Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2007

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:
(Please tick or highlight)
- Mr Ireland
- Mr Barrett
- Mr Lowe
- Mr Rezcallah
- Mr Ee
- Mr Trenwith
- Mr Weiss

Student Number:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	12	12	12	12	12	12	12	12	12	12	120	100

Question 1 (12 marks) Marks

(a) Evaluate to 2 decimal places $\dfrac{19+4.6}{\sqrt{9-1.4^2}}$ 2

(b) Express $\sqrt{27}+\sqrt{48}$ in the form $a\sqrt{3}$ where a is an integer. 2

(c) Solve for x: $|x-4|<6$ 2

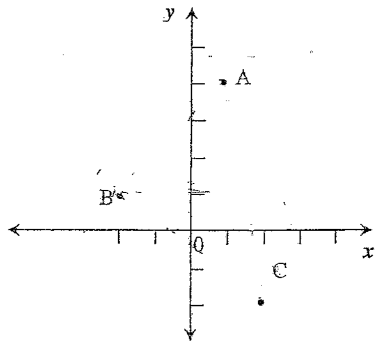
(d) Find the exact value of $\tan\dfrac{2\pi}{3}$ 2

(e) A coin is tossed three times. Find the probability of tossing one head 2

(f) Solve $2x-\dfrac{3x+4}{5}=9$ 2

Question 2 (12 marks) Use a separate page **Marks**

(a) The co-ordinates of points A, B and C are A(1, 4), B(−2, 1) and C(2, −2).



(i) Show that the equation of the line AB is $x-y+3=0$ — 2

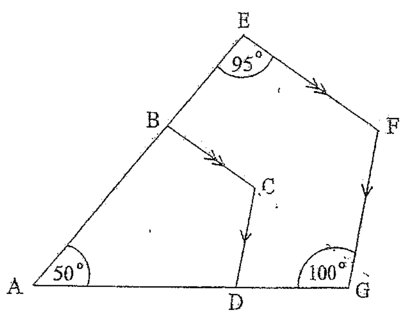
(ii) Find the perpendicular distance from C to AB — 2

(iii) Find the distance AB. (Leave answer in simplified surd form) — 1

(iv) Hence find the area of ΔABC, correct to one decimal place. — 1

(v) Find the equation of the median BM in ΔABC. — 3

(b)



NOT TO SCALE

In the diagram BC is parallel to EF and CD is parallel to FG.
$\angle BAD = 50°$, $\angle BEF = 95°$ and $\angle FGD = 100°$.

Find the size of $\angle BCD$, giving reasons. — 3

Question 3 (12 marks) Use a separate page **Marks**

(a) Differentiate with respect to x:

(i) $y=\dfrac{x}{e^x}$ — 2

(ii) $y=(3-\cos x)^{10}$ — 2

(b) (i) Evaluate $\displaystyle\int_0^4 e^{\frac{x}{4}}\,dx$ — 2

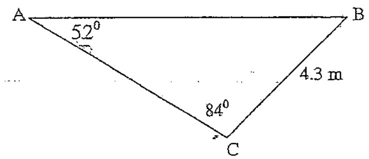
(ii) Find $\displaystyle\int \frac{x^2}{x^3+2}\,dx$ — 2

(iii) Evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\pi} \sin 3x\,dx$ — 3

(c) Show that $\ln 2$, $\ln 4$, $\ln 8$, $\ln 16$ form an arithmetic sequence. — 1

Question 4 (12 marks) Use a separate page **Marks**

(a)



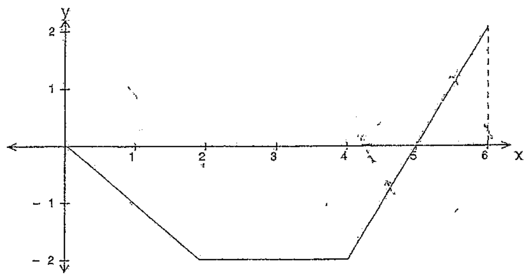
NOT TO SCALE

In the diagram ABC is a triangle where $BC = 4{\cdot}3$ m, $\angle ACB = 84^0$ and $\angle BAC = 52^0$.

Find the length of AC correct to one decimal place. 3

(b) If $\tan^2\theta = 3$ find the values for θ where $0 \leq \theta \leq 2\pi$ 3

(c) The diagram represents a function $y = f(x)$.



Evaluate $\int_0^6 f(x)\,dx$. 2

(d) For the parabola $x^2 = 5y + 10x$

i) Find the focal length of this parabola. 2

ii) Calculate the coordinates of the vertex. 1

iii) Find the equation of the directrix of this parabola. 1

Question 5 (12 marks) Use a separate page **Marks**

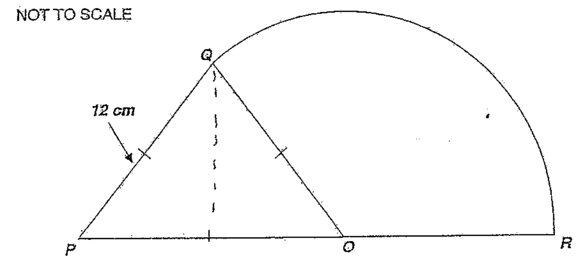
(a) Consider the curve $y = 3x^4 - 8x^3 + 6$

i) Find the turning points of the curve and determine their nature. 3

ii) Find the points of inflexion of the curve. 2

iii) Sketch the curve showing the turning points and the points of inflexion. (Do not find the x-intercepts). 2

(b)



ΔOPQ is an equilateral triangle of sides 12 cm. PR is a straight line.
QR is an arc of a circle, centre O. Find:

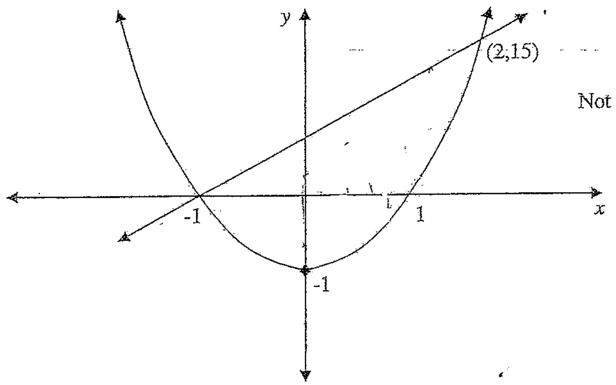
i) The perimeter of the whole region $PQRO$ (Leave your answer in exact form). 2

ii) The area of the whole region $PQRO$ (Leave your answer in exact form). 2

(c) Solve $\log_3(3 - x) = 1$ 1

Question 6 (12 marks) Use a separate page **Marks**

(a)



Not to scale

The diagram shows the curve $y = x^4 - 1$ and the line $y = 5x + 5$

(i) Show that the line and curve intersect at the points $(-1, 0)$ and $(2, 15)$ **2**

(ii) Calculate the area between the curve and the line. **3**

(b) A rotary club sell exactly 100 raffle tickets. When the prizes are drawn the first ticket drawn wins first prize, the second ticket drawn wins second prize and the third ticket drawn wins the third, and final, prize.
If Joseph buys two tickets, what is the probability that:

i) he wins first prize only ? **1**

ii) he wins the first and the last prize ? **1**

iii) he wins at least one prize ? **2**

c) i) Show that the derivative of $\log_e(\tan x)$ is $\sec x \operatorname{cosec} x$. **2**

ii) Hence, evaluate $\int 2 \sec x \operatorname{cosec} x \, dx$ **1**

Question 7 (12 marks) Use a separate page **Marks**

(a) A pool is being drained and the number of litres of water, L, in the pool at time t minutes is given by the equation

$$L = 160(60 - t)^2$$

(i) Find the initial volume of water in the pool. **1**

(ii) At what rate is the water draining out of the pool when $t = 8$ minutes ? **2**

(iii) How long will it take for the pool to be completely empty? **1**

(b) The gradient function for a curve is $\frac{dy}{dx} = 6x + \frac{1}{x}$.
Find the equation of the curve if it passes through the point (1, 8). **2**

(c) A plant is 65 cm high when first observed. In the first month of observation it grows 8 cm, and in each succeeding month the growth in height is 75% of the previous growth. What will be the ultimate height of the plant? **2**

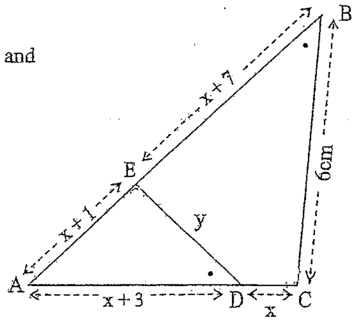
(d) The region enclosed between the curve $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, the x axis and the lines $x = 5$ and $x = 11$ is rotated about the x axis.

Calculate the volume of the solid generated. **4**

Question 8 (12 marks) Use a separate page **Marks**

(a) In the diagram ΔAED is similar to ΔACB and ∠EBC = ∠EDA.

Find the values of 'x' and 'y'. **3**



(b) Show that the equation of the tangent to $y = x\log_e x$ at the point where $x = e$ is $y = 2x - e$. **2**

(c) The roots of the quadratic equation $x^2 + (k+4)x + 5k = 0$ are α and β and $k \neq 0$.

Find the value of: (i) $\alpha + \beta$ **1**

(ii) $\alpha^2 + \beta^2$ **1**

(iii) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ **1**

(d) A decagon (10- sided polygon) has its angle sizes in arithmetic sequence, The smallest being 117°.

(i) Show that the angle sum of the decagon is 1440°. **1**

(ii) Find the common difference of the sequence. **2**

(iii) Find the size of the largest angle. **1**

Question 9 (12 marks) Use a separate page **Marks**

(a) Find the values of k for which the quadratic equation $x^2 - (k+2)x - 3k - 11 = 0$ has real roots. **3**

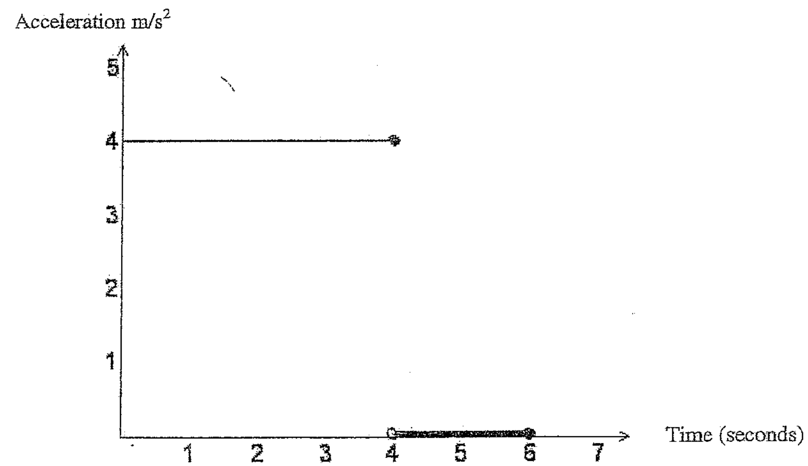
(b) The value $V of a machine is given by the formula $V = Ae^{-kt}$, where A and k are constants and t is the time measured in years. The initial cost of the machine is $120 000. Two years later the value was $95 000.

(i) Calculate the value of the constants A and k. **2**

(ii) Calculate the value of the machine after 10 years. **1**

(iii) If the machine was bought on 1st January 2000, in which year will the value of the machine fall below $60 000? **2**

(c) This graph shows the acceleration of a particle during a 6 second interval.



Initially the particle is at rest at the origin.

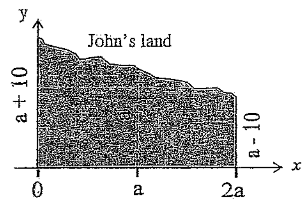
i) Sketch, accurately, the velocity time graph of the particle for $0 \leq t \leq 6$. **2**

ii) Calculate the distance travelled by the particle during this 6 seconds. **2**

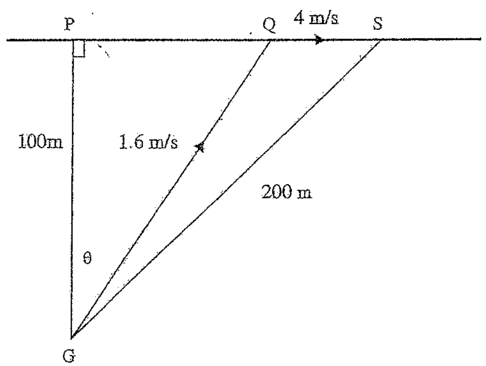
Question 10 (12 marks) Use a separate page **Marks**

a) The shaded area shown in the diagram below represents John's land. Its dimensions are given in terms of 'a'. 3

Given that the area of this land is 3200m^2, use Simpson's rule with 3 function values to find an estimate the value of '**a**'.



b) A swimmer at G in a calm lagoon is 100 metres from the beach at P when he decides to return for a snack to S, which is on the beach 200 metres from him. He can swim at 1.6 metres per second and jog along the beach at 4 metres per second.



i) Show that $QS = 100\,(\sqrt{3} - \tan\theta)$ where $\theta = \angle PGQ$ 2

ii) Show that the total time of travel T (in seconds) for the trip sections is given by: 2

$$T = \frac{125}{2\cos\theta} + 25\,(\sqrt{3} - \tan\theta)$$

iii) In which direction θ should he start swimming to reach the snack in the shortest time? 4

iv) How long till he reaches his destination? 1

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1}, n \neq -1;\ x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax\, dx$	$= \frac{1}{a}\sin ax,\ a \neq 0$
$\int \sin ax\, dx$	$= -\frac{1}{a}\cos ax,\ a \neq 0$
$\int \sec^2 ax\, dx$	$= \frac{1}{a}\tan ax,\ a \neq 0$
$\int \sec ax \tan ax\, dx$	$= \frac{1}{a}\sec ax,\ a \neq 0$
$\int \frac{1}{a^2+x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}} dx$	$= \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}} dx$	$= \ln(x + \sqrt{x^2-a^2}),\ x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}} dx$	$= \ln(x + \sqrt{x^2+a^2})$

NOTE : $\ln x = \log_e x, \quad x > 0$

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Solutions - Marking scale

Marking:

Q1 & Q2 – Mr Ireland

Q3 & Q4 – Mr Ireland

Q5 & Q6 – Mr Barrett

Q7 & Q8 – Mr Lowe

Q9 & Q10 – Mr Rezcallah

Marking Guidelines: Mathematics Solutions

Answers:

Question 1	Question 1
1.(a) $\dfrac{19+4.6}{\sqrt{9-1.4^2}}=8.89458$ $=8.89(2\,dec\ places)$	(a) 1 for approximation 1 f or 2 dp
(b) $\sqrt{27}+\sqrt{48}=3\sqrt{3}+4\sqrt{3}=7\sqrt{3}$	(b) 1 for simplifying a surd 1 for the simplification.
(c) $\lvert x-4\rvert<6$ $\therefore -6<x-4<6$ $-2<x<10$	(c) 1 for end points 1 for correct inequality
(d) $\tan\dfrac{2\pi}{3}=\tan\left(\pi-\dfrac{\pi}{3}\right)=-\tan\dfrac{\pi}{3}=-\sqrt{3}$	(d) 1 for $-\tan\dfrac{\pi}{3}$ 1 for exact answer
(e) outcomes possible are HTT,THT,TTH since there are 8 outcome $\therefore P(1\text{H})=\dfrac{3}{8}$	(e) 1 for indicating 8 outcomes 1 for correct answer
(f) $2x-\dfrac{3x+4}{5}=9$ $10x-3x-4=45$ $7x=49$ $x=7$	(f) 1 for multiplying by 5 and correct signs 1 for answer $x=7$
Question 2	**Question 2**
2(a) (i) $y-4=\dfrac{1-4}{-2-1}(x-1)$ $y-4=x-1$ where m = 1 $0=x-y+3$	(a)(i) 1 for gradient m=1 1 for correct equation formula to get the given answer
(ii) $d=\dfrac{\lvert 1\times2-1\times-2+3\rvert}{\sqrt{1^2+(-1)^2}}$ $=\dfrac{\lvert7\rvert}{\sqrt{2}}=\dfrac{7}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{7\sqrt{2}}{2}$	(ii) 1 for sub. into formula 1 for correct answer

(iii) $d=\sqrt{(-2-1)^2+(1-4)^2}$ $=\sqrt{18}$ $=3\sqrt{2}$	(iii) 1 for correct answer (do not penalize for decimal answer if formula shown)
(iv) $A=\frac{1}{2}\times 3\sqrt{2}\times\frac{7\sqrt{2}}{2}$ $=10.5\,\text{u}^2$	(iv) 1 for correct answer
(v) Mid point M of AC: $M(\frac{1+2}{2},\frac{4-2}{2})$ $=M(\frac{3}{2},1)$	(v) 1 for correct midpt M
Gradient of BM: $m=\dfrac{1-1}{\frac{3}{2}+2}=0$	1 for m = 0
Equation of BM : $y = 1$	1 for equation y = 1 Award full marks if students show with valid reasoning, drawings or workings that y = 1 N.B: 2 marks only if another median is found correctly
(b) ∠BAD = 50°	(b)
∠ABC = ∠BEF = 95° (corresponding ∠s of // ∠ADC= ∠FGD = 100° lines are =)	1 for correct values of ∠BEF and ∠FGD 1 for correct reason(s)
∠BCD = 360° -(50°+ 95°+100°) (∠ sum of quad = 115° ABCD)	1 for correct ∠BCD value with working or reason

Answers:

Question 3	
3(a)	3 (a)
(i) $y=\frac{x}{e^x}$ $y'=\frac{1(e^x)-x(e^x)}{(e^x)^2}$ $=\frac{(1-x)(e^x)}{e^x.e^x}=\frac{(1-x)}{e^x}$ Accept also product rule answer: $y=e^{-x}-xe^{-x}$	(i) 1 for applying correct rule 1 for correct answer
(ii) $y=(3-\cos x)^{10}$ $y'=10(--\sin x)(3-\cos x)^9$ $=10(\sin x)(3-\cos x)^9$	(ii) 1 for applying chain rule 1 for simplification
(b) (i) $\int_0^4 e^{\frac{x}{4}}dx=\left[4e^{\frac{x}{4}}\right]_0^4=4e-4$	(b) (i) 1 for integrating 1 for correct answer
(ii) $\int\frac{x^2}{x^3+2}dx=\frac{1}{3}\int\frac{3x^2}{x^3+2}dx$ $=\frac{1}{3}\log_e(x^3+2)+c$	ii) 1 for correct method 1 for correct answer
(iii) $\int_{\frac{\pi}{3}}^{\pi}\sin 3x\,dx=\left[\frac{-\cos 3x}{3}\right]_{\frac{\pi}{3}}^{\pi}$ $=\left(\frac{-\cos 3\pi}{3}\right)-\left(\frac{-\cos\pi}{3}\right)$ $=\frac{1}{3}-\frac{1}{3}$ $=0$	(iii) 1 for integrating 1 for substituting 1 for correct answer
(c) $t_2-t_1=t_3-t_2\Rightarrow\log 4-\log 2=\log 8-\log 4$ $2\log 2-\log 2=3\log 2-2\log 2$ $\log 2=\log 2$ ∴ an arithmetic sequence	(c) 1 for correctly showing it's AS

Answers

Question 4

4 (a)

(i) $\angle ABC = 44^0$

$$\frac{AC}{\sin 44^0} = \frac{4.3}{\sin 52^0}$$

$$AC = \frac{4.3 \sin 44^0}{\sin 52^0}$$

$$= 3.790...$$

$$= 3.8 \,(1dp)$$

(b) $\tan^2\theta = 3 \quad \therefore \tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3} \Rightarrow \quad \theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$= \frac{\pi}{3}, \frac{4\pi}{3}$

$\tan\theta = -\sqrt{3} \Rightarrow \quad \theta = \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}, \frac{5\pi}{3}$

(c)

For $\int_0^6 f(x)\,dx$: students can identify that the solution simplifies to be the area of a trapezium under the x-axis.

Hence $\int_0^6 f(x)\,dx = -\left\{\frac{2}{2}(4+2)\right\}$

$= -6$

(d)

$x^2 = 5y + 10x$

$x^2 - 10x + 25 = 5y + 25$

$(x-5)^2 = 5(y+5)$ *of the form*

$(x-h)^2 = 4a(y-k)$

i) $4a = 5 \Rightarrow$ focal length $= \frac{5}{4}$

ii) $h = 5, \quad k = -5$

Vertex $(5, -1)$

iii) Directrix $y = -6\frac{1}{4} = -\frac{25}{4}$

Question 4 (marking)

4 (a)

1 using correct $\angle ABC$

1 using sine rule correctly

1 for correct answer

(b)

1 for finding 2 equations

1 for correct 2 answers

1 for other 2 correct answers

(c)

1 for finding areas between the curve and the x-axis (above & below); or simplifies the question as in solution

1 for correct final answer of -6.

(d)

1 for changing the equation to the known form

i) 1 for correct a

ii) 1 for correct V

iii) 1 for correct directrix

Answers:

Question 5

5 (a)

i) $y = 3x^4 - 8x^3 + 6$

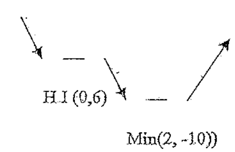
$\frac{dy}{dx} = 12x^3 - 24x^2 = 0$

$12x^2(x-2) = 0$

$\therefore x = 0, \quad 2$

x	-1	0	1	2	3
$\frac{dy}{dx}$	-36	0	-12	0	108

y



The curve has a horizontal point of inflexion at(0,6) and a Minimum at (2,-10).

ii)

$\frac{d^2y}{dx^2} = 36x^2 - 48x = 0$

$12x\,(3x-4) = 0$

$\therefore x = 0, \quad \frac{4}{3}$

x	-1	0	1	4/3	2
y''	84	0	-12	0	48

Y. ∪ ∩ ∪

Conc up Conc down Conc up

The curve has a horizontal point of inflexion at(0,6)

and another point of inflexion at $\left(\frac{4}{3}, -3\frac{13}{27}\right)$

Question 5 (marking)

a)

i)

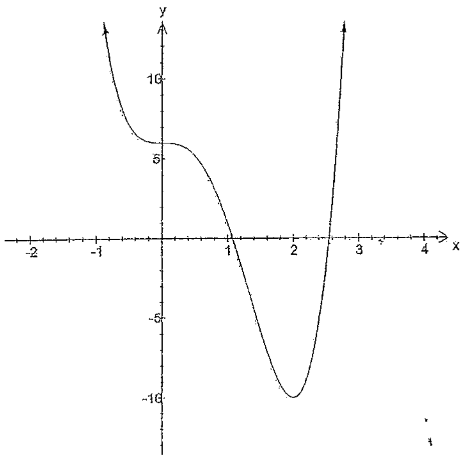
1 for correct derivative = 0

1 for testing each of the 2 points

1 for each point's coordinates and nature

1 for finding each of the 2 inflexions

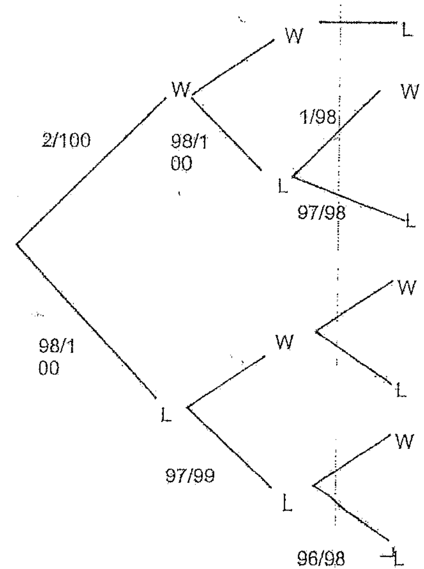
1 for testing the non zero inflexion

iii)  The 3 points should be labeled on the diagram .	iii) for correct shape 1 for the 3 points found earlier shown on the diagram
b) i) $\angle QOR = 120^0$ (straight line) Arc $QR = r\theta$ $= 12(\frac{2\pi}{3})$ $= 8\pi$ $\therefore$ Perimeter = 12 + 12 + 12 + arc QR $= 36 + 8\pi$ cm	b) i) 1 for correct arc QR 1 for correct answer or their length of arc QR + 36.
ii) Area= Triangle + Sector $= \frac{1}{2} ab \sin C + \frac{1}{2} r^2 \theta$ $= \frac{1}{2}.12.12.\sin 60^0 + \frac{1}{2}.12^2.\frac{2\pi}{3}$ $= \frac{1}{2}.12.12.\frac{\sqrt{3}}{2} + \frac{1}{2}.144.\frac{2\pi}{3}$ $= 36\sqrt{3} + 48\pi \ cm^2$	ii) 1 for correct answer for area of triangle. 1 for correct answer for area of sector (using their calculation for $\angle QOR$)in radians.
iii) $\log_3 (3 - x) = 1 = \log_3 3$ $3 - x = 3$ $x = 0$	iii) 1 for correct answer

Answers

Question 6	Question 6
6 (a) (i) sub $(-1,0)$ and $(2,15)$ into $y = x^4 - 1$ $0 = 1 - 1$ true and $15 = 16 - 1$ true sub $(-1,0)$ and $(2,15)$ into $y = 5x + 5$ $0 = -5 + 5$ true and $15 = 10 + 5$ true Or use algebra (factorization)	(6) (a) (i) 1 for showing (-1,0) lies on the intersection. 1 for showing (2,15) lies on the intersection.
(ii) $A = \int_{-1}^{2} (5x+5) - (x^4 - 1) \, dx$ $= \int_{-1}^{2} -x^4 + 5x + 6 \, dx$ $= \left[\frac{-x^5}{5} + \frac{5x^2}{2} + 6x \right]_{-1}^{2}$ $= \left(\frac{-32}{5} + 10 + 12 \right) - \left(\frac{1}{5} + \frac{5}{2} - 6 \right)$ $= 15\frac{3}{5} - \left(-3\frac{3}{10} \right) = 18.9 \, u^2$	(ii) 1 correct integration 1 for correct sub. 1 for correct answer

(b)



Question 6(b) Continued	Question 6(b)
(i) $P(WLL)=\frac{2}{100}\times\frac{98}{99}\times\frac{97}{98}=\frac{97}{4950}$	(i) 1 for correct answer
(ii) $P(WLW)=\frac{2}{100}\times\frac{98}{99}\times\frac{1}{98}=\frac{1}{4950}$	(ii) 1 for correct answer
(iii) P(win at least 1 prize) = 1 –P(no prizes)	(iii) 1 for correct method
$=1-P(LLL)=1-\frac{98}{100}\times\frac{97}{99}\times\frac{96}{98}=\frac{49}{825}$	1 for correct answer
(c) i) $y=\log_e(\tan x)$ $\frac{dy}{dx}=\frac{\sec^2 x}{\tan x}$	(c) i) 1 for correct derivative
$=\frac{1+\tan^2 x}{\tan x}=\frac{1}{\tan x}+\tan x$ $=\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}$ $=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}=\frac{1}{\sin x\cos x}$ $=\sec x\cos ecx$	1 for correct method
Alternatively, $\frac{dy}{dx}=\frac{\sec^2 x}{\tan x}$ $=\frac{\sec^2 x}{\tan x}=\frac{1}{\cos^2 x}\div\frac{\sin x}{\cos x}$ $=\frac{1}{\cos^2 x}\times\frac{\cos x}{\sin x}$ $=\frac{1}{\sin x\cos x}$ $=\sec x\cos ecx$	
ii) $\int 2\sec x\cos ecx\,dx=$ $2\ln(\tan x)+C$	ii) 1 for correct integration

Question 7	Question 7
7(a) (i) $L=160(60-t)^2$ let $t=0$ $\therefore L=160(60-0)^2$ $\therefore L=576\,000$ Litres	(7) (a) (i) 1 for correct answer
(ii) $\frac{dL}{dt}=-320(60-t)$	(ii) 1 for correct derivative
when $t=8$ $\frac{dL}{dt}=-16\,640$ $\therefore$ draining out at 16 640 litres per minute	1 for correct answer
(iii) $0=160(60-t)^2$ $\therefore t=60$ minutes	(iii) 1 for correct answer
(b) $\frac{dy}{dx}=6x+\frac{1}{x}$ $y=\int(6x+\frac{1}{x})\,dx=3x^2+\ln x+c$	1 for correct integration
Sub in (1,8) $8=3(1)+0+c$ $c=5$ $y=3x^2+\ln x+5$	1 for correct c value
(c) Application of Limiting Sum $\text{height}=65+8+\frac{3}{4}\times 8+\frac{3}{4}\times\frac{3}{4}\times 8+\ldots$ $=65+\frac{8}{1-\frac{3}{4}}$ $=65+32=97$ cm	c) 1 for establishing the G.P. 1 for correct answer

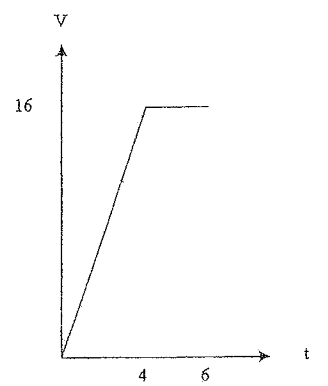
Question 7 Continued	Question 7 Continued
(d)	(d)
$v=\pi\int y^2 dx$	
$=\pi\int_5^{11}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$	1 for correct integration statement
$=\pi\int_5^{11} x+2+\frac{1}{x}dx$	1 for correct integration
$=\pi\left[\frac{x^2}{2}+2x+\log x\right]_5^{11}$	1 for substitution
$=\pi\left[\frac{121}{2}+22+\log 11-\left(\frac{25}{2}+10+\log 5\right)\right]=\pi\left(60+\log\frac{11}{5}\right)u^3$	1 for simplification.
Accept also $V=190.97\ u^3$	

Question 8	
a) Since ΔAED is similar to ΔACB	
$\frac{AE}{AC}=\frac{AD}{AB}$	
$\frac{x+1}{2x+3}=\frac{x+3}{2x+8}$	1 for correct ratio in terms of x
$(x+1)(2x+8)=(x+3)(2x+3)$	
$2x^2+10x+8=2x^2+9x+9$	
$\therefore x=1\ \ cm$	1 for correct answer x=1 cm
$\frac{ED}{BC}=\frac{AD}{AB}$	
$\frac{y}{6}=\frac{x+3}{2x+8}=\frac{4}{10}$	
$10y=24$	
$y=2.4\ \ cm$	1 for correct answer y=2.4 cm

(b) $y=x\ln x$	(b)
$y'=\ln x+x(\frac{1}{x})=\ln x+1$	1 for correct derivative
$m=\ln e+1=1+1=2$	
At $x=e, y=e\ln e=e$	
$y-e=2(x-e)$	1 for showing the given equation
$y=2x-2e+e=2x-e$	
(c) (i) $x^2+(k+4)x+5k=0$ $\alpha+\beta=-(k+4)$	(c)(i) 1 mark for correct answer
ii) $\alpha\beta=5k$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(k+4)^2-10k$ $=k^2-2k+16$	(ii) 1 mark for correct answer
(iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$ $=\frac{k^2-2k+16}{5k}$	(iii) 1 mark for correct answer
(d)(i) Angle sum $=(10-2)\,180°$ $=1440°$	(d)(i) 1 for showing the angle sum
(ii) $\frac{10}{2}[234+(10-1)d]=1440$ $234+9d=288$ $9d=54$ $d=6$	(ii) 1 for correct application of the sum formula 1 mark for correct d
(iii) $T_{10}=117+(10-1)6$ $=171°$	(iii) 1 mark for correct answer

Answers

Question 9	Question 9
(9)(a)	
$x^2-(k+2)x-3k-11=0$ For real roots $\Delta \geq 0$ $(k+2)^2-4\times 1\times(-3k-11)\geq 0$ $k^2+4k+4+12k+44\geq 0$	(a) 1 for $\Delta \geq 0$ 1 for quad. inequation
$k^2+16k+48\geq 0$ $(k+12)(k+4)\geq 0$ $k\leq -12, k\geq -4$	1 for correct inequality answers.
(b) (i) $V=Ae^{-kt}$ $t=0, V=120\,000 \therefore A=120\,000$ $t=2, V=95\,000$	(b) (i) 1 for A
$\therefore 95\,000=120\,000e^{-2k}$ $\frac{19}{24}=e^{-2k}$ $\log\frac{19}{24}=-2k$ $\therefore k=\frac{-1}{2}\log\frac{19}{24}$ $=0.1168..$ $=0.117(3dp)$	1 for k
(ii) When $t=10$ $V=120000e^{-10\times 0.1168...}$ $=37315.82$	(ii) 1 for correct ans.
(iii) $120\,000e^{-kt}<60\,000$ $e^{-kt}<\frac{1}{2}$	(iii)
$-kt<\log\frac{1}{2}$ $t>\frac{\log\frac{1}{2}}{-k}$	1 for taking logs
$t>5.92$ $\therefore$ during 2005	1 for answer

(c) i) For $0<t<4$, $a=4$ m/s^2 $v=4t+c$ @ $t=0$, $v=0$ $\therefore c=0$ $\therefore v=4t$ for $0<t<4$ @ $t=4$, $v=4(4)=16$ m/s Alternatively to find the velocity @ $t=4$s, v= Area of rectangle of sides 4 by 4= 16 m/s. Then the particle travels with this constant velocity as a =0 when $4<t<6$. $\therefore$ $v=16$ m/s for $4<t<6$	(c) i) 1 for $v=16$ m/s
	1 for correct graph with 0 and 16 shown clearly in the 2 phases of motion
ii) $x=A_1+A_2$ $=\frac{4\times 16}{2}+16\times 2$ $=32+32=64m$	
Alternatively, $x=\int_0^4 4t\,dt+\int_4^6 16\,dt$ $=\left[2t^2\right]_0^4+\left[\frac{16t}{1}\right]_4^6$	1 for Area of one section correct
$=32+16(6)-16(4)$ $=32+32$ $=64m$	1for correct answer

Answers

Question 10

a) h=a

x	0	a	2a
y	a+10	a	a-10

$$A=\frac{h}{3}(y_1+4y_2+y_3)$$

$$3200=\frac{a}{3}(a+10+4a+a-10)$$

$$3200=\frac{a}{3}(6a)$$

$$2a^2=3200$$

$$a^2=1600$$

$$a=\pm 40 \quad (a>0)$$

$$\Rightarrow a=40$$

1 for correct simpson's equation applied in this qusetion

1 for solving equation

1 for a = 40 (0 for ±)

b) (i)

Also $PQ+QS=PS$ where $PS^2=200^2-100^2=30\,000$ (Pythagoras' theorem)

Hence $PS=100\sqrt{3}$ and as $PQ=100\tan\theta$ we find $QS=100(\sqrt{3}-\tan\theta)$.

b) (i) 1 for PS

1 for PQ

(ii)

Now $100=GQ\cos\theta$ so $GQ=\dfrac{100}{\cos\theta}$

(ii) 1 for GQ

The time of travel T (in seconds) is given by $\dfrac{\text{distance travelled}}{\text{speed}}$ for the trip sections

$$T=\frac{GQ}{1.6}+\frac{QS}{4}$$

$$T=\frac{100}{1.6\cos\theta}+\frac{100}{4}(\sqrt{3}-\tan\theta)$$

$$T=\frac{125}{2\cos\theta}+25(\sqrt{3}-\tan\theta)$$

1 for showing the given time equation

(iii)

$$\frac{dT}{d\theta}=\frac{100}{1.6}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{25}{\cos^2\theta}$$

for a stationary point

$\Leftrightarrow$ $100\sin\theta=25\times 1.6$

$\Leftrightarrow$ $\sin\theta=0.4$

so $\theta=23.57°$ or $\doteqdot 23°35'$

(iii) 1 for $\dfrac{dT}{d\theta}$

1 for getting $\sin\theta=0.4$

1 for θ

Testing:

$\theta=0,\quad \dfrac{dT}{d\theta}=0-\dfrac{25}{1}=-25$

$\theta=30,\quad \dfrac{dT}{d\theta}=\dfrac{100}{1.6}\left(\dfrac{0.5}{0.75}\right)-\dfrac{25}{0.75}\cong 8.33$

θ	0	23.57°	30°
$\frac{dT}{d\theta}$	-25	0	8.33
T	↘	Min	↗

1 for testing

(iv)

The minimum time occurs when $\sin\theta=0.4$ and hence $\cos\theta=\sqrt{0.84}$

This is about 100.6 seconds.

Also Accept 1 min 36 seconds

(iv) 1for correct answer