NORTH SYDNEY GIRLS

Year 11 Term 4 Assessment

2005

MATHEMATICS

Name: ____________________

Class: ____________________

Time Allowed: 1 hour + 2 minutes reading time

Instructions:

- Start each question on a new page
- Write on one side of the page only
- Write your name on each sheet of your own paper
- Hand in each question stapled separately with the question number clearly marked including a sheet with the question number and your name on it for non-attempts
- Each question is worth equal marks
- **Answer all 8 questions**

This task is worth 10% of the HSC Assessment Mark

Question 1 (6 Marks)

a. If α and β are roots of the quadratic equation $2x^2 - 3x - 1 = 0$. Find

i. $\alpha + \beta$ (1)

ii. $\alpha\beta$ (1)

iii. $\alpha^2\beta^3 + \alpha^3\beta^2$ (1)

iv. $\alpha^{-1} + \beta^{-1}$ (1)

b. Find the values of x for which

$x^2 - 7x + 10 > 0$ (2)

Question 2 (6 Marks)

a. Find the range of values of p for which the equation $x^2 - 3 = 2x + 2p$ has no real roots (2)

b. Find the values of k for which the equation $x^2 - (k + 3)x + 4k = 0$ has

   i. equal roots (2)

   ii. roots which are equal in magnitude but of opposite sign (2)

Question 3 (6 Marks)

a. Find the minimum value of the function $f(x) = 2x^2 + 6x - 5$ and state where this occurs (2)

b. Find values of a, b and c for which

$3x^2 + 5x - 1 \equiv ax(x + 3) + bx^2 + c(x + 1)$ (3)

c. Explain why $y = 2x^2 - 4x + 9$ is positive definite. (1)

Question 4 (6 Marks)

a. Solve $1 + \dfrac{4}{x} + \dfrac{4}{x^2} = 0$ (2)

b. Solve $(2m - 3)^2 - 8(2m - 3) + 15 = 0$ (2)

c. Solve $2^{2x} - 9.2^x + 8 = 0$ (2)

Question 5 (6 Marks)

a. Derive the equation of the locus of a point $P(x,y)$ that moves so that it is always 9 units from the point $(5, -2)$ (3)

b. Derive the equation of the locus of a point $P(x,y)$ that moves so that it is always equidistant from the point $(2, -3)$ and the line $y = 7$. (3)

Question 6 (6 Marks)

a. Find the centre and radius of the circle with equation given by

$x^2 + 2x + y^2 + 12y - 12 = 0$ (3)

b. Write the equation of the parabola with focus at $(0,4)$ and directrix $y = -4$ (1)

c. Find the focus and directrix of the parabola

$x^2 - 2x + 2y - 3 = 0$ (2)

Question 7 (6 Marks)

a. An integer is chosen at random from the numbers 1 to 21. Find the probability that the integer is

   i. divisible by 3 (1)

   ii. divisible by 3 and 5 (1)

   iii. divisible by 3 or 5 (1)

b. A bag contains three black balls and one white ball. A second bag contains two black balls and three white balls. Andrew takes one ball at random from each bag and places them in a third bag.

   i. draw a probability tree to show all possible outcomes (1)

   ii. what is the probability that the third bag contains

      1. two black balls (1)

      2. one white and one black ball (1)

Question 8 (6 Marks)

a. A box contains n marbles, p of which are green, and the remaining q are red. If one marble is drawn at random, what is the probability that it is green. (1)

b. Three points are marked on a circle on the ground. Amy stands in one of them, draws a marble from the box in part (a), and then returns it to the box. If the marble drawn is green, Amy moves clockwise to the next point. If red, she moves anticlockwise to the next point. She then continues the process. Show that

   i. the probability P of Amy being back where she started after two moves is given by $P=\frac{2pq}{n^2}$ (1)

   ii. the probability that, after three moves, Amy has yet to return to her starting point is $\frac{P}{2}$ (2)

   iii. the probability that Amy will not be back at her starting point after three moves is $\frac{3P}{2}$ (2)

Hint: Draw a tree diagram

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Solutions

Question 1

a i) $\alpha+\beta = \frac{-b}{a}$
$= \frac{3}{2}$.

ii) $\alpha\beta = \frac{c}{a}$.
$= \frac{-1}{2}$

iii) $\alpha^2\beta^3+\alpha^3\beta^2 = \alpha^2\beta^2(\beta+\alpha)$
$=(\alpha\beta)^2(\alpha+\beta)$
$= (-\frac{1}{2})^2(\frac{3}{2})$
$= \frac{3}{8}$

iv) $\alpha^{-1}+\beta^{-1} = \frac{1}{\alpha}+\frac{1}{\beta}$.
$= \frac{\alpha+\beta}{\alpha\beta}$.
$= \frac{3}{2} \div \frac{-1}{2}$.
$= -3$

b. $x^2-7x+10>0$
$(x-5)(x-2)>0$.
$x>5$ or $x<2$

Question 2

a. $x^2-2x-2p-3=0$
$x^2-2x-(2p+3)=0$.
$\Delta = b^2-4ac$.
$= (-2)^2+4(2p+3)$.
$= 4+8p+12$.
$= 8p+16$.
for unreal roots $\Delta<0$.
$8p+16<0$.
$8p<-16$
$p<-2$

b. $x^2-(k+3)x+4k=0$.
i) $\Delta = [-(k+3)]^2-4(4k)$.
$= (k+3)^2-16k$.
$= k^2+6k+9-16k$.
$= k^2-10k+9$.
for equal roots $\Delta=0$
$k^2-10k+9=0$.
$(k-1)(k-9)=0$.
$\therefore k = 1$ or 9.

ii) let the roots be α and $-\alpha$.
$\alpha-\alpha = \frac{-[-(k+3)]}{1}$
$0 = k+3$.
$\therefore k=-3$.

Question 3

a. $f(x) = 2x^2+6x-5$.
$x = \frac{-b}{2a}$.
$= \frac{-6}{4}$.
$= -\frac{3}{2}$.

$f(-\frac{3}{2}) = 2(-\frac{3}{2})^2+6(-\frac{3}{2})-5$
$= -\frac{19}{2}$. or $-9\frac{1}{2}$

The minimum value of the function is $-\frac{19}{2}$ and occurs at $(-\frac{3}{2}, -\frac{19}{2})$.
or where $x=-\frac{3}{2}$.

b.
$3x^2+5x-1 \equiv ax(x+3)+bx^2+c(x+1)$
RHS $= ax(x+3)+bx^2+c(x+1)$
$= ax^2+3ax+bx^2+cx+c$.
$= (a+b)x^2+(3a+c)x+c$

$a+b=3$. ①
$3a+c=5$ ②
$c=-1$ ③
substituting in ② for c.
$3a-1=5$
$3a=6$
$a=2$
substituting in ① for a.
$2+b=3$.
$b=1$
hence $a=2$, $b=1$, $c=-1$

c. $a>0$.
$\Delta = b^2-4ac$.
$= (-4)^2-(4(2)(9)$.
$= 16-72$.
$= -56$.
$\therefore y=2x^2-4x+9$ is positive definite as $a=2$
$\therefore a>0$ and $\Delta<0$.

Question 4

a. $1+\frac{4}{x}+\frac{4}{x^2}=0$.
$x^2+4x+4=0$
$(x+2)^2=0$.
$\therefore x=-2$

$(2m-3)^2 - 8(2m-3) + 15 = 0$

Let $x = 2m-3$.

$x^2 - 8x + 15 = 0$.

$(x-5)(x-3) = 0$.

$\therefore x = 5$ or 3.

$2m-3 = 5$ or $2m-3 = 3$

$m = 8$ $\quad 2m = 6$.

$m = 4$ $\quad m = 3$.

$\therefore m = 3$ or 4.

$2^{2x} - 9 \cdot 2^x + 8 = 0$

Let $y = 2^x$

$y^2 - 9y + 8 = 0$.

$(y-8)(y-1) = 0$.

$\therefore y = 8$ or 1

$2^x = 8$ or $2^x = 1$

$2^x = 2^3$. $2^x = 2^0$

$\therefore x = 0$ or 3.

Question

a. let $P($ $)$ be a point on the loc

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$9 = \sqrt{(x-5)^2 + (y-(-2))^2}$

$9 = \sqrt{(x-5)^2 + (y+2)^2}$.

$(x-5)^2 + (y+2)^2 = 81$.

or

$x^2 - 10x + y^2 + 4y - 52 = 0$.

b. let $P(x,y)$ be a point on the locus.

$|y-7| = \sqrt{(x-2)^2 + (y-(-3))^2}$

$(y-7)^2 = (x-2)^2 + (y+3)^2$.

$y^2 - 14y + 49 = x^2 - 4x + 4 + y^2 + 6y + 9$

$-20y = x^2 - 4x - 36$.

$20y = 36 + 4x - x^2$.

Question 6

a.

$x^2 + 2x + y^2 + 12y - 12 = 0$.

$x^2 + 2x + y^2 + 12y = 12$.

$x^2 + 2x + 1 + y^2 + 12y + 36 = 12 + 1 + 36$

$(x+1)^2 + (y+6)^2 = 49$

Centre $(-1,-6)$

radius 7.

b.

$x^2 = 4($

$x^2 = 4(4)y$.

$x^2 = 16y$.

c.

$x^2 - 2x + 2y - 3 = 0$

$x^2 - 2x + 1 + 2y - 3 - 1 = 0$

$(x-1)^2 + 2y - 4 = 0$.

$(x-1)^2 = -2(y-2)$.

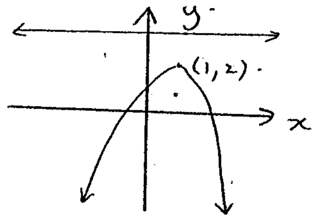
$(x-h)^2 = -4a(y-k)$

$a = \frac{1}{2}$

Vertex is $(1,2)$

focus is $(1, 1\frac{1}{2})$

directrix is $y = 2\frac{1}{2}$.



Question 7

a.

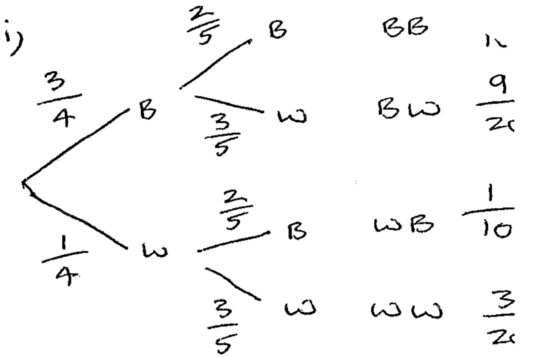
i) $\frac{7}{21} = \frac{1}{3}$.

ii) $\frac{1}{21}$

iii) $\frac{10}{21}$

b.

i)



ii)

1. $\frac{3}{10}$.

2. $\frac{9}{20} + \frac{1}{10} = \frac{11}{20}$.

Question 8

a.

$\frac{p}{n}$.

b.

i) $P(\text{at starting point}) = P(CA) + P(AC$

$= \frac{p}{n} \times \frac{q}{n} + \frac{q}{n} \times$

$= \frac{pq}{n^2} + \frac{pq}{n^2}$.

$= \frac{2pq}{n^2}$ as requi

yet to return) $= P(CCA) + P(AAC) \, . \, \underline{<0}$

$$= \frac{p}{n} \times \frac{p}{n} \times \frac{q}{n} + \frac{q}{n} \times \frac{q}{n} \times \frac{p}{n}.$$

$$= \frac{p^2q}{n^3} + \frac{q^2p}{n^3}.$$

$$= \frac{pq(p+q)}{n^3}$$ but $p+q=n$.

$$= \frac{pqn}{n^3}.$$

$$= \frac{pq}{n^2}.$$

$$= \frac{p}{2}.$$ as required

ii)

'(will not be back) $= 1 - P(\text{will be back})$.

'(will be back) $= P(CCC) + P(AAA)$.

$$= \frac{p}{n} \times \frac{p}{n} \times \frac{p}{n} + \frac{q}{n} \times \frac{q}{n} \times \frac{q}{n}.$$

$$= \frac{p^3}{n^3} + \frac{q^3}{n^3}.$$

$$= \frac{p^3+q^3}{n^3}.$$

$$= \frac{(p+q)(p^2-pq+q^2)}{n^3}$$ but $p+q=n$

$$= \frac{(p^2-pq+q^2)}{}$$

$$= \frac{(p+q)^2 - 3pq}{n^2}.$$

$$= \frac{(p+q)^2}{n^2} - \frac{3pq}{n^2}.$$ but $p+q=n$.

$$= \frac{n^2}{n^2} - \frac{3pq}{n^2}.$$

$$= 1 - \frac{3p}{2}.$$

$P(\text{will not be back}) = 1 - \left(1 - \frac{3p}{2}\right)$

$$= \frac{3p}{2}.$$ as required.