1. Sketch the following relations, showing main features, and stating the domain & range.

(a) $xy = 1$	(b) $y = \|x\|$	(c) $y = x^2 - 4$
(d) $x^2 + y^2 = 36$	(e) $y = 2^x$	(f) $y = x^3 - 4x^2$
(g) $y = -\sqrt{49 - x^2}$	(h) $y = -\sqrt{3 - 2x}$	(i) $y = 1 + \frac{3}{x+4}$
(j) $(x-1)^2 + (y+2)^2 = 1$	(k) $y = 1 + \|x+1\|$	(l) $y = x^3 - 4x^2 + 3x$

2. Determine the equation of the axis of symmetry and the coordinates of the vertex of the parabola $y = 3x^2 - 6x + 1$. Hence sketch the parabola and write down its range.

3. For the domain $-1 \leq x \leq 3$, determine the maximum value and the minimum value of each of the functions defined by $f(x) = 3x^2$, $g(x) = 4x - x^2$.
Find all values of x in the interval $-1 \leq x \leq 3$ for which $f(x) = g(x)$.
Sketch the curves $y = f(x)$ and $y = g(x)$, for $-1 \leq x \leq 3$, on the same diagram. Indicate clearly the points where maximum and minimum values are attained and the points of intersection of the two curves.

4. A function is defined by the following rule:

$$f(x) = \begin{cases} 0 & \text{if } x \le -2 \\ -1 & \text{if } -2 < x < 0 \\ x & \text{if } x \ge 0 \end{cases}$$

Find: (a) $f(-2) + f(-1) + f(0)$; (b) $f(a^2)$.

5. The function $f(x)$ is defined by the rule $f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$

Sketch the function $f(x)$, from $x = -2$ to $x = 2$.

6. If $f(x) = ax^2 + bx + c$ find a simplified expression for $f(x) - f(-x)$.

7. (a) What is the equation of the circle whose centre is at the origin and which passes through the point $(5, -7)$?

(b) A circle, of radius 5 units, has its centre at the point $(-3, 4)$. What are the coordinates of the two points at which the circle cuts the y axis?

(c) Find the equation of the parabola with vertex $(-1, -4)$ which passes through the origin and whose axis is parallel to the y-axis.

8. The parabola $y = ax^2 - c$, and the circle $x^2 + y^2 = 16$ meet on both the x and y axes. If a and c are both positive, what are their values?

9. (i) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes.

(ii) Find the coordinates of the point of intersection of these two graphs.

10. State the natural (ie. the largest possible) domain of the function given by $y = \sqrt{1+x} - \sqrt{1-x}$.

11. Find the range of the function f given by $f(x) = \dfrac{1}{1+x^2}$ over the domain of all real numbers.

12. Consider $f(x) = \dfrac{1-x^2}{x^4}$, $x \neq 0$.

(i) State the zeros of this function, ie. the values of x where $f(x) = 0$.

(ii) Describe how this function behaves near $x = 0$.

(iii) Describe how this function behaves for large x.

(iv) Give a rough sketch of this function.

13. Find the maximum value of $2x\,(1 - x)$.

14. The function $f(x)$ is defined by the rule $f(x) = 9x(x - 2)^2$ in the domain $-1 \leq x \leq 3$. Draw a sketch of the graph of $y = f(x)$, showing clearly the intercepts with the x and y axes, and the values at the endpoints of the domain.

15. (a) Sketch the curve $y = x^3 + x^2 - x - 1$ over the domain $-1 \leq x \leq 2$.

(b) Given that one of the turning points of this curve occurs when $x = \frac{1}{3}$, find the range of this function over the stated domain.

16. Sketch graphs for

(a) $y = \dfrac{x^2 + x}{x}$

(b) $y = \dfrac{x^2 + x}{x + 1}$

(c) $y = \dfrac{x^3 + 2x^2 + x}{x + 1}$

17. $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x+1}$. For what values of x does

(a) $g(x^2) = [g(x)]^2$?

(b) $f(x^2) = [f(x)]^2$?

18. Find $f(x)$ if

(a) $f(x+1) = x + 4$

(b) $f(x+1) = 3x - 2$

(c) $f(x+1) = 1 - x^2$

(d) $f(2a) = a^2 - 2a - 1$

19. $f(x) = 3x - 4$ and $g(x) = x^2 - 1$, show that

(a) $f(a) + f(b) = f(a+b)$

(b) $g(a) + g(b) \neq g(a+b)$

20. Write down the range of the function $f(x) = \dfrac{1}{x^2 + 4x + 7}$.

21. Sketch the graph of the function $y = \sqrt{16 - (x - 3)^2} + 1$. State the domain and range of the function.

22. If $f(x) = 3x + 5$ and $g(x) = \frac{x - 5}{3}$, find a simplified expression for $f[g(x)] + g[f(x)]$.

23. The parabola $y = ax^2 + bx + c$ has its vertex at $(2, 1)$ and passes through the point $(0, 0)$. Find a, b and c.

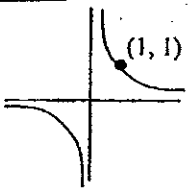
24. Given that $Q(x) = ax^2 + bx + c$ for all x, and that $Q(0) = 4$, $Q(1) = 23$, $Q(-1) = 1$, determine the constants a, b, c,

25. Sketch $y = \sqrt{(x+2)^2}$

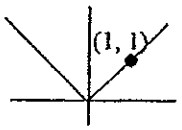
26. (a) By first sketching $y = x^2 - 1$, draw a sketch of $y = \dfrac{1}{x^2 - 1}$.

(b) Hence write down the domain and range of the function $y = \dfrac{1}{x^2 - 1}$.

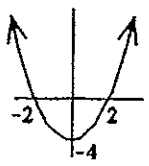
ANSWERS:

1. (a) 

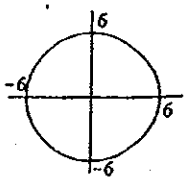
D : $x \neq 0$
R : $y \neq 0$

(b) 

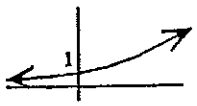
D : all real x
R : $y \geq 0$

(c) 

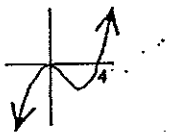
D : all real x
R : $y \geq -4$

(d) 

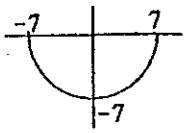
D : $-6 \leq x \leq 6$
R : $-6 \leq y \leq 6$

(e) 

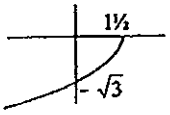
D : all real x
R : $y > 0$

(f) 

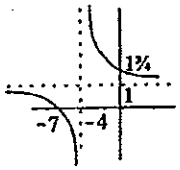
D : all real x
R : all real y

(g) 

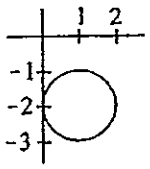
D : $-7 \leq x \leq 7$
R : $-7 \leq y \leq 0$

(h) 

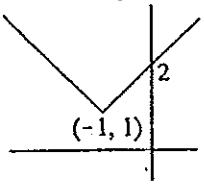
D : $x \leq 1\frac{1}{2}$
R : $y \leq 0$

(i) 

D : $x \neq -4$
R : $y \neq 1$

(j) 

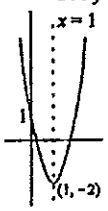
D : $0 \leq x \leq 2$
R : $-3 \leq y \leq -1$

(k) 

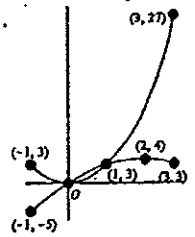
D : all real x
R : $y \geq 1$

(l) 

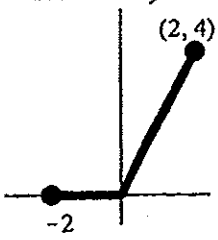
D : all real x
R : all real y

2. 

Axis of Sym: $x = 1$
Range: $y \geq -2$

3. 

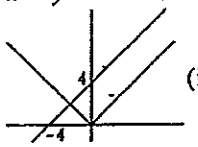
4. (a) -1 (b) a^2

5. 

6. $2bx$

7. (a) $x^2 + y^2 = 74$ (b) (0, 0), (0, 8) (c) $y = 4(x + 1)^2 - 4$

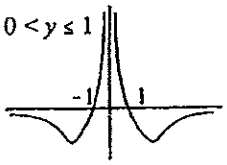
8. $a = \frac{1}{4}$, $c = 4$

9. (i) 

(ii) (-2, 2)

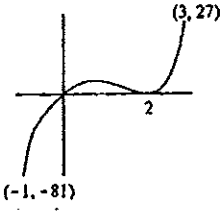
10. $-1 \leq x \leq 1$

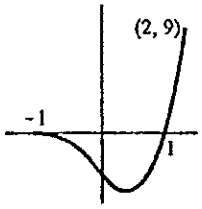
11. $0 < y \leq 1$

(iv) 

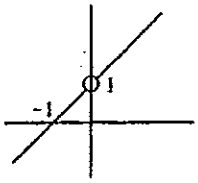
12. (i) $x = \pm 1$ (ii) approaches infinity (iii) approaches 0 from below

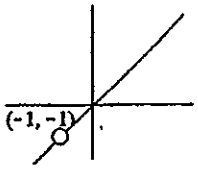
13. $\frac{1}{2}$

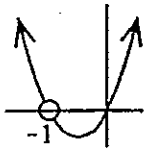
14. 

15. (a) 

(b) $y \geq -\frac{32}{27}$

16. (a) 

(b) 

(c) 

17. (a) $x = 0$ (b) $x \geq 0$

18. (a) $f(x) = x + 3$ (b) $f(x) = 3x - 5$ (c) $f(x) = 2x - x^2$ (d) $f(x) = \frac{1}{4}x^2 - x - 1$