NORTH SYDNEY BOYS HIGH SCHOOL

2006 YEAR 12 HSC
ASSESSMENT TASK 2

Mathematics

Examiner: G. Rezcallah

General Instructions

- Working time – 1 hour
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- A table of standard integrals is included in the answer booklet.
- Each new question is to be started on a **new page.**

Total Marks (60)

- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Ms Silverman
- O Mr Weiss
- O Mr Ee
- O Ms Jackson
- O Mr Lowe
- O Mr Rezcallah
- O Mr Barrett
- O Mr Trenwith

Student Number: .....

Question	1	2	3	4	5	6	Total	Total
Mark	9	9	11	10	10	11	60	100

Question 1 (9 marks) - **Start a new page** — Marks

a) Find:

(i) $\int (2x^3 - 5x^2 + 1)\,dx$ — 2

(ii) $\int \left(6 - \frac{1}{\sqrt{x}}\right) dx$ — 2

(iii) $\int (t+1)(t-1)\,dt$ — 2

b) Evaluate $\int_1^5 (3y - 2)\,dy$ — 3

Question 2 (9 marks) - **Start a new page**

a) Write down the equation of the circle with centre (0, - 3) and radius 4. — 1

b) Sketch each parabola, indicating the focus, vertex and directrix:

(i) $x^2 = -y$ — 3

(ii) $(y-1)^2 = 16(x+3)$ — 3

c) Find the equation of the parabola whose focus is at the point (-1,2) and whose directrix is the line y = 6. — 2

Question 3 (11 marks) - **Start a new page**

a) The points A(1,4) and B(-3,2)are given. **Describe** each locus of the point P(x, y) geometrically and find the equation of each locus if:

(i) P is 3 units from the point B. — 2

(ii) P is equidistant from point A and point B. — 3

(iii) P is equidistant from point A and the x axis. — 3

b) If $\int_{-1}^{a} x\,dx = 4$, evaluate a. — 3

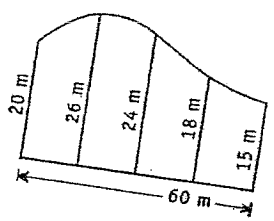
Question 4 (10 marks) - **Start a new page** Marks

a)

(i) Find the equation of the locus of a point P(x,y) so that its distance from the point M(-2,-1) is always twice its distance from the point N(4, 2). 3

(ii) Show that the locus is a circle and find its centre and radius. 3

b) David is going to make a feature garden at the front of his country property. He measured the area and made this sketch of the garden.



(i) Use all the measurements on the diagram and the trapezoidal rule to calculate the approximate area of David's feature garden. 3

(ii) Michael plans to top dress the garden with 3 cm of quality soil. How many cubic metres will he need? 1

Question 5 (10 marks) - **Start a new page**

a) Let $y = f(x)$ be a function who se graph consists of straight lines connecting the points P(0, 3), Q(3, -3), M(4, 3) and N(5, 3).

(i) Sketch an accurate graph of $y = f(x)$, showing the x- intercepts. 2

(ii) Hence, find $\int_0^5 f(x)dx$. 3

b) (i) Evaluate $\int_0^6 (9-x^2)dx$. 2

(ii) Find the area bounded by the curve $y = 9 - x^2$ and the x axis and the lines $x = 0$ and $x = 6$. 3

Question 6 (11 marks) - **Start a new page**

a) Consider the parabola $x^2 = 36y$.

(i) Write down the coordinates of the focus and sketch the parabola. 2

(ii) A line is drawn through the focus parallel to the x axis. Calculate the area contained within the parabola below this line. 2

(iii) The section of the parabola between $x = 0$ and $x = 6$ is rotated about the x axis. Calculate the exact volume so formed. 3

b) Every point on a given curve satisfies the relation $\frac{d^2y}{dx^2} = (x+5)^{\frac{1}{5}}$.

The curve's point of inflexion is (- 5, 0) and the gradient to the curve at this point is 2.

Find the equation of the curve. 4

End of Task

Go Back & Check Your Work!

MAN!

Very good

57/60

Question 1.

a) i) $\int (2x^3 - 5x^2 + 1)\,dx$

$= \frac{2x^4}{4} - \frac{5x^3}{3} + x + C$

$= \frac{x^4}{2} - \frac{5x^3}{3} + x + C$ ✓

ii) $\int \left(6 - \frac{1}{\sqrt{x}}\right) dx$ $\int (6 - x^{-1/2})\,dx$.

$= 6x - 2\sqrt{x} + C$ ✓ $6x - 2x^{1/2}$

iii) $\int (t+1)(t-1)\,dt$

$\int t^2 - 1 \, . \, dt$

$= \frac{t^3}{3} - t + C$ ✓

b) $\int_1^5 (3y - 2)\,dy$

$= \left[\frac{3y^2}{2} - 2y\right]_1^5$

$F(5) = \frac{75}{2} - 10$

$= 27\frac{1}{2}$

$F(1) = -\frac{1}{2}$

$F(5) - F(1) = 28\ u^2$ ✓

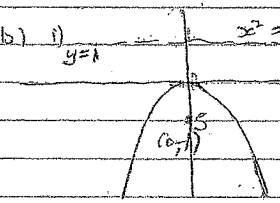
Question 2.

a)

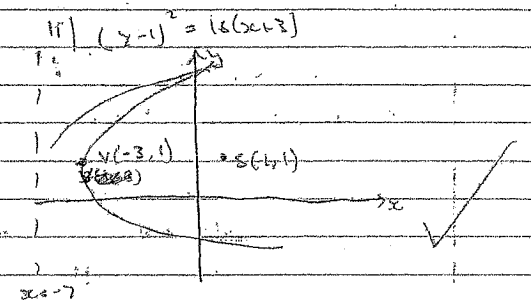
Centre $(0, -3)$ radius 4

$x^2 + (y+3)^2 = 16$ ✓

b) i) $x^2 = -y$



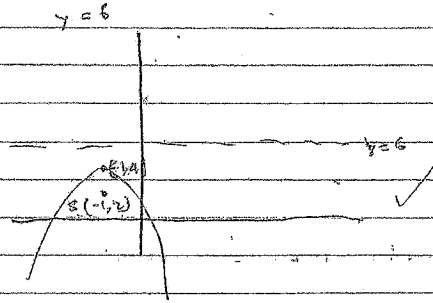
ii) $(y-1)^2 = 16(x+3)$



c) $S(-1,2)$

$y = 6$

$(x+1)^2 = -24(y-4)$ ✓

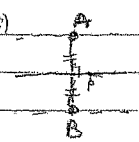


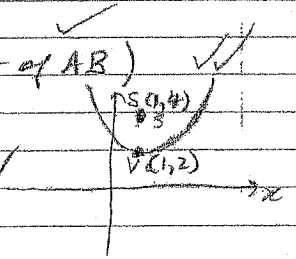
Question 3.

a) i) circle rad = 3 : $(x+3)^2 + (y-2)^2 = 9$ ✓

ii) $2x + y - 1 = 0$ ✓

(Perpendicular bisector of AB) ✓





iii) Parabola $(x-1)^2 = 8(y-2)$ ✓✓

b) $\int_{-1}^{a} x\,dx = 4$

$\left[\frac{x^2}{2}\right]_{-1}^{a} = 4$

$F(a) = \frac{a^2}{2}$

$F(-1) = \frac{1}{2}$

$F(a) - F(-1) = \frac{a^2}{2} - \frac{1}{2} = 4$ $a^2 - 1 = 8$ $a^2 = 9$

$a = \pm 3$

Question 4.

a) i) $PM = 2PN$ $\Rightarrow PM^2 = 4PN^2$ M(−2, −1) N(4, 2)

$PM^2 = (y+1)^2 + (x+2)^2$
$= y^2 + 2y + 1 + x^2 + 4x + 4$
$= y^2 + 2y + x^2 + 4x + 5$

$PN^2 = 4\left[(y-2)^2 + (x-4)^2\right]$
$= 4(y^2 - 4y + 4 + x^2 - 8x + 16)$
$= 4(y^2 - 4y + x^2 - 8x + 20)$
$= 4y^2 - 16y + 4x^2 - 32x + 80$

i) $y^2 + 2y + x^2 + 4x + 5 = 4y^2 - 16y + 4x^2 - 32x + 80$
$0 = 3x^2 + 3y^2 - 36x - 18y + 75$
$(\div 3)$ $x^2 + y^2 - 12x - 6y + 25 = 0$

ii) $x^2 - 12x + y^2 - 6y + 25 = 0$
$(x-6)^2 + (y-3)^2 = -25 + 36 + 9$
$(x-6)^2 + (y-3)^2 = 20$
A circle
centre (6, 3)
radius $= \sqrt{20} = 2\sqrt{5}$

b) i)

x	0	15	30	45	60
f(x)	20	26	24	19	15

$\frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 \ldots)\right]$

$h = 15$ ✓

$= \frac{15}{2}\left[(20 + 15) + 2(26 + 24 + 19)\right]$

$= \frac{15}{2}(35 + 136)$

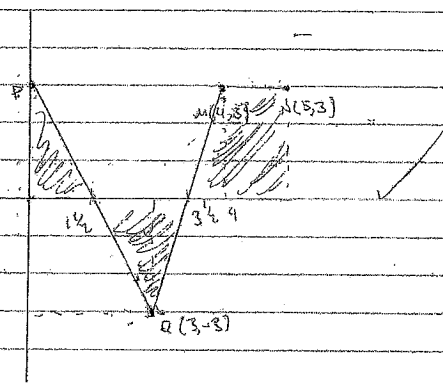
$= 1282\frac{1}{2}\ m^2$ ✓

ii) $1282\frac{1}{2} \div 0.03 = 38475\ m^2$
$1282\frac{1}{2} \times 0.03 = 38.475\ m^2$

Question 5.

a) P(0, 3), Q(3, −3), M(4, 3) and N(5, 3)

i) Sketch $y = f(x)$



ii) $\int_0^3 f(x)\,dx$ $\left(\frac{1}{2}\times\frac{3}{2}\times 3\right) - \left(\frac{1}{2}\times 2\times 3\right) + \left(\frac{1}{2}\times\frac{1}{2}\times 3\right) + (1\times 3)$

$= \frac{9}{4} - 3 + \frac{3}{4} + 3$
$= 3$ ✓

b) i) $\int_0^6 (9 - x^2)\,dx$ ii) $y = 9 - x^2$ $(3-x)(3+x)$

$= \left[9x - \frac{x^3}{3}\right]_0^6$

$F(6) = 54 - 72$
$= -18$
$F(0) = 0$
$F(6) - F(0) = -18$ ✓

$A = \left|2\int_0^3 9 - x^2\,dx\right| + \left|\int_3^6 9 - x^2\,dx\right|$

$= 2\left[9x - \frac{x^3}{3}\right]_0^3 + \left[9x - \frac{x^3}{3}\right]_3^6$

$F(3) = 27 - 9$
$= 18$
$F(0) = 0$
$F(3) - F(0) = 18 \times 2 = 36$

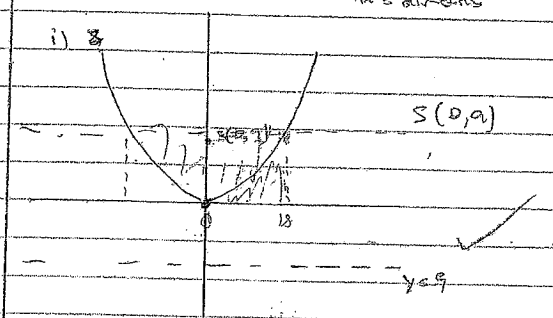
$F(6) = 54 - 72$
$= -18$
$F(3) = 27 - 9$
$= 18$

$|F(6) - F(3)| = |-36|$
$A = 36 + 36$ ✓
$= 72\ u^2$

Question 6.

a) $x^2 = 36y$ $\quad 4a = 36$ $\quad a$ = focal length, $-a$ = directrix $\quad a = 9$

i)



ii) $x^2 = 36(9)$

$x = \pm 18$ ✓

$$\int_0^{18} \frac{x^2}{36}\,dx$$

$$= \left[\frac{x^3}{108}\right]_0^{18}$$

$F(18) = 54$ ✓

$F(0) = 0$

Area under curve $= 54 \times 2$ ✓

$= 108$

Required

Area $= (36 \times 9) - 108$

$= 216u^2$ ✓

iii) $y = \frac{x^2}{36}$

$V_x = \pi \int y^2 \cdot dx$

$$V = \pi \int_0^6 \left(\frac{x^2}{36}\right)^2 \cdot dx$$

$$= \pi \left[\frac{x^4}{1296}\right]_0^6$$

$= \pi\ u^3$ ✓

b) $y'' = (x+5)^{3/5}$

$$y' = \frac{5(x+5)^{8/5}}{8} + C$$

$$2 = \frac{5(-5+5)^{8/5}}{8} + C$$

$C = 2$

$$\therefore y' = \frac{5(x+5)^{8/5}}{8} + 2$$ ✓

$$y = \frac{25(x+5)^{11/5}}{88} + 2x + k$$ sub $(-5, 10)$

$0 = -10 + k$

$10 = k$

$$y = \frac{25(x+5)^{11/5}}{88} + 2x + 10$$ ✓