NSW INDEPENDENT SCHOOLS

PRELIMINARY EXAMINATION

2000

MATHEMATICS

(2 UNIT)

Time Allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board-approved calculators may be used.
- Each question attempted is to be handed in separately clearly marked Question 1, Question 2 ,... etc..
- ***The question paper must be handed to the supervisor at the end of the examination.***

STUDENT NUMBER / NAME..

Question 1

(a) Find the value of: $\sqrt{40-(mn)^2}$ correct to three significant figures given that $m = 4.6 \times 10^{-3}$ and $n = 8.1 \times 10^{-2}$. **2**

(b) Factorise completely: $w^2x - x - y + w^2y$. **1**

(c) Express $0.002\dot{3}$ as a simple fraction. **2**

(d) Expand and simplify: $(xy-3)(x^2y^2+3xy+9)$. **1**

(e) Solve: $x = 1 + \dfrac{6}{x}$ **2**

(f) From a point in the Olympic Stadium 160 metres from the base of the vertical tower holding the Olympic Flame, the angle of elevation to the top of the tower is 63°. **2**

Draw a sketch showing this information and calculate the height of the tower.

(g) Simplify: $\dfrac{x^2+y^2+2xy}{-y^2+x^2}$ **2**

Question 2. *(Start a new page)*

(a) Find rational numbers a and b such that : **2**

$$\frac{1}{3-\sqrt{5}} = a + b\sqrt{5}$$

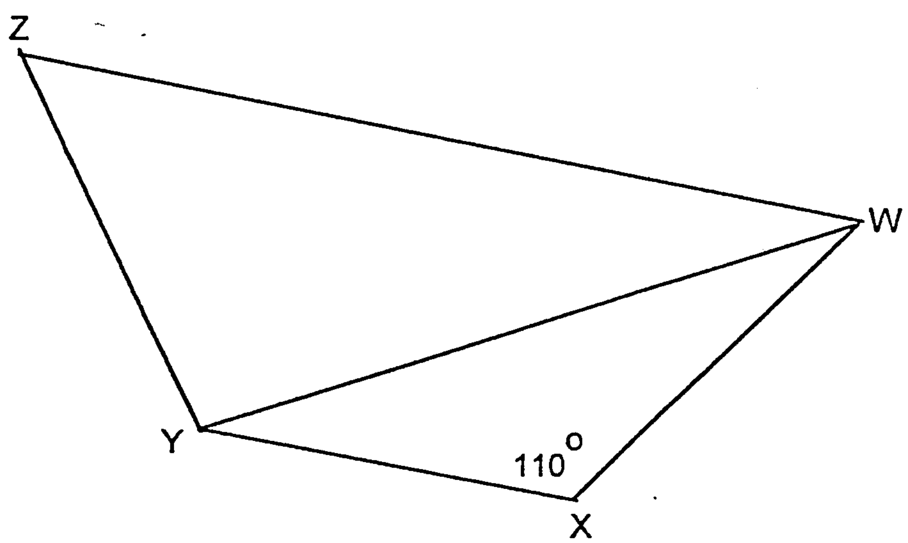
(b) The function $y = f(x)$ is defined as follows.

$$f(x) = \begin{cases} -2 \text{ for } x \le -1 \\ x+1 \text{ for } -1 < x < 2 \\ x^2+1 \text{ for } x \ge 2 \end{cases}$$

(i) Evaluate $f(-1) + f(2)$. **1**

(ii) Write an expression for $f\left(a^2+2\right)$. **1**

(c) In the diagram below YZ = WY, XY = WX and ZW // YX **3**



Copy the diagram onto your worksheet.

Find ∠WYZ, giving all reasons.

(d) On the same set of axes, sketch the graphs of : **3**

$$x^2 + y^2 = 9 \text{ and } y = 3 - x.$$

Shade the region on your graph where :

$$x^2 + y^2 \le 9$$
$$y \le 3 - x \text{ and}$$
$$x \ge 0$$

(e) Solve : $|2-3x| < 5$ **2**

Question 3. *(Start a new page)*

(a) Differentiate each of the following :

(i) $2-6x+13x^4$ **2**

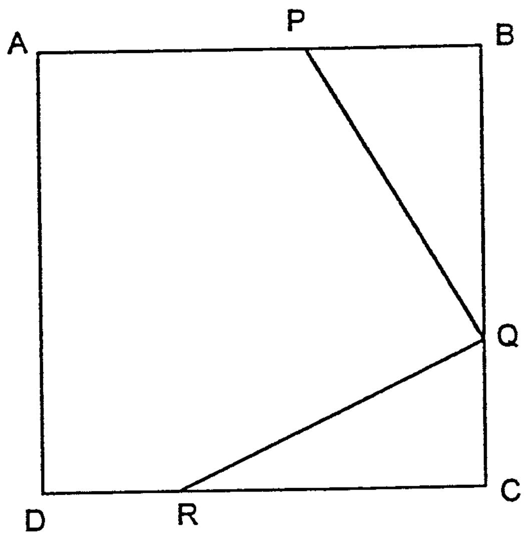
(ii) $3\sqrt[3]{x}$. **1**

(iii) $\dfrac{2x-3}{3x-2}$. **2**

(iv) $(2x-3)^9$. **1**

(b) If $f(x)=15x^{-2}-9x^3$, find the value of $f'(-1)$. **2**

(c) In the diagram below, ABCD is a square. P, Q and R are points on sides AB, BC and CD respectively such that PB = QC = RD.



Copy the diagram onto your worksheet.

(i) Prove that $\Delta BPQ \equiv \Delta CQR$. **2**

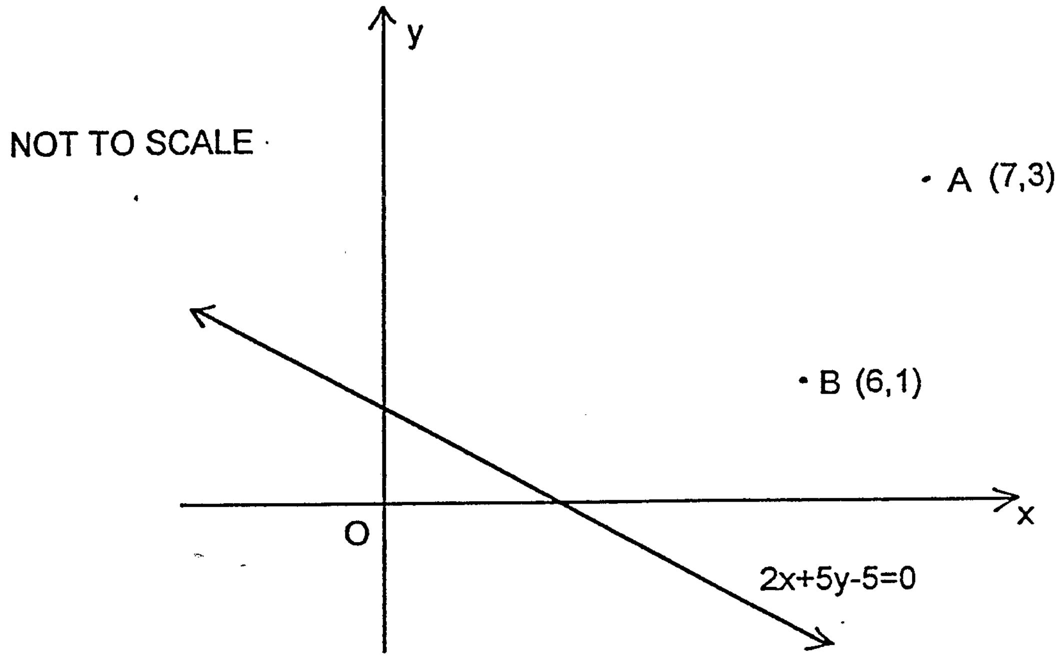
(ii) Hence or otherwise prove that PQ = RQ. **1**

(iii) Deduce that $\angle PQR = 90°$. **1**

Question 4. *(Start a new page)*

(a) Solve for x: $x^4 - 5x^2 - 36 = 0$. **2**

(b)



The diagram above shows the line $2x + 5y - 5 = 0$ and the points A (7,3) and B (6,1).

Copy the diagram onto your worksheet.

(i) Find the equation of the line AB. **2**

(ii) Find the coordinates of the point of intersection, P, of the line $2x + 5y - 5 = 0$ and the line AB. **1**

(iii) Find the shortest distance from P to the line $y + 2 = 0$. **1**

(c) The equation $2x^2 - 7x + 12 = 0$ has roots α and β. Find the value of;

(i) $\alpha + \beta$. **1**

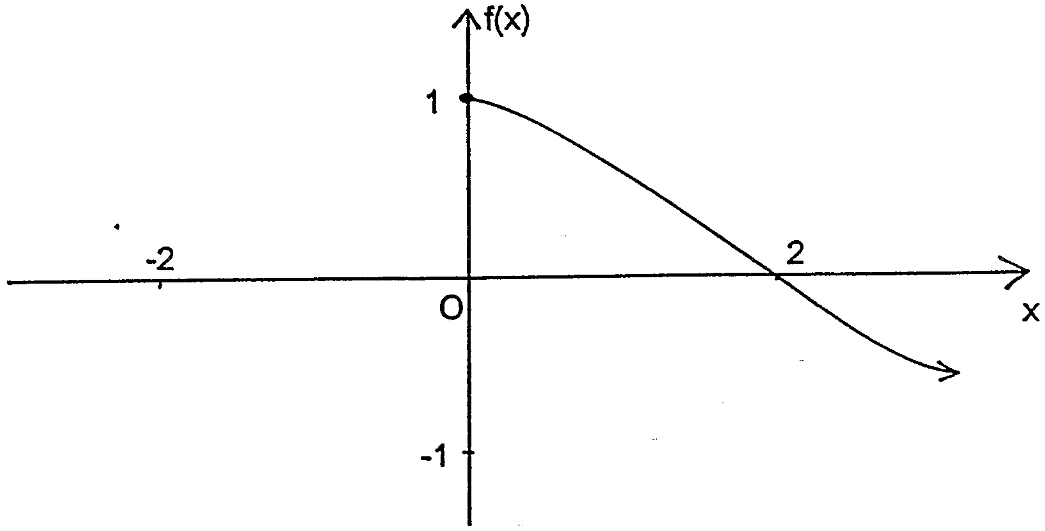
(ii) $\alpha\beta$. **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$. **1**

(d) Find A, B and C such that $A(x-1)^2 + Bx + C = x^2$ **3**

Question 5. *(Start a new page)*

(a) Part of the graph of the function $y = f(x)$ is shown below.



Draw three (3) neat copies of this graph and label them (A), (B) and (C).

Complete the graphs of $y = f(x)$ on each sketch so that :

(i) In (A), $y = f(x)$ is an EVEN function. **1**

(ii) In (B), $y = f(x)$ is an ODD function. **1**

(iii) In (C), $y = f(x)$ is neither ODD nor EVEN. **1**

(b) Prove that $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$. **2**

(c) (i) Solve for x : $\sin x = -\frac{1}{2}$: $0^0 \le x \le 360^0$. **1**

(ii) Sketch the graph $y = \sin x$: $0^0 \le x \le 360^0$. **1**

(iii) On the same set of axes draw $y = -\frac{1}{2}$. **1**

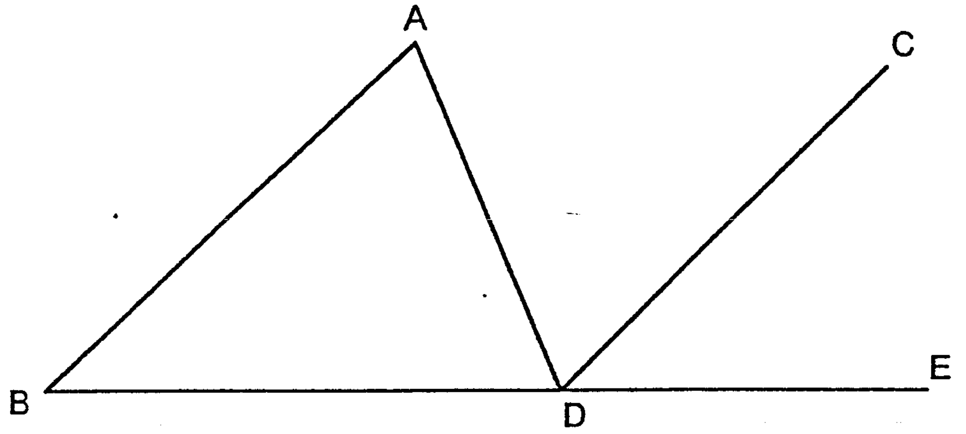
(iv) Write down the coordinates of the points of intersection of the graphs $y = \sin x$ and $y = -\frac{1}{2}$ for the given domain. **1**

(d) Draw a neat sketch of the graph $y = 4 - x^2$, showing the x and y intercepts. **3**

Hence or otherwise, solve $4 - m^2 \ge 0$.

Question 6. *(Start a new page)*

(a) In the diagram below, AB // CD. BD is produced to E and CD bisects ∠ADE. **3**



Prove that ΔABD is isosceles.

(b) Given the parabola $8y = x^2 - 6x - 23$

(i) Write the equation in the form $(x-h)^2 = -4a(y-k)$. **1**

(ii) Find the coordinates of the vertex and focus. **2**

(iii) Find the equation of the axis of symmetry of the parabola. **1**

(iv) Draw a neat sketch of the parabola showing the above information. **2**

(c) Consider the points M (-1,10) and N (6,11). The point P $(k,2k)$ lies on MN.

(i) Find the gradient of MN. **1**

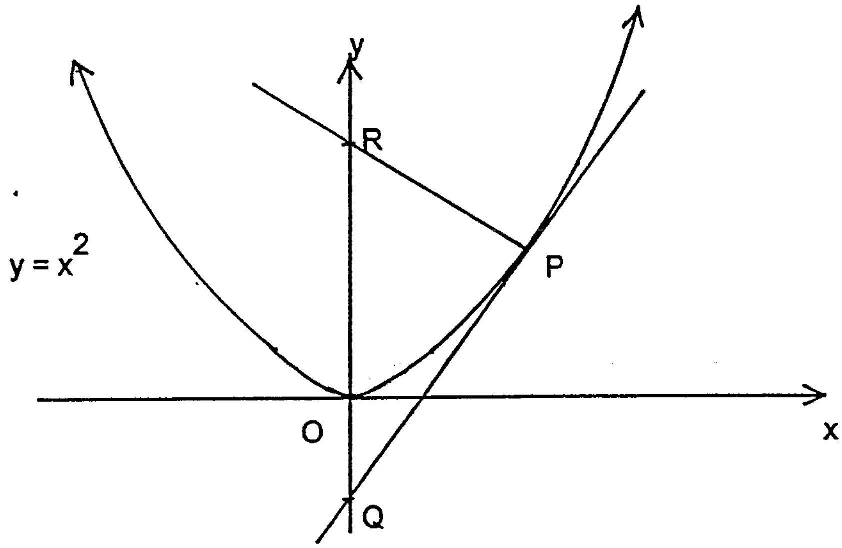
(ii) Show that the gradient of MP is $\dfrac{2k-10}{k+1}$. **1**

(iii) Hence find the value of k. **1**

Question 7. *(Start a new page)*

(a) Find the values of m, given that $(3m-1)x^2-4mx+2=0$ has equal roots. **2**

(b)



The diagram above shows the parabola $y=x^2$. The tangent and normal at P (4,16) cut the y-axis at Q and R respectively.

(i) Show that the equation of the tangent at P is $y=8x-16$. **2**

(ii) Find the equation of the normal at P. **2**

(iii) Find the coordinates of Q and R. **1**

(iv) Hence find the area of triangle PQR. **1**

(c) A seaplane flies from a Port (P) to a Quay (Q), 600 kilometres away, on a bearing of 050°T. It then flies on a course of 120°T to a Rock (R), a distance of 850 kilometres from the Quay.

(i) Draw a diagram on your worksheet showing this information. **1**

(ii) Show that $\angle PQR$ is 110°. **1**

(iii) Calculate the shortest distance from the Port to the Rock. (to the nearest kilometre). **1**

(iv) Find the bearing of the Rock from the Port (to the nearest degree). **1**

End of Paper

Q1

(a) 6.324555309

$\Rightarrow \underline{6.32}$

(b) $w^2(x+y) - 1(x+y)$

$(w^2-1)(x+y)$

$\underline{(w-1)(w+1)(x+y)}$

(c) Let $A = 0.002\dot{3}$

$1000A = 2.\dot{3}$ — ①

$10000A = 23.\dot{3}$ — ②

② − ① $9000A = 21$

$A = \frac{21}{9000}$

$= \underline{\frac{7}{3000}}$

(d) $\underline{x^3y^3 - 27}$

(e) $x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\underline{x = 3, -2}$

(f) $\tan 63° = \frac{h}{160}$

$\underline{h = 314\text{ m}}$

(g) $\frac{(x+y)^2}{(x+y)(x-y)} = \underline{\frac{x+y}{x-y}}$

Q2

(a) $\frac{1}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{3+\sqrt{5}}{9-5}$

$= \frac{3}{4} + \frac{1}{4}\sqrt{5}$

$a = \frac{3}{4} \quad b = \frac{1}{4}$

(b) (i) $f(-1) + f(2)$

$= -2 + (2^2+1)$

$= \underline{3}$

(ii) $a^2 + 2 \geq 2$

$\therefore f(a^2+2) = (a^2+2)^2 + 1$

$= \underline{a^4 + 4a^2 + 5}$

(c) Let $X\hat{W}Y = x°$

$\therefore X\hat{Y}W = x°$

(angles opposite = sides)

$2x° + 110° = 180°$

(∠ sum of △XYW)

$x° = 35°$

$Z\hat{W}Y = X\hat{Y}W = 35°$

(alt ∠'s, ZW ∥ YX)

$Y\hat{Z}W = 35°$

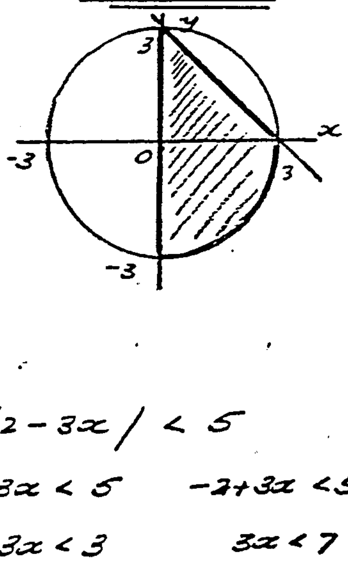
(angles opp = sides)

$W\hat{Y}Z + 35° + 35° = 180°$

(Ang sum of △ZYW)

$\therefore \underline{W\hat{Y}Z = 110°}$

(d)



(e)

$|2-3x| < 5$

$2-3x < 5 \qquad -2+3x < 5$

$-3x < 3 \qquad 3x < 7$

$x > -1 \qquad x < \frac{7}{3}$

$\underline{-1 < x < \frac{7}{3}}$

Q3

(a) (i) $\underline{-6 + 52x^3}$

(ii) $\frac{d}{dx}(3x^{\frac{1}{3}})$

$= \underline{x^{-2/3}}$

(iii) $\frac{(3x-2)2 - (2x-3)3}{(3x-2)^2}$

$= \underline{\frac{5}{(3x-2)^2}}$

(iv) $\underline{18(2x-3)^8}$

(b) $f'(x) = -30x^{-3} - 27x^2$

$f(-1) = \frac{-30}{(-1)^3} - 27(-1)^2$

$= 30 - 27$

$= \underline{3}$

(c) (i) In △'s BPQ, CQR.

$\hat{B} = \hat{C} = 90°$ (∠'s in square)

PB = QC (data)

Also. BC = DC (sides in sq)

RD = QC (data)

$\therefore$ BC − QC = DC − RD

$\therefore$ BQ = CR

$\therefore \underline{\triangle BPQ \equiv \triangle CQR \text{ (SAS)}}$

(ii) $\therefore$ PQ = RQ

(corr sides in cong △'s)

(iii) $B\hat{P}Q + P\hat{Q}B + 90° = 180$

$\therefore B\hat{P}Q + P\hat{Q}B = 90°$

$R\hat{Q}C = B\hat{P}Q$ (corr ∠'s in cong △'s)

ie $R\hat{Q}C + B\hat{P}Q = 90°$

$R\hat{Q}C + P\hat{Q}R + B\hat{P}Q = 180°$

($B\hat{Q}C$ is a str angle)

$\therefore \underline{P\hat{Q}R = 90°}$

Q4

(a) $x^4 - 5x^2 - 36 = 0$

$(x^2 - 9)(x^2 + 4) = 0$

$x^2 = 9 \quad x^2 \neq -4$

$\underline{x = \pm 3}$

(b) (i) $x = 2\frac{1}{2}, \ y = 1$

(ii) $\frac{y-1}{x-6} = \frac{2}{1}$

$y - 1 = 2x - 12$

$\underline{y = 2x - 11}$

(iii) $2x + 5(2x - 11) + 5 = 0$

$12x = 60$

$x = 5$

$y = -1$

$\underline{P(5, -1)}$

(iv) $\underline{1 \text{ unit}}$

(c) $\alpha + \beta = \frac{7}{2}$

$\alpha\beta = \underline{6}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{7}{12}$

(d) Coeff x^2 . $A = 1$

$x = 1 \quad B + C = 1$

$x = 0 \quad A + C = 0$

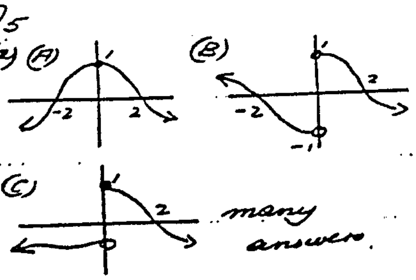
$C = -1$

$B = 2$

$\underline{A = 1}$

Q5

(a) (A) (B) (C)



(b) $(\sin^2 A + 2\sin A \cos A + \cos^2 A) + (\sin^2 A - 2\sin A\cos A + \cos^2 A)$

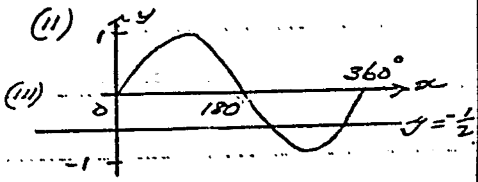
$= 2(\sin^2 A + \cos^2 A)$

$= 2 \times 1 = 2$

(c) (i) $x = 210°, 330°$

(ii)

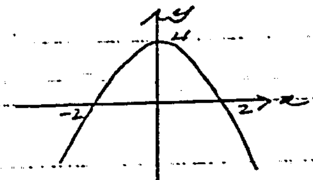
(iii)



(iv) $(210°, -\frac{1}{2})$

$(330°, -\frac{1}{2})$

(d)



$4 - m^2 \geq 0$

$\underline{-2 \leq m \leq 2}$

Q6

(a) Let $\angle C\hat{D}E = x°$

$\therefore C\hat{D}A = x°$ (CD bisects $A\hat{D}E$)

$C\hat{D}E = A\hat{B}D = x°$

(corr ∠'s, AB ∥ CD)

$C\hat{D}A = D\hat{A}B = x°$

(alt ∠'s, AB ∥ CD)

$\therefore D\hat{A}B = A\hat{B}D$

$\therefore \underline{\Delta ABD \text{ is isosceles.}}$

(b) $-8y + 32 = x^2 - 6x + 9$

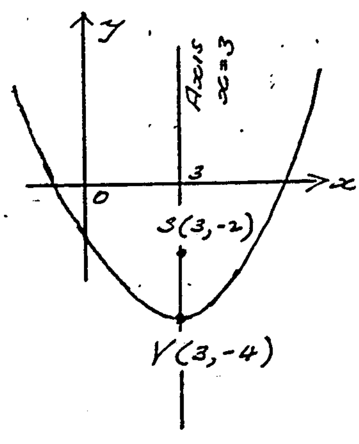
(i) $(x - 3)^2 = 4 \times 2(x + 4)$

(ii) $V(3, -4)$

$S(3, -2)$

(iii) Axis $x = 3$

(iv)



(c) (i) $\frac{11 - 10}{6 - (-1)} = \underline{\frac{1}{7}}$

(ii) $\frac{2k - 10}{k - -1} = \frac{2k - 10}{k + 1}$

(iii) $\frac{2k - 10}{k + 1} = \frac{1}{7}$

$14k - 70 = k + 1$

$13k = 71$

$\underline{k = \frac{71}{13}}$

Q7

(a) $\Delta = (-4m)^2 - 4(3m - 1) \cdot 2$

$= 16m^2 - 24m + 8$

$= 8(2m - 1)(m - 1)$

$\Delta = 0$ when $m = \frac{1}{2}, 1$

$\therefore$ Real Roots when

$\underline{m = \frac{1}{2}, 1}$

(b) (i) $y' = 2x$

at P $m_1 = 8$

$y - 16 = 8(x - 4)$

$\underline{y = 8x - 16}$

(ii) $m_2 = -\frac{1}{8}$

$y - 16 = -\frac{1}{8}(x - 4)$

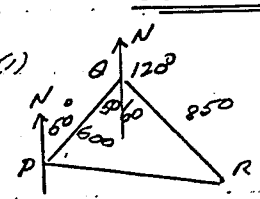
$\underline{x + 8y - 132 = 0}$

(iii) $\underline{Q(0, -16) \quad R(0, 16\frac{1}{2})}$

(iv) $A = \frac{1}{2} \cdot 32\frac{1}{2} \cdot 4$

$= \underline{65 \text{ units}^2}$

(c)(i)



(iii) $PR^2 = 600^2 + 850^2 - 2 \cdot 600 \cdot 850 \cdot \cos 110°$

$PR = \underline{1196 \text{ km}}$

(iv) $\cos O\hat{P}R = \frac{600^2 + 1196^2 - 850^2}{2 \times 650 \times 1196}$

$O\hat{P}R = 42°$

Bearing $\underline{092° T}$