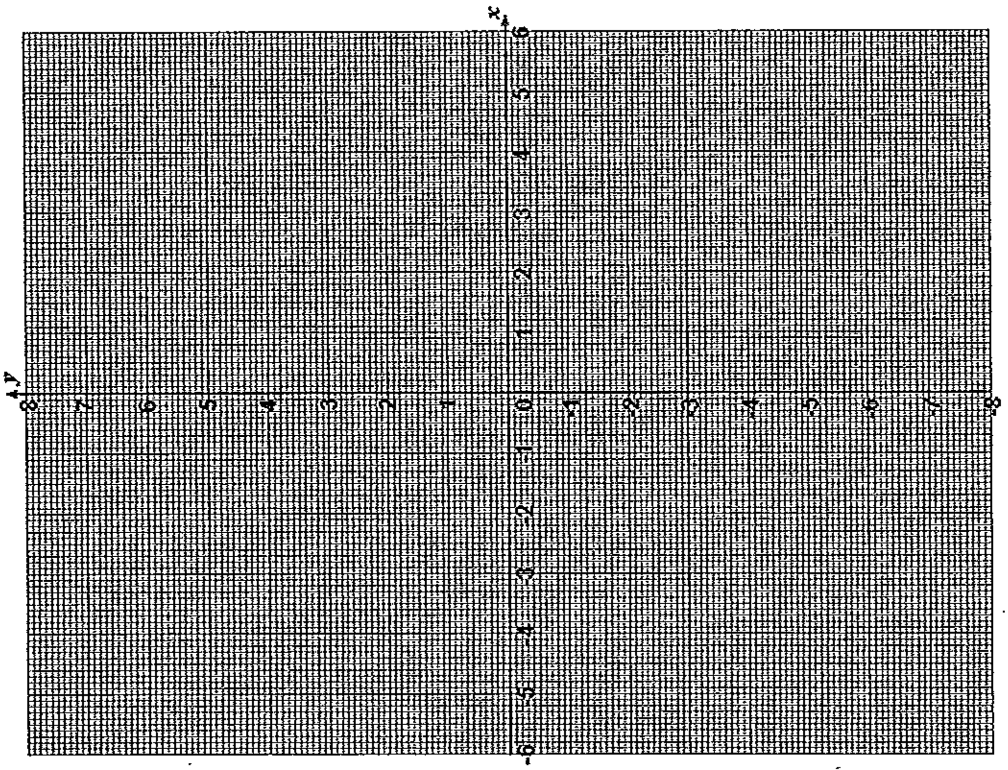


Graph Name is:

(2)

$y = x^2 - 4$

x	-3	-2	-1	-.5	0	.5	1	2	3
y									

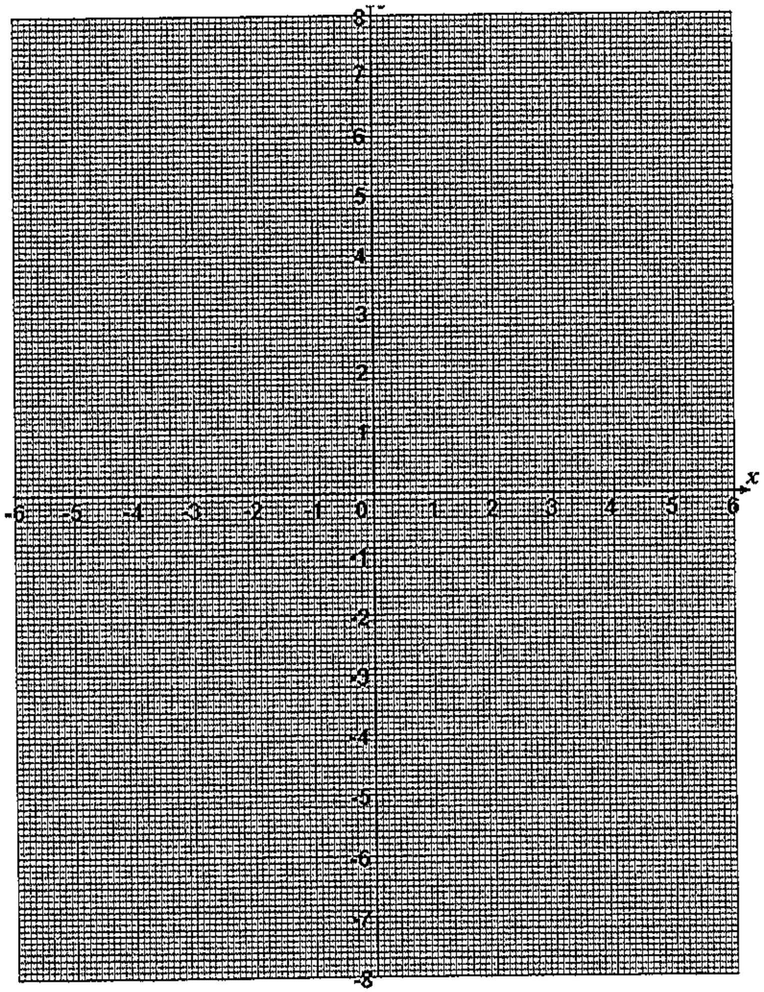


Graph Name is:

(3)

$x^2 + y^2 = 25$

x	-5	-4	-3	-2	0	2	3	4	5
y									

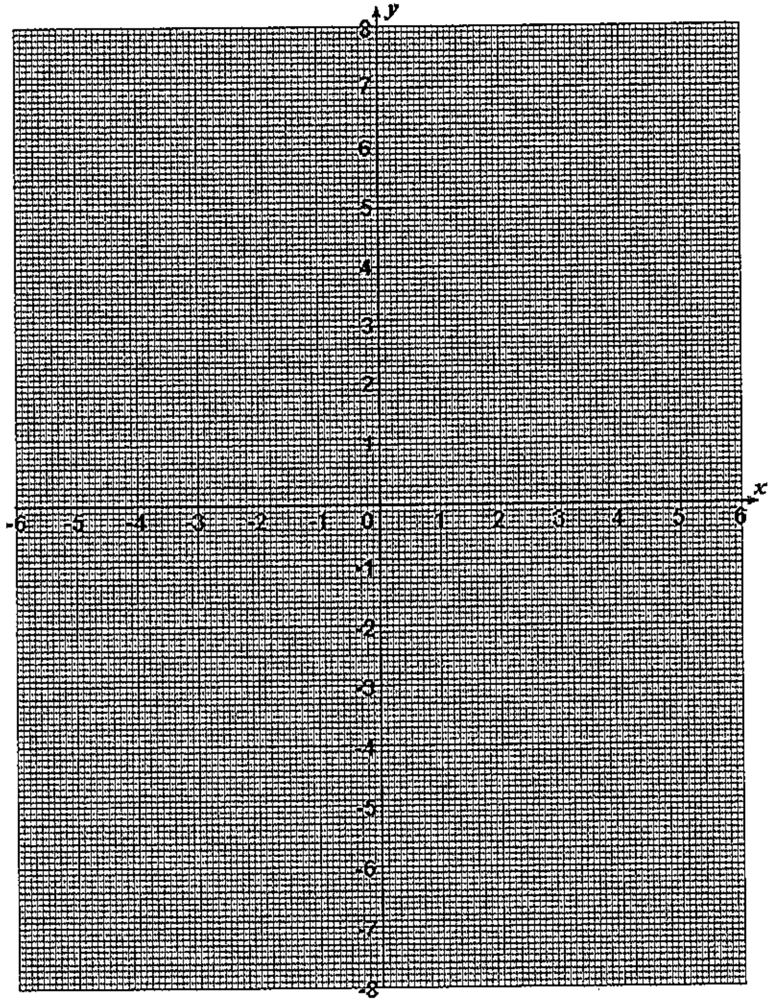


Graph Name is:

(4)

$y = {}^2/_x$

x	-5	-4	-2	-1	-·8	-·4	·4	·8	1	2	4	5
y												

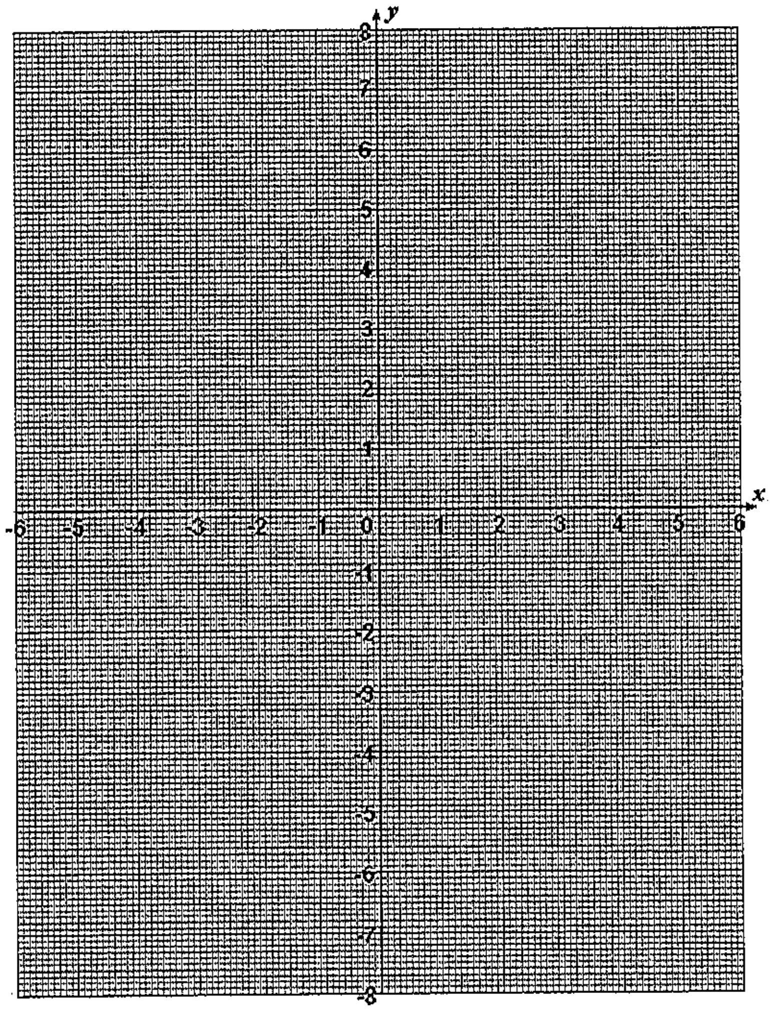


Graph Name is:

(5)

$y = \sqrt{x}$

x	0	0·25	1	2·25	4
y					

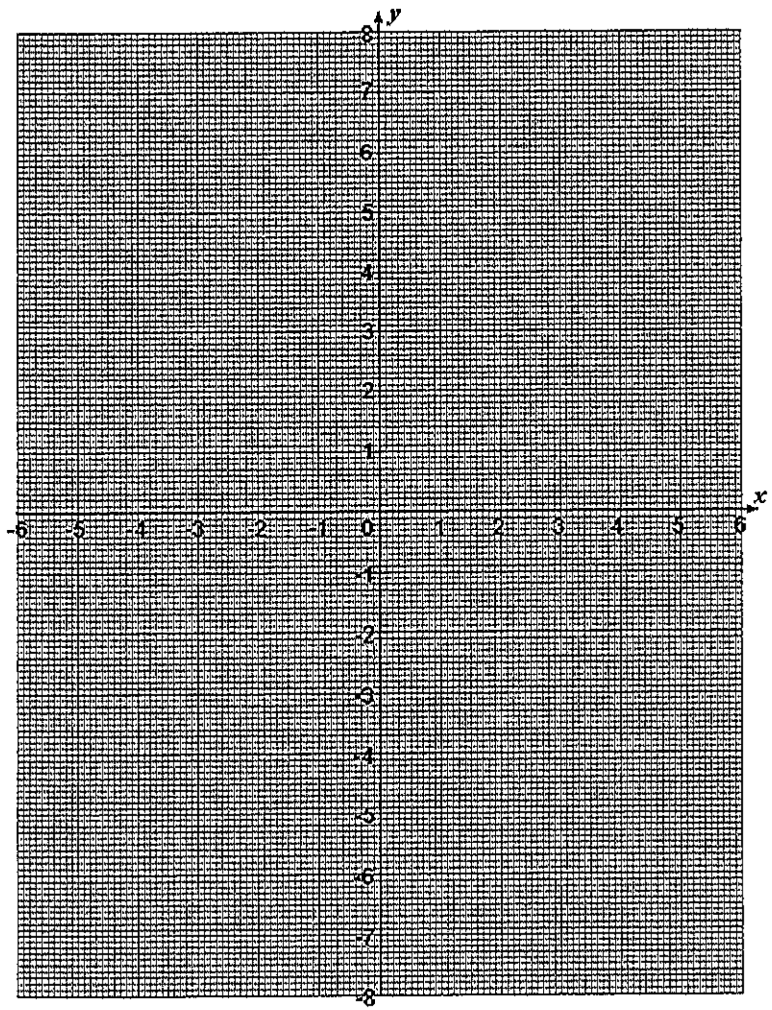


Graph Name is:

(6)

$y = x^3 - 4x$

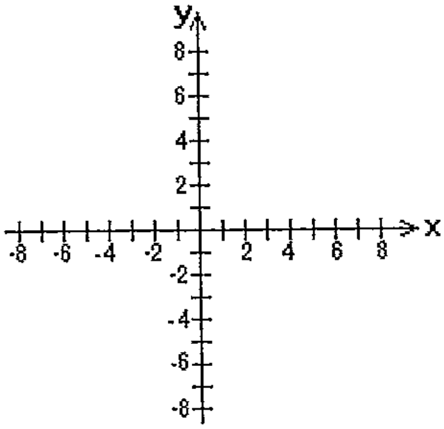
x	-2.5	-2	-1	-0.5	0	0.5	1	2	2.5
y									



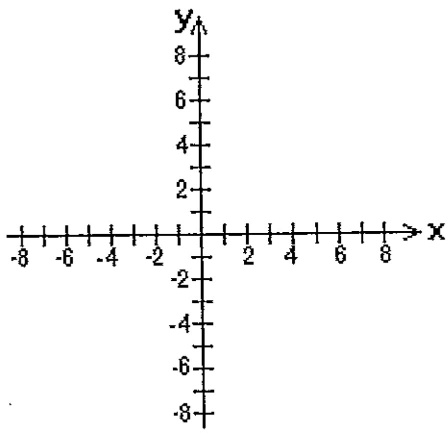
Graph Name is:

Sketch each graph below on the number plane provided. Name each graph.

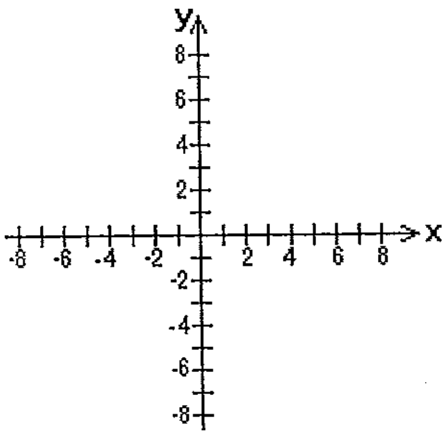
(7) $y = x^2 - 1$



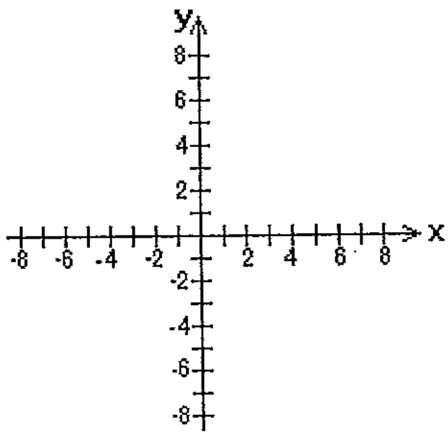
(8) $y = -2^x$



(9) $y = -\frac{1}{x} + 1$



(10) $y = (x-3)^2$

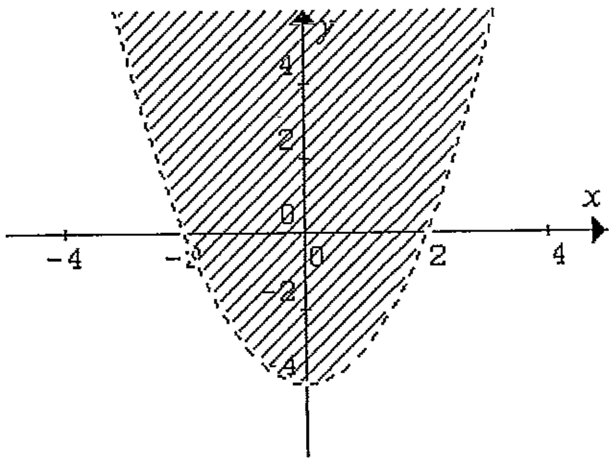


REGIONS:

Inequalities (or inequations) containing 2 variables have an infinite set of solutions which, when graphed, produce a *region* on the Number Plane.

Example: $y > x^2 - 4$

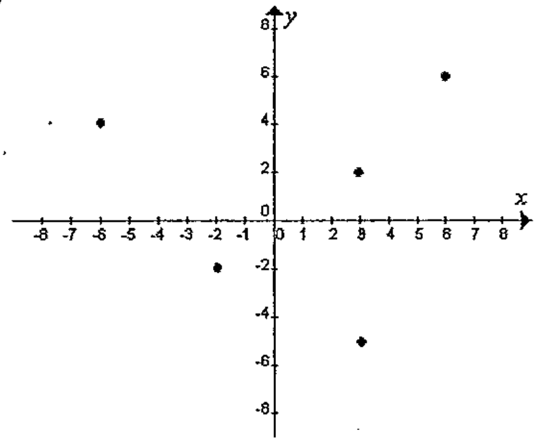
The edge of this region is called the *boundary* of the region, and here it has the equation $y = x^2 - 4$



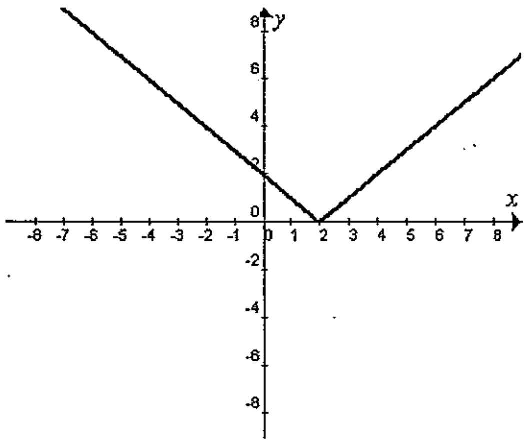
HOMEWORK SHEET (17)

1. Which of the following graphs represent functions?

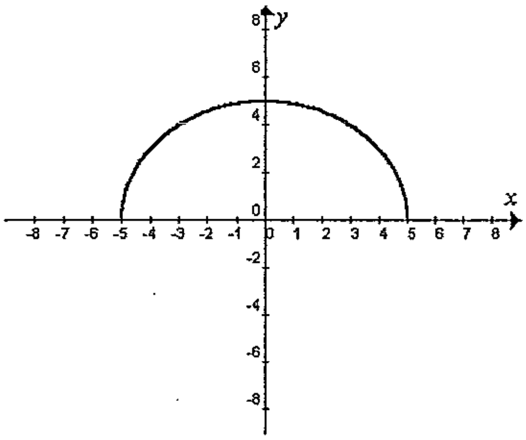
(a)



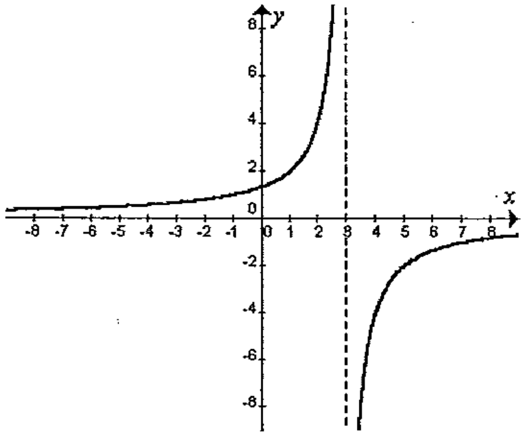
(b)



(c)



(d)



2. Find the domain $\mathcal{D}$ and Range $\mathcal{R}$ of each of the graphs in question1 above.

(a) $\mathcal{D}$ = { }
$\mathcal{R}$ = { }

(b) $\mathcal{D}$ = { }
$\mathcal{R}$ = { }

(c) $\mathcal{D}$ = { }
$\mathcal{R}$ = { }

(d) $\mathcal{D}$ = { }
$\mathcal{R}$ = { }

3. Given the function $f(x) = 2x^2 - 7x + 6$, evaluate:

(a) $f(0) =$

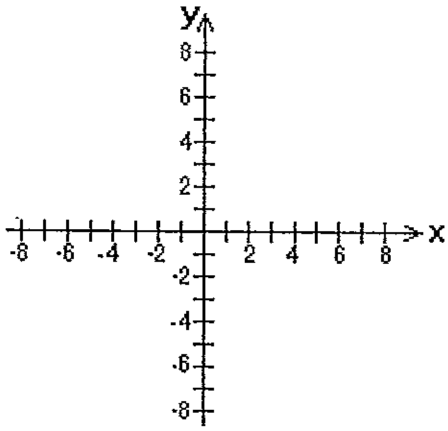
(b) $f(3) =$

(c) $f(-1) =$

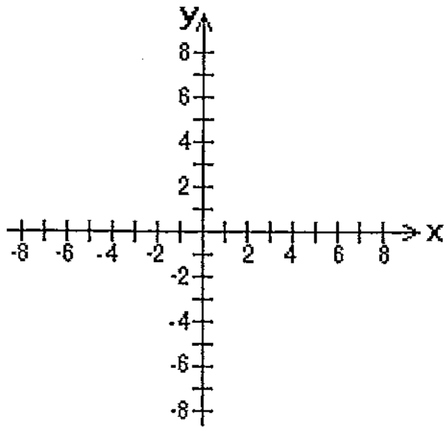
(d) $f(-2) =$

HOMEWORK SHEET (18)

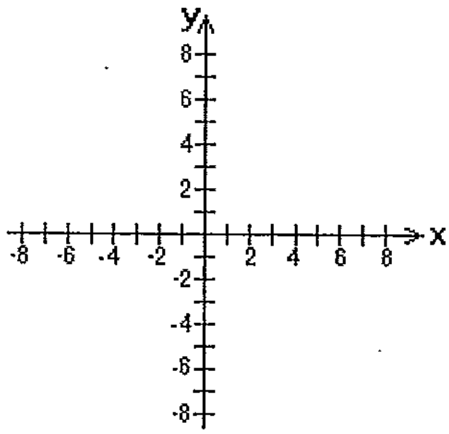
1. (a) $y = -x^2$



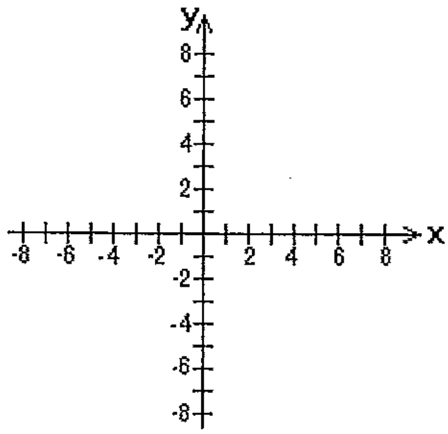
(b) $y = 2^x - 4$



(c) $y = \dfrac{1}{x+2}$



(d) $y = |x| + 3$



2. What are the names of the above functions?

(a) .... (b) ....

(c) .... (d) ....

3. Draw the graph of the inequation:

$y < 4 - 2x$

(note: this graph is a *region*)