NSW INDEPENDENT SCHOOLS

2001
Higher School Certificate
Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used
- Write using black or blue pen
- Draw diagrams using pencil
- Write your student number and/or name at the top of every page

Attempt ALL questions

All questions are of equal value

All necessary working should be shown in every qeustion. Marks may be deducted for careless or badly arranged work

This paper MUST NOT be removed from the examination room

STUDENT NAME/NUMBER ______________________

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QUESTION 1

Marks

(a) Factorise completely:

(i) $a^2 - 4a + 4$ **1**

(ii) $a^2 - 4a + 4 - 9b^2$ **1**

(b) Express $\sqrt{243} + 2\sqrt{75}$ in the form $a\sqrt{b}$, where a and b are integers. **1**

(c) A Sports Jacket is on sale at George's Menswear for \$198. To arrive at this selling price, George takes the cost price of the shirt from the manufacturer, adds 20% profit and to this he then adds 10% GST. What was the cost price of the shirt? **2**

(d) From the top of one of the Sydney Harbour Bridge pylons, the angle of depression to a ferry moored at Circular Quay is 38°. The horizontal distance from the base of the pylon to the ferry is 2.6 kilometres.

(i) Draw a neat sketch showing this information. **1**

(ii) Calculate the height of the pylon (correct to the nearest metre) **1**

(e) Express $\dfrac{7}{\sqrt{5} - 2}$ with a rational denominator. **2**

(f) Simplify: $\dfrac{1}{x-1} - \dfrac{1}{x+1}$ **1**

(g) Solve for x: $|2 - 3x| \geq 5$. **2**

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QUESTION 2 *(Start a new page)* **Marks**

(a) Consider the function :

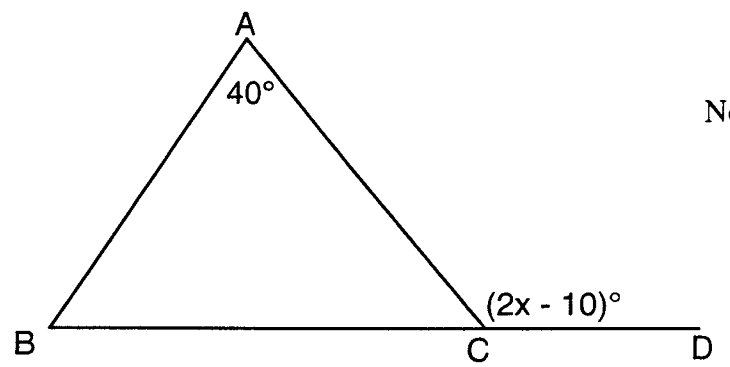
$$f(x)=\begin{cases}5 \text{ for } x<0\\ x^2+1 \text{ for } x\geq 0\end{cases}$$

(i) Find the value of $f(-3)$ **1**

(ii) Write down the value of $f(a^2)$ **1**

(iii) Draw a neat sketch of the graph of the function $y=f(x)$ for the domain : $-3\leq x\leq 3$. **2**

(b) **2**



Not to Scale

In the diagram above ΔABC is isosceles, with AB = AC.
Copy the diagram onto your worksheet.

Find the value of *x*, giving reasons.

(c) On separate diagrams, draw neat sketches of each of the following functions, showing any intercepts with the *x* and *y*-axes

(i) $y=4-x$ **1**

(ii) $y=4-x^2$ **1**

(iii) $y=\sqrt{4-x^2}$ **1**

(iv) $y=\dfrac{4}{x},\ x\neq 0$. **1**

(d) State the domain and range of the function in (c)(iii). **2**

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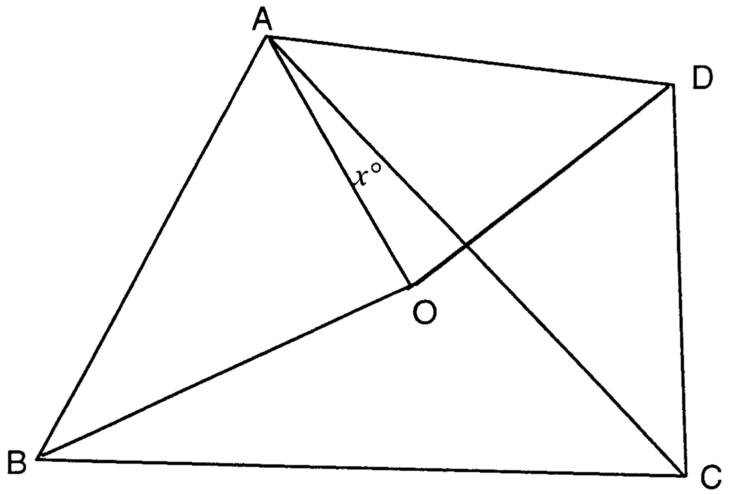
QUESTION 3 *(Start a new page)* **Marks**

(a)



Not to Scale

In the diagram above, triangles ABC and ADO are equilateral. $\angle OAC = x°$.

Copy the diagram onto your worksheet.

(i) Show that $\angle BAO = \angle DAC$ **1**

(ii) Prove that $\Delta AOB \equiv \Delta ADC$ **2**

(b) For the parabola $(y-3)^2 = -12(x+1)$

(i) State the coordinates of the vertex. **1**

(ii) State the coordinates of the focus. **1**

(iii) Write down the equation of the directrix. **1**

(iv) Draw a neat sketch of the parabola clearly showing the vertex, focus and directrix. **1**

(c) A(a,6) and B(-5,b) are two points on the number plane. The midpoint of AB is M(3,-2). Find the values of a and b. **2**

(d) Solve the simultaneous equations: **2**

$$2x + y = -2$$
$$4x - 3y = 31$$

(e) A function $f(x) = x^n + x^{3n}$ is an odd function. Give a possible value of n ? **1**

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QUESTION 4 *(Start a new page)* **Marks**

(a) Prove that : $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=2\sec A$ **3**

(b) For what values of K, does the equation $x^2+2x+4-K=0$, have real, unequal roots? **2**

(c) A (2,3), B(4,2) and C(a,6) are points on the number plane.

(i) Find the gradient of AB. **1**

(ii) Show that the equation of AB is $x+2y-8=0$. **1**

(iii) The perpendicular distance from C to AB is $2\sqrt{5}$ units. Find the two possible values of a. **2**

(d) (i) Solve the equation : $6x^2+13x=5$ **2**

(ii) Hence or otherwise solve $6x^2+13x\le 5$ **1**

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QUESTION 5 *(Start a new page)* **Marks**

(a) Given that $\sin\alpha = \frac{3}{4}$ and $90° < \theta < 180°$, write down the exact value of $\sec\theta$. **2**

(b) Differentiate with respect to x.

(i) $\left(x^2 - 5x\right)^4$ **2**

(ii) $\dfrac{2x-3}{5-x}$ **2**

(iii) x^{n^2} **1**

(iv) $(x^2 - 5x + 7)(3x + 2)$ **2**

(c) The course for a yacht race starts from Twofold Bay (Point T). The first leg is due South for a distance of 18 kilometres to Point P. The second leg is on a bearing of 160° for a distance of 12 kilometres to point Q. The third and final leg is in a straight line from Q to T.

(i) Draw a neat sketch showing this information. **1**

(ii) Calculate length of the third leg correct to the nearest one-tenth of a kilometre. **2**

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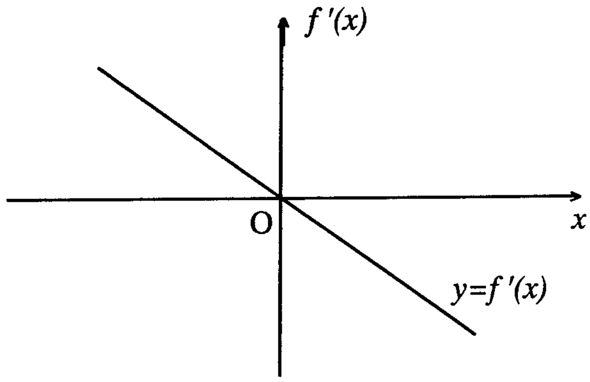
QUESTION 6 *(Start a new page)* **Marks**

(a) Given the function $y = 10x^3 - 4$. Find the value(s) of x for which $\frac{dy}{dx} = 10$ **2**

(b) **2**



The diagram above shows the graph of $y = f\,'(x)$. Given that $f(0) = 2$, sketch a graph for $y = f(x)$

(c) Solve : $2\sin 2\beta = \sqrt{3}$, for $0° \leq \beta \leq 360°$. **3**

(d) α and β are the roots of the quadratic equation $x^2 + 5x + 9 = 0$.

Find the values of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) $(\alpha - \beta)^2$ **1**

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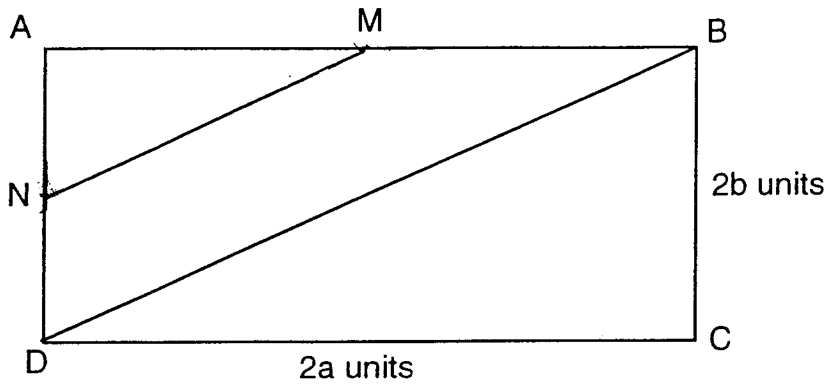
QUESTION 7 *(Start a new page)* **Marks**

(a)



In the diagram above, ABCD is a rectangle. BC = 2b units and CD = 2a units. M and N are the midpoints of AB and AD respectively. Copy the diagram onto your worksheet.

(i) Find the area of ΔANM. **1**

(ii) Show that MN is parallel to DB. **1**

(iii) What type of quadrilateral is NMBD ? **1**

(iv) Find the ratio of the area of NMBD to the area of ABCD. **2**

(b) Given the function $y = x^2 - 3x$.

(i) Draw a neat sketch of the graph of the function. **1**

(ii) Find the gradient of the tangent to the curve at the point (-1,4). **2**

(iii) Find the equation of the normal to the curve at this point. **2**

(iv) At what point does this normal meet the axis of symmetry of the curve? **2**

NSW INDEPENDENT TRIAL EXAMS
SUGGESTED SOLUTIONS
2U MATHEMATICS 2001 PRELIMINARY

Q1 (a) (i) $\underline{(a-2)^2}$

(ii) $(a-2)^2 - (3b)^2$

$= \underline{(a-2-3b)(a-2+3b)}$

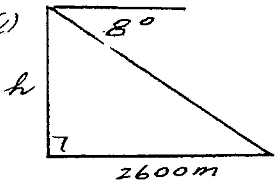
(b) $9\sqrt{3} + 2 \times 5\sqrt{3}$

$= \underline{19\sqrt{3}}$

(c) $\frac{5}{6}$ of $\frac{10}{11}$ of \$198

$= \underline{\$150}$

(d) (i)



(ii) $\tan 8° = \dfrac{h}{2600}$

$\underline{h = 365\text{ m}}$

(e) $\dfrac{7}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \dfrac{7\sqrt{5}+14}{5-4}$

$= \underline{7\sqrt{5}+14}$

(f) $\dfrac{(x+1)-(x-1)}{(x+1)(x-1)} = \underline{\dfrac{2}{x^2-1}}$

(g) $2-3x \geq 5$ $\quad -2+3x \geq 5$

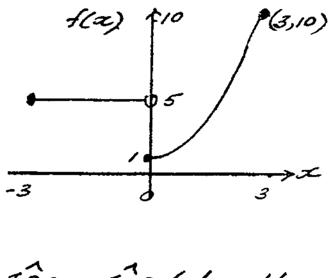
$-3x \geq 3$ $\quad 3x \geq 7$

$\underline{x \leq -1 \text{ OR } x \geq 2\frac{1}{3}}$

Q2 (a) (i) $f(-3) = \underline{5}$

(ii) $f(a^2) = \underline{a^2+1}$

(iii)



(b) $A\hat{B}C = A\hat{C}B$ (base ∠'s isos Δ)

$A\hat{B}C + A\hat{C}B + 40° = 180°$ (∠sum ΔABC)

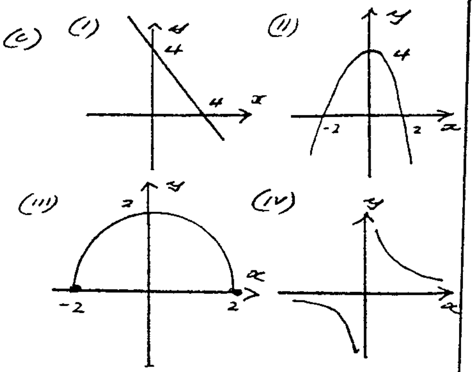
$A\hat{C}B = 70°$

$A\hat{C}B + (2x-10)° = 180°$ (BCD straight ∠)

$\therefore 70 + 2x - 10 = 180$

$\underline{x = 60}$

(c)



(d) D $\underline{-2 \leq x \leq 2}$

R $\underline{0 \leq y \leq 2}$

Q3 (a) (i) $B\hat{A}C = 60°$ (eq ΔABC)

$\therefore B\hat{A}O = (60-x)°$

sim $D\hat{A}C = (60-x)°$

$\therefore \underline{B\hat{A}O = D\hat{A}C}$

(ii) In Δ's AOB, ADC

$B\hat{A}O = D\hat{A}C$ (i)

$AO = AD$ (eq ΔAOD)

$AB = AC$ (eq ΔABC)

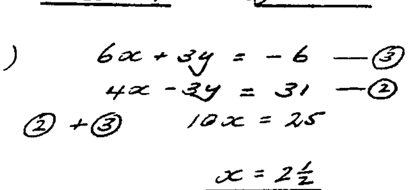
$\therefore \underline{\Delta AOB \equiv \Delta ADC \text{ (SAS)}}$

(b) (i) V $\underline{(-1,3)}$

(ii) S $\underline{(-4,3)}$

(iii) Dir $\underline{x = 2}$

(iv)



(c) $\dfrac{a-5}{2} = 3$ $\quad \dfrac{6+b}{2} = -2$

$\underline{a = 11}$ $\quad \underline{b = -10}$

(d) $6x + 3y = -6$ — ③

$4x - 3y = 31$ — ②

② + ③ $\quad 10x = 25$

$\underline{x = 2\frac{1}{2}}$

sub ① $\quad 5 + y = -2$

$\underline{y = -7}$

(e) n must be ODD

Q4 (a)

$\dfrac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$

$= \dfrac{2 + 2\sin A}{\cos A(1+\sin A)}$

$= \dfrac{2}{\cos A}$

$= \underline{2\sec A}$

(b) $\Delta > 0$

$2^2 - 4(1)(4-k) > 0$

$4 - 16 + 4k > 0$

$\underline{k > 3}$

(c) (i) $m = \dfrac{2-3}{4-2}$

$\underline{m = -\frac{1}{2}}$

(ii) $y - 3 = -\frac{1}{2}(x-2)$

$2y - 6 = -x + 2$

$\underline{x + 2y - 8 = 0.}$

(iii) $\left|\dfrac{a + 2(6) - 8}{\sqrt{1^2+2^2}}\right| = 2\sqrt{5}$

$|a+4| = 10$

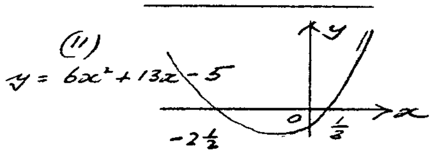
$\underline{a = 6, -14}$

(d) (i) $6x^2 + 13x - 5 = 0$

$(3x-1)(2x+5) = 0$

$\underline{x = \frac{1}{3}, -2\frac{1}{2}}$

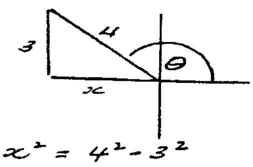
(ii)



$6x^2 + 13x - 5 \leq 0$

$\underline{-2\frac{1}{2} \leq x \leq \frac{1}{3}}$

Q5 (a)



$x^2 = 4^2 - 3^2$

$x = {}^{-}\sqrt{7}$

$\cos\theta = -\sqrt{7}/4$

$\sec\theta = \underline{-4/\sqrt{7}}$

(b) (i) $\underline{4(x^2-5x)^3(2x-5)}$

(ii) $\dfrac{(5-x)2 - (2x-3)(-1)}{(5-x)^2}$

$= \underline{\dfrac{7}{(5-x)^2}}$

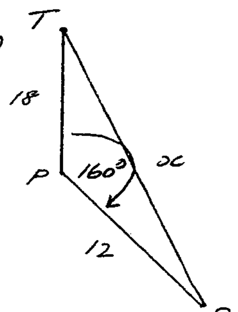
(iii) $\underline{n^2 x^{n^2-1}}$

(iv) $(x^2-5x+7)3 +$

$(3x+2)(2x-5)$

$= \underline{9x^2 - 26x + 11}$

(c) (i)



(ii)

$x^2 = 18^2 + 12^2 - 2 \times 18 \times 12 \times \cos 160°$

$x \doteqdot \underline{29.6 \text{ km}}$

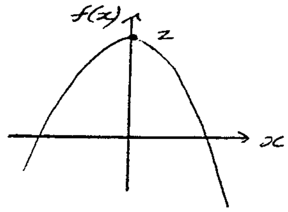
Q6 (a)

$\dfrac{dy}{dx} = 30x^2 = 10$

$x^2 = \frac{1}{3}$

$\underline{x = \pm \dfrac{1}{\sqrt{3}}}$

(b)



(c) $\sin 2\beta = \sqrt{3}/2$

$0 \le 2\beta \le 720°$

Quads 1, 2

$\therefore\ 2\beta = 60°, 120°, 420°, 480°$

$\beta = \underline{30°, 60°, 210°, 240°}$

(d) (i) $\alpha + \beta = -\dfrac{b}{a} = \underline{-5}$

(ii) $\alpha\beta = \dfrac{c}{a} = \underline{9}$

(iii) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= (-5)^2 - 2(9)$

$= \underline{7}$

(iv) $(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 7 - 2(9)$

$= \underline{-11}$

Q7 (a)

(i) AM = a units

AN = b units

∴ Area ANM $= \underline{\frac{1}{2}ab}$.

(ii) Line joining mid pts of 2 sides is // 3rd side

(iii) Trapezium

(iv) ABCD = 4ab.

ABD = 2ab.

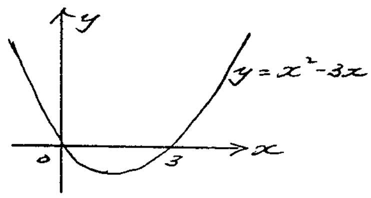
∴ NMBD = ABD − AMN

$= \frac{3}{2}ab$

∴ NMBD : ABCD $= \frac{3}{2} : 4$

$= \underline{3 : 8}$

(b) (i)



(ii) $y' = 2x - 3$

At $(-1, 4)$ $m = 2(-1) - 3$

$= -5$

(iii) Normal $m_2 = +\frac{1}{5}$

$\therefore\ y - 4 = \frac{1}{5}(x+1)$

$5y - 20 = x + 1$

$\underline{x - 5y + 21 = 0}$

(iv) Axis $x = 1\frac{1}{2}$

$\therefore\ 1\frac{1}{2} - 5y + 21 = 0$

$\underline{y = 4\frac{1}{2}}$