Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2007
ASSESSMENT TASK 2

Mathematics

Examiner: G. Rezcallah

General Instructions

- Working time – 65 minutes
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Mr Ireland
- ○ Mr Lowe
- ○ Mr Rezcallah
- ○ Mr Fletcher
- ○ Mr Ee
- ○ Mr Trenwith
- ○ Mr Weiss

Student Number: /

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total
Mark	/13	/12	/11	/13	/11	/60	/100

Question 1 (13 marks) — Marks

(a) Write down the primitive function of $\sqrt{x}$ — 2

(b) Evaluate:

(i) $\int_0^1 (5x^4 - 3x^2 + 7)\,dx$ — 2

(ii) $\int_{-1}^1 (2y-1)^5\,dy$ — 3

(c) The curve $y = f(x)$ has a gradient function $\frac{dy}{dx} = 3 - 4x$. — 3

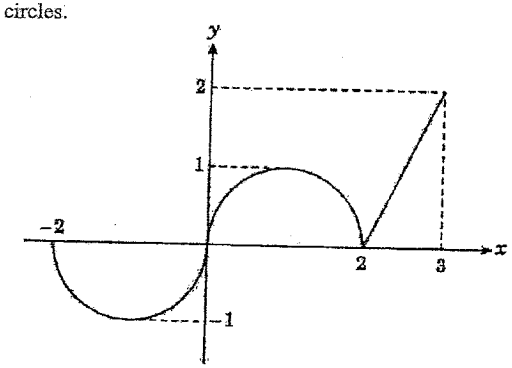
The curve passes through the point (1, -1). Find the equation of the curve.

(d) Find the value of A if — 1

$$2x^2 - 3x + 5 \equiv A(x-1)^2 + x + 3$$

(e)

The diagram below illustrates a function $y = f(x)$ for $-2 \le x \le 3$. It consists of 1 line segment and 2 semi circles.



Evaluate $\int_{-2}^{3} f(x)\,dx$. — 2

Question 2 (12 marks) Start a NEW Page. **Marks**

(a) Sketch the locus of point P(x, y) which moves so that it is always a distance of 2 units from the point (-2, 0). Hence write down its equation. 2

(b) Solve the equation $x^4 - 7x^2 + 12 = 0$ 3

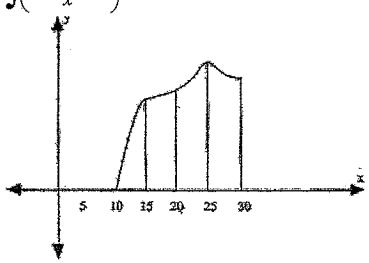
(c) A point Q(x, y) moves so that it is equidistant from the point (1,2) and the line $y = -2$.
Describe the locus of point Q geometrically. (**Do not find its equation.**). 1

(d) If α and β are the roots of the equation $2x^2 - 7x - 5 = 0$, find the values of
(i) $\alpha + \beta$ 1
(ii) $\alpha\beta$ 1
(iii) $(\alpha + 1)(\beta + 1)$ 2
(iv) $(\alpha + 1)^{-1} + (\beta + 1)^{-1}$ 2

Question 3 (11 marks) Start a NEW Page.

(a) Find:
(i) $\int (2x - 1)(2x + 1)dx$ 2
(ii) $\int \left(\frac{2x^5 + 3}{x^5} \right) dx$ 2

(b)

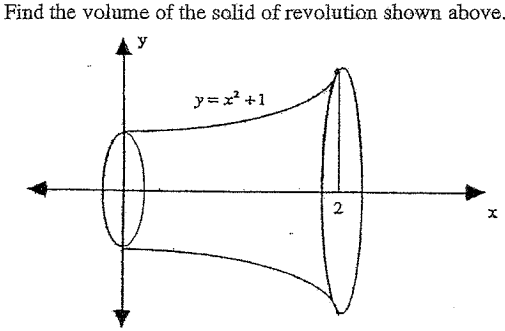


x	10	15	20	25	30
y	0	13	17	22	16

3

Find the approximate area under the curve shown above using the Trapezoidal rule.

(c) Find the volume of the solid of revolution shown above.



4

Question 4 (13 marks) Start a NEW Page. **Marks**

(a) A parabola has vertex V(3,1) and directrix $y = -1$
Find the equation of the parabola. 2

(b) Find the values of k in the quadratic equation $x^2 - 5x + k - 1 = 0$ if
(i) one root is equal to 2 2
(ii) one root is the reciprocal of the other. 2

(c) For the parabola $x^2 = 8y - 24$, find the coordinates of the focus. 2

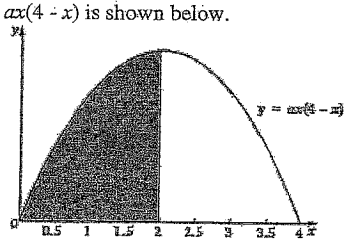
A (-3, 1) and B (3, -1) are two fixed points. The point P(x, y) is a variable point which moves such that $PA^2 + PB^2 = 70$
(i) Find the equation of the locus of point P. 3
(ii) Describe this locus in geometrical terms, stating its important features. 2

Question 5 (11 marks) Start a NEW Page.

(a) (i) Solve: $(4 + k)(1 - k) < 0$ 1
(ii) For what values of k is the quadratic expression $kx^2 + 4x + (k + 3)$ positive definite?(Hint: part (i) may be useful.) 3

(b) (i) Differentiate $(x^2 + 3)^5$ 1
(ii) Hence, find $\int x(x^2 + 3)^4 dx$ 1

(c) The graph with equation $y = ax(4 - x)$ is shown below.



3

The area of the shaded region is 40 square units. Find the value of a.

(d) If $\int_{-1}^{5} g(x)dx = 7$, find the value of : (i) $\int_{5}^{-1} g(x)dx$ 1

(ii) $\int_{-1}^{5} [3g(x) + 2]\,dx$ 2

Solution of 2 Unit Task 2 - 2007

Question 1:

(a) $\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

$= \frac{2}{3}x\sqrt{x} + C.$

(b) (i) $\int_0^1 (5x^4 - 3x^2 + 7)\,dx = \left[x^5 - x^3 + 7x\right]_0^1$

$= (1 - 1 + 7) - 0 = 7.$

(ii) $\int_{-1}^{1} (2y-1)^5\,dy = \left[\frac{(2y-1)^6}{2\times 6}\right]_{-1}^{1}$

$= \frac{1}{12}\left[(2-1)^6 - [-2-1]^6\right]$

$= \frac{1}{12}\left(1 - 3^6\right) = -\frac{728}{12} = -\frac{182}{3}.$

(c) $y = \int (3-4x)\,dx = 3x - 2x^2 + C.$

$(1,-1)$ $\quad -1 = 3(1) - 2(-1)^2 + C.$

$-1 = 3 - 2 + C \Rightarrow C = -2$

$y = 3x - 2x^2 - 2.$

(d) $2x^2 \equiv Ax^2 \ldots \Rightarrow A = 2$

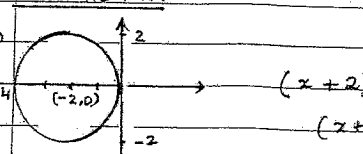
(e) $\int_{-2}^{3} f(x) = -A + A + \frac{1}{2}(1)(2)$ where A = Area of semi-circle

$= 1.$

Marks
(a) 0 marks for differentiation
✓ for
✓ for +C if integration is attempted correctly.
(b)
✓
✓ for 7.
(ii)
✓ for correct ∫ion.
✓ for subing correctly.
✓ for $-\frac{728}{12} = -\frac{182}{3}$
(c)
✓
✓ for method of finding C.
✓ for correct y.
(d)
✓ for A = 2.
(e)
✓ for correct working
✓ for 1.
No marks for adding the areas.

Question 2:

(a)



$(x+2)^2 + (y-0)^2 = 2^2$

$(x+2)^2 + y^2 = 4$

(b) $x^4 - 7x^2 + 12 = 0.$

$(x^2-4)(x^2-3) = 0.$

$x^2 = 4$ or $x^2 = 3$

$x = \pm 2$ or $x = \pm\sqrt{3}.$

(c) The Locus of point Q is a parabola of focus (1,2) and directrix $y = -2.$

Marks
(a)
✓ for sketch
✓ for equation.
(b)
✓ for factoring.
✓ for each 2 correct
✓ for other 2 correct.
(c)
✓ for mentioning parabola

(d) $2x^2 - 7x - 5 = 0.$

(i) $\alpha + \beta = -\frac{b}{a} = \frac{7}{2}.$ (ii) $\alpha\beta = -5/2$

(ii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= -\frac{5}{2} + \frac{7}{2} + 1$

$= 2.$

(iii) $(\alpha+1)^{-1} + (\beta+1)^{-1} = \frac{1}{\alpha+1} + \frac{1}{\beta+1}$

$= \frac{\beta+1+\alpha+1}{(\alpha+1)(\beta+1)}$ ← part (ii)

$= \frac{(\alpha+\beta)+2}{2}$

$= \frac{\frac{7}{2}+2}{2} = \frac{11}{4} = 2.75$

Marks.
(i) ✓ for 7/2
(ii) ✓ for $\alpha\beta = -5/2$
(iii) ✓ for correct expansion
✓ for correct answer
(iii)
✓ for correct setting
✓ for $\frac{11}{4}$ or 2.75.

Question 3

(a) (i) $\int (2x-1)(2x+1)\,dx = \int (4x^2 - 1)\,dx.$

$= \frac{4x^3}{3} - x + C.$

(ii) $\int \frac{2x^5+3}{x^5}\,dx = \int (2 + 3x^{-5})\,dx.$

$= 2x + \frac{3x^{-4}}{-4} + C.$

or $= 2x - \frac{3}{4x^4} + C$

(b) $A = \frac{h}{2}\left[1\times y_1 + 2(y_2+y_3+y_4) + y_5\right]$ where $h = 5$

$= \frac{5}{2}\left[0 + 2(13+17+22) + 16\right]$

$= \frac{5}{2}\times 120 = 300 \text{ unit}^2.$

(c) $V = \pi\int_0^2 y^2\,dx = \pi\int_0^2 (x^2+1)^2\,dx.$

$= \pi\int_0^2 (x^4 + 2x^2 + 1) = \pi\left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_0^2$

$= \pi\left[\frac{32}{5} + \frac{16}{3} + 2 - 0\right]$

$= \frac{206\pi}{15} \text{ unit}^3$ or 43.14 unit^3 or $13\frac{11}{15}\pi \text{ unit}^3.$

N.B: For (c) No marks will be given for $V = \pi\int x^2\,dy$; No marks for finding areas.

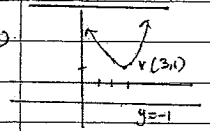
Marks.
(a) (i)
✓ for multiplying.
✓ for answer.
(ii)
✓ for simplifying correctly leading to ∫ion.
✓ for answer.
N.B: Do not penalise for +C as it was done in 1(a)
(b) ✓ for h = 5.
✓ for correct rule.
✓ for 300 unit²
✓ for correct V.
✓ for integration including squaring.
✓ for correct subing.
✓ for answer.
N.B: 1 mark will be deducted if no units are shown in both (b) and

Solution of 2 Unit Task 2 – 2007

Marks

Question 4:

(a)



$a = 2$.

$(x-h)^2 = 4a(y-k)$

$(x-3)^2 = 8(y-1)$

or $x^2 - 6x + 17 = 8y$.

(a) ✓ for $a = 2$.

✓ for correct equation

(b) $x^2 - 5x + k - 1 = 0$.

(i) $x = 2 \Rightarrow 2^2 - 5(2) + k - 1 = 0$.

$4 - 10 - 1 + k = 0$

$k = 11 - 4 = 7$.

(b) (i) ✓ for subing $x = 2$

✓ for $k = 7$.

(ii) $\alpha = \frac{1}{\beta} \Rightarrow \alpha\beta = 1$.

$\frac{k-1}{1} = 1 \Rightarrow k = 2$.

(ii) ✓ for $\alpha\beta = 1$

✓ for $k = 2$.

(d) $PA^2 + PB^2 = 70$.

(i) $(x+3)^2 + (y-1)^2 + (x-3)^2 + (y+1)^2 = 70$

$x^2 + 6x + 9 + y^2 - 2y + 1 + x^2 - 6x + 9 + y^2 + 2y + 1 = 70$.

$2x^2 + 2y^2 + 20 = 70$.

$2(x^2 + y^2) = 50$.

$x^2 + y^2 = 25$.

(i) ✓ for correct equ

✓ for correct expa

✓ for simplifying

(ii) The Locus is a circle of centre $(0,0)$ and radius $= 5$.

✓ for circle

✓ for centre & rad

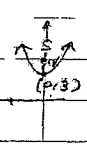
or correct Locus from working

(c) $x^2 = 8y - 24$

$x^2 = 8(y-3)$ $V(0,3)$

$4a = 8 \Rightarrow a = 2$.

Focus $(0,5)$



✓ for $a=2$ or $V(0$

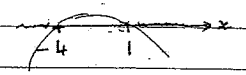
✓ for focus.

Solution of 2 Unit Task 2 – 2007

Marks.

Question 5: (12 marks).

(a) (i) $(4+k)(1-k) < 0$.



$k < -4$ or $k > 1$.

(a) (i)

✓ for answer.

(ii) $\Delta = 4^2 - 4k(k+3)$.

$= 16 - 4k^2 - 12k$.

$= 4(4 - 3k - k^2)$

$= 4(4+k)(1-k) < 0$.

$\Delta < 0 \Rightarrow k < -4$ or $k > 1$.

But $a = k > 0$

$\therefore \underline{k > 1}$

(ii) ✓ for stating $\Delta < 0$ and $a > 0$.

✓ for answer of $\Delta < 0$

✓ for correct $k > 1$

(b) (i) $\frac{d}{dx}(x^2+3)^5 = 5(2x)(x^2+3)^4$.

$= 10x(x^2+3)^4$.

(b)

✓ correct answer.

(ii) $\int 10x(x^2+3)^4\,dx = (x^2+3)^5 + C$

$\int x(x^2+3)^4\,dx = \frac{1}{10}(x^2+3)^5 + C$.

✓ for correct $\int$ (Do not penalise for $+C$)

(c) $\int_0^2 ax(4-x) = 40$.

$a\int_0^2 (4x - x^2) = 40 \Rightarrow a\left[2x^2 - \frac{x^3}{3}\right]_0^2 = 40$.

$a\left[2\times 4 - \frac{8}{3}\right] = 40 \Rightarrow \frac{16a}{3} = 40$.

$\Rightarrow a = \frac{120}{16} = \underline{7.5}$.

(c)

✓ for correct integral.

✓ for correct working.

✓ for correct answer.

(d) Given: $\int_{-1}^{5} g(x)\,dx = 7$

(i) $\int_5^{-1} g(x)\,dx = -7$

(d)

(i) for -7.

(ii) $\int_{-1}^{5}[3g(x) + 2] = 3\int_{-1}^{5} g(x)\,dx + \int_{-1}^{5} 2\,dx$.

$= 3\times 7 + \left[2x\right]_{-1}^{5}$

$= 21 + 10 - (-2) = 21 + 12 = \underline{33}$.

(ii)

✓ for correct setting.

✓ for final answer.