2 UNIT TEST NUMBER 8

1996

Trigonometric Functions.

QUESTION 1. (7 marks)

Marks

(a) Express $\frac{2\pi}{3}$ radians in degrees. **1**

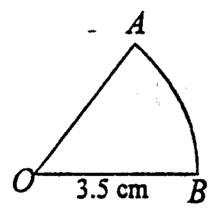
(b) Express 258° in radians, correct to 4 significant figures. **1**

(c) A sector *AOB* of a circle has a radius of 3.5 cm. Its perimeter is 9.5 cm. **5**

(i) Find the length of the arc *AB*.

(ii) Find the size of $\angle AOB$.

(iii) Find the area of the sector *AOB*.



QUESTION 2. (15 marks)

(a) Differentiate: **3**

(i) $\cos 2x$

(ii) $\sin(5-x)$

(iii) $\tan^3 x$.

(b) Find: **5**

(i) $\int \cos 3x \, dx$

(ii) $\int \sec^2\left(\frac{x}{2}\right) dx$

(iii) $\int \cot x \, dx$ by writing $\cot x$ in terms of $\sin x$ and $\cos x$.

(c) Evaluate $\int_{0.6}^{1.5} \cos 2x \, dx$ correct to 4 decimal places. **2**

(d) (i) Differentiate $x \sin x$. **2**

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$. **3**

QUESTION 3. (7 marks) **Marks**

(a) State the period and amplitude of the function $y = 3\sin 2x$. **2**

(b) On the same diagram, in the domain $0 \le x \le 2\pi$, draw the graphs of: **4**
$y = 3\sin 2x$ and $y = 1 - \cos x$.

(c) For the equation $3\sin 2x = 1 - \cos x$, how many solutions are there in the domain $0 \le x \le 2\pi$? **1**

QUESTION 4. (11 marks)

(a) For the function $y = \sin x$, find the value of $\frac{dy}{dx}$ when $x = 2$ (correct to 3 decimal places). **2**

(b) (i) By substitution, show that the graphs $y = \sin 2x$ and $y = \sin x$ intersect at points whose x-coordinates are $x = 0$ and $x = \frac{\pi}{3}$. **6**

(ii) Find the area between the two graphs for $0 \le x \le \frac{\pi}{3}$.

(c) The curve $y = \sec x$, for $0 \le x \le \frac{\pi}{3}$, is rotated about the x-axis. **3**
Find the volume of the solid formed. (Leave answer as an exact value.)

2 UNIT TEST NUMBER 8

1996

SUGGESTED SOLUTIONS

QUESTION 1

(a) $\frac{2\pi}{3}^c = \frac{2}{3} \times 180° = 120°$ — 1 — *Note :* π radians $= 180°$

(b) $258° = 258 \times \frac{\pi}{180}$ radians

$= 4.503$ radians (4 sig. fig.) — 1

(c) (i) $\text{arc } AB + 2 \times 3.5 = 9.5$

$\text{arc } AB = 2.5$ cm — 1

(ii) Using $l = r\theta$ — *Note :* θ is in radians in this formula.

$2.5 = 3.5 \times \theta$ — 1

$\theta = \frac{2.5}{3.5}$

$\angle AOB = \frac{5}{7}$ radians (or 41°) — 1 — **Total = 2**

(iii) $A = \frac{1}{2}r^2\theta$ — *Note :* θ is in radians in this formula.

$= \frac{1}{2} \times (3.5)^2 \times \frac{5}{7}$ — 1

$= 4.375$

Area of sector $= 4.375 \text{ cm}^2$ — 1 — **Total = 2**

QUESTION 2

(a) (i) $\frac{d}{dx}\cos 2x = -2\sin 2x$ — 1 — Using $\frac{d}{dx}\cos f(x) = -f'(x)\sin f(x)$.

(ii) $\frac{d}{dx}\sin(5-x) = (-1)\cos(5-x)$ — 1 — Using $\frac{d}{dx}\sin f(x) = f'(x)\cos f(x)$.

$= -\cos(5-x)$

(iii) $\frac{d}{dx}\tan^3 x = \frac{d}{dx}(\tan x)^3$ Using $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \times f'(x)$.

$= 3(\tan x)^2 \times \sec^2 x$

$= 3\tan^2 x \sec^2 x$ 1

(b) (i) $\int \cos 3x \, dx = \frac{1}{3}\sin 3x + C$ 1

(ii) $\int \sec^2\left(\frac{x}{2}\right) dx = 2\tan\left(\frac{x}{2}\right) + C$ 2 *Note:* $\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + C$.

(iii) $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$ 1

$= \ln(\sin x) + C$ 1 Using $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$.

Total = 2

(c) $\int_{0.6}^{1.5} \cos 2x \, dx = \left[\frac{1}{2}\sin 2x\right]_{0.6}^{1.5}$ 1

$= \frac{1}{2}\sin 3 - \frac{1}{2}\sin 1.2$

$= -0.3955$ (correct to 4 d.p.) 1 *Note:* Use radian mode on your calculator.

Total = 2

(d) (i) $\frac{d}{dx} x\sin x = \sin x + x\cos x$ 2 *Note:* By product rule.

(ii) Make $x\cos x$ the subject of this equation.

$x\cos x = \frac{d}{dx}x\sin x - \sin x$ [from (i)] 1

$\int_0^{\frac{\pi}{2}} x\cos x \, dx = \int_0^{\frac{\pi}{2}} \left(\frac{d}{dx}x\sin x - \sin x\right) dx$ *Note:* Differentiation and integration are inverse processes i.e. $\int \frac{d}{dx} x\sin x \, dx = x\sin x$.

$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$ 1

$= \left[\frac{\pi}{2} \times 1 + 0\right] - [0 + 1]$

$= \frac{\pi}{2} - 1$ 1 **Total = 3**

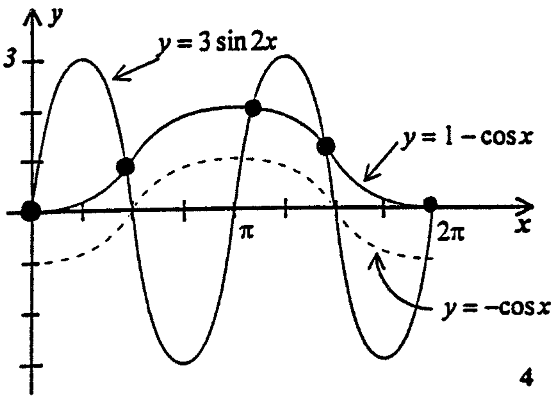
QUESTION 3

(a) $y = 3\sin 2x$

Period $= \frac{2\pi}{2} = \pi$ 1 *Note:* Formula $T = \frac{2\pi}{n}$.

Amplitude $= 3$ 1 **Total = 2**

(b)



4

Note : $y = 1 - \cos x$
i.e. $y = -\cos x + 1$.

To draw $y = 1 - \cos x$, draw $y = -\cos x$ and move it up one unit.

(c) $3 \sin 2x = 1 - \cos x$
has 5 solutions in $0 \le x \le 2\pi$
because there are 5 points of intersection of
the two curves (large dots on the diagram). 1

QUESTION 4

(a) $y = \sin x$

$\frac{dy}{dx} = \cos x$ 1

When $x = 2$, $\frac{dy}{dx} = \cos 2$

$= -0.416$ (3 dec.pl.) 1

Note : Use radian mode on the calculator.

Total = 2

(b) (i) $y = \sin 2x$, $y = \sin x$

When $x = 0$, $\sin 2x = \sin 0 = 0$

$\sin x = \sin 0 = 0$ 1

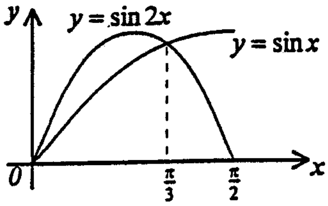
When $x = \frac{\pi}{3}$, $\sin 2x = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin x = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ 1

Since both graphs have the same y-value for
both $x = 0$ and $x = \frac{\pi}{3}$, the graphs intersect at
$x = 0$ and $x = \frac{\pi}{3}$

Total = 2

(ii)



$A = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\, dx$ **1**

$= \left[-\frac{1}{2}\cos 2x + \cos x\right]_0^{\frac{\pi}{3}}$ **1**

$= \left[-\frac{1}{2}\cos\frac{2\pi}{3} + \cos\frac{\pi}{3}\right] - \left[-\frac{1}{2}\cos 0 + \cos 0\right]$

$= \left[-\frac{1}{2} \times -\frac{1}{2} + \frac{1}{2}\right] - \left[-\frac{1}{2} \times 1 + 1\right]$ **1**

$= \frac{3}{4} - \frac{1}{2}$

Area = 0.25 unit^2 **1** **Total = 4**

(c) $V = \pi \int_0^{\frac{\pi}{3}} y^2\, dx$

$= \pi \int_0^{\frac{\pi}{3}} \sec^2 x\, dx$ **1**

$= \pi[\tan x]_0^{\frac{\pi}{3}}$ **1**

$= \pi\left[\tan\frac{\pi}{3} - \tan 0\right]$

$= \pi \times \sqrt{3}$

Volume is $\sqrt{3}\,\pi$ unit^3 **1** **Total = 3**