NSW INDEPENDENT SCHOOLS

MATHEMATICS

2/3 UNIT (COMMON)

PRELIMINARY EXAMINATION

1997

Time Allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be handed in separately clearly marked Question 1, Question 2.....etc..
- ***The question paper must be handed to the supervisor at the end of the examination.***

Write your Student Number/Name on every page.

STUDENT NUMBER / NAME.....

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Question 1 **Marks**

(a) Lead can be flattened into very thin sheets 1.04×10^{-3} millimetres thick. How many such sheets are in a pile which is 5 centimetres high? **1**

(b) Express in simplest surd form : **2**

(i) $\sqrt{54}+\sqrt{6}-\sqrt{24}$.

(ii) $\left(\sqrt{3}-1\right)^2$.

(c) Solve the following equations. **4**

(i) $x-2(3x-5)=11$

(ii) $\dfrac{x}{2400-x}=\dfrac{2}{3}$

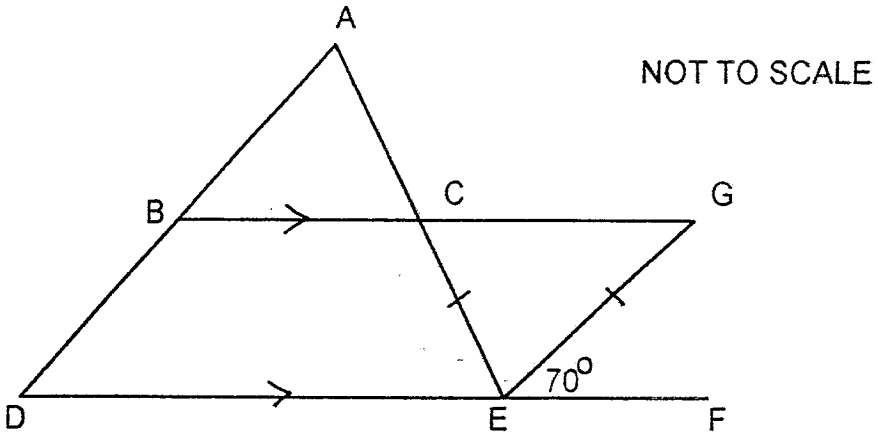
(d) (i) Sketch the lines $x-y-1=0$ and $3x+y-6=0$ on the same set of axes. **3**

(ii) Shade on your sketch the region which is satisfied by the inequalities $x-y-1\geq 0$, $3x+y-6\leq 0$ and $y\geq 0$.

(e) Simplify : $\dfrac{x}{x-y}-\dfrac{y}{x+y}$ **2**

Question 2 *(Start a new page)* **Marks**

(a) **3**

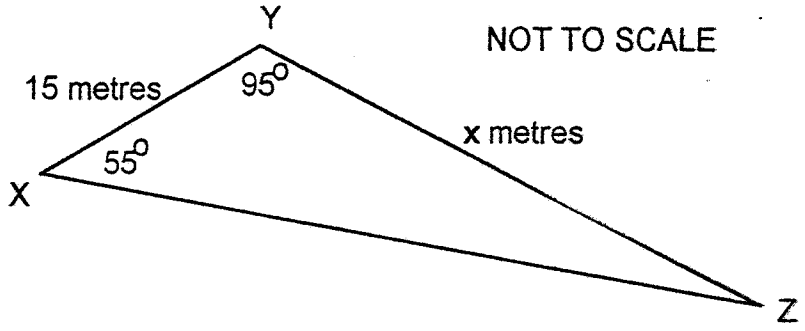


In the diagram above $BG \| DF$, $CE = EG$ AND $\angle GEF = 70^0$.
Copy the diagram onto your answer sheet.

Find the size of $\angle ACB$.

(b) Simplify : $\dfrac{a^2+2ab+b^2}{2a+2b}$ **2**

(c) **2**



Find the value of x correct to one decimal place.

Cont .....

Question 2 (cont.) **Marks**

(d) Factorise completely : **3**

(i) $8-t^3$.

(ii) x^3-x^2-x+1.

(e) Solve the quadratic equation : $x^2-5x-36=0$ **2**

Question 3 *(Start a new page)*

(a) Find the point of intersection of the lines $y=3x+4$ and $x+2y-1=0$ **2**

(b) Find the equation of the line parallel to $2x+3y-6=0$ passing through the point (1,-2). **2**

(c) Find the equation of the line with x and y intercepts 2 and -3 respectively. **1**

(d) L is the line $2x+y-5=0$. The point P has coordinates (1,-2). **2**

(i) Show that P does not lie on L.

(ii) Find the shortest distance from P to L.

(e) A (1,-3) and B (-2,0) are two points on the number plane. **3**

(i) Calculate the length of AB.

(ii) The line $y=mx+2$ is perpendicular to AB. Find the value of m.

(f) Find the exact value of : $\sin 210^0 \times \text{cosec} 135^0$ **2**

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Question 4 *(Start a new page)* **Marks**

(a) On separate sets of axes, draw neat sketches of the following showing any important features. **2**

(i) $xy = 3$

(ii) $y = \sqrt{9 - x^2}$

(b) Differentiate each of the following : **7**

(i) $y = \dfrac{x+1}{x-1}$

(ii) $y = \frac{1}{2}\left(x^4 - 4x^3 - 1\right)$

(iii) $y = \sqrt{x+1}$

(iv) $y = \left(x^2 + 2x\right)^4$

(c) If α and β are the roots of the equation $x^2 - 2x - 3 = 0$, find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(d) Given that $\tan\theta = T$, express $\tan\left(180^0 - \theta\right)$ in terms of T. **1**

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Question 5 *(Start a new page)* **Marks**

(a) From the top of a 40 metre high Mobile Phone tower, the angle of depression to the front gate of a nearby school is 42^{o}. Assuming that the ground is level; **4**

(i) Draw a diagram showing this information.

(ii) How far is the school gate from the base of the tower ?

(iii) Calculate the angle of elevation to the top of the tower from a point midway between the school gate and the tower base.

(b) Show that if $x^2 - 2x - 2m + 3$ is positive definite, then $m < 1$. **3**

(c) Solve the inequation $x(x-4) \leq 12$ **2**

(d) (i) Sketch the graph of $y = 2\sin x$, $0^0 \leq x \leq 360^0$ **3**

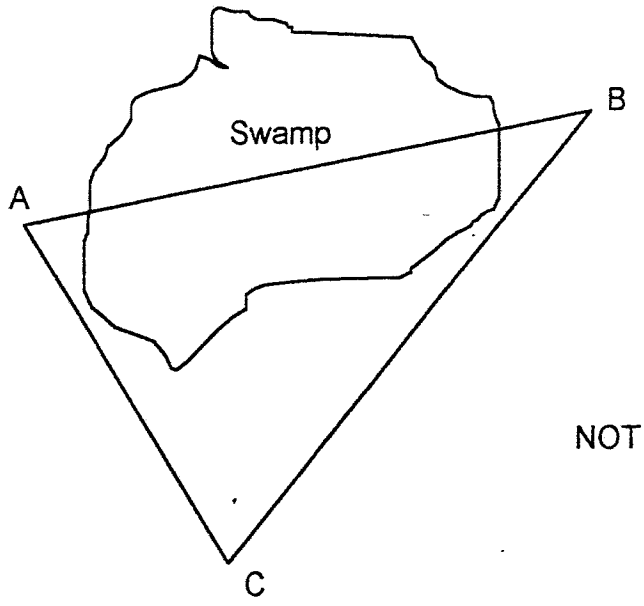
(ii) State the range of this function.

(iii) Which straight line graph would you need to draw with $y = 2\sin x$ to solve the equation $2\sin x + 1 = 0$ (Do NOT draw the graph)

Question 6 *(Start a new page)* **Marks**

(a) Find the equation of the tangent to the curve $y = \frac{1}{2}x^2 - 5x$ at the point where it cuts the y-axis. **3**

(b) **3**



NOT TO SCALE

The diagram shows two points A and B on opposite sides of a crocodile infested swamp. From a point C a surveyor notes that the bearings to A and B are 340°T and 036°T respectively. The distances AC and BC were then measured and found to be 180 metres and 212 metres respectively.

(i) Copy the diagram showing this information.

(ii) Calculate the distance from A to B across the swamp.

(c) Solve $2\cos\theta = 1$ for the domain $0^\circ \leq \theta \leq 360^\circ$. **2**

(d) Find the locus of the point P which moves so that the distances from the points A (2,1) and B (-1,-4) are equal. **2**

(e) Solve : $|x+3| = 2x-1$ **2**

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Question 7 *(Start a new page)* **Marks**

(a) Find the value of $\lim_{x \to 4} \left(\frac{x^2 - x - 12}{x - 4} \right)$ **2**

(b) Prove that : $\sin^2 \alpha \cdot \cos^2 \beta - \cos^2 \alpha \cdot \sin^2 \beta = \sin^2 \alpha - \sin^2 \beta$ **2**

(c) A function is defined by : **5**

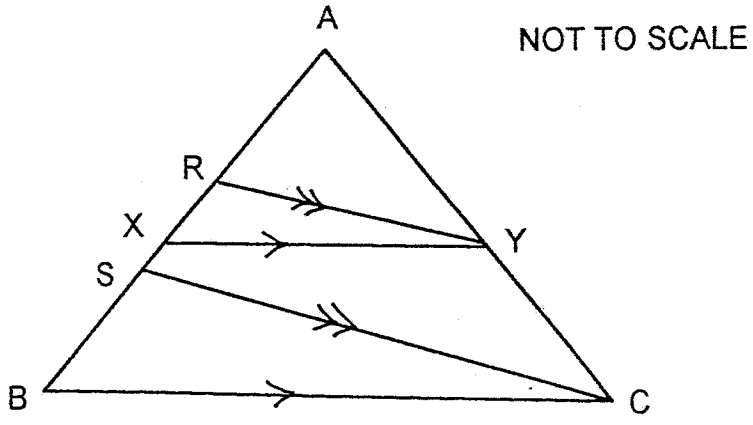
$$f(x) = \begin{cases} 1 - x \text{ for } -2 \le x < 0 \\ x + 1 \text{ for } 0 \le x \le 2 \end{cases}$$

(i) Write down the values of $f\left(\frac{1}{2}\right)$ and $f\left(-\frac{1}{2}\right)$

(ii) Show that $f(-x) = f(x)$ and state whether the function is odd, even or neither.

(iii) Sketch the graph this function.

(d) **3**



Given that $XY \| BC$ and $RY \| SC$

Prove that $AX : XB = AR : RS$

MATHEMATICS

2 UNIT PRELIMINARY 1997

Question 1

(a) $50 \div (1.04 \times 10^{-3}) \doteq 48077$

(b) (i) $3\sqrt{6} + \sqrt{6} - 2\sqrt{6} = 2\sqrt{6}$

(ii) $3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$

(c) (i) $x - 6x + 10 = 11$

$-5x = 1$

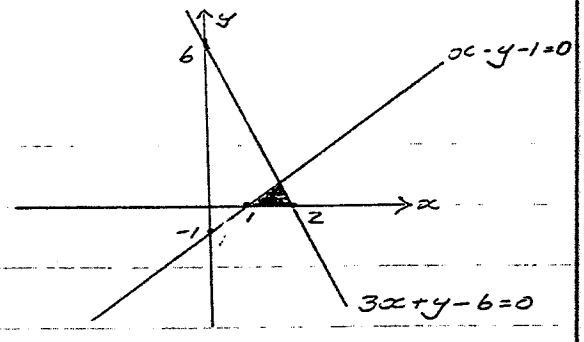
$x = -\frac{1}{5}$

(ii) $3x = 4800 - 2x$

$5x = 4800$

$x = 960$

(d)



Test (0,0) $0 - 0 - 1 > 0$ ✗

$3(0) + 0 - 6 < 0$ ✓

(e) $\frac{x(x+y) - y(x-y)}{x^2 - y^2}$

$= \frac{x^2 + xy - xy + y^2}{x^2 - y^2}$

$= \frac{x^2 + y^2}{x^2 - y^2}$

(b) $\frac{(a+b)^2}{2(a+b)} = \frac{a+b}{2}$

(c) $\hat{Z} = 180^\circ - (95^\circ + 55^\circ)$ (∠ sum of △)

$= 30^\circ$

$\therefore \frac{15}{\sin 30^\circ} = \frac{x}{\sin 55^\circ}$

$x = \frac{15 \sin 55^\circ}{\sin 30^\circ} \doteq 24.57\ldots$

$x \doteq 24.6$ metres

(d) (i) $(2-t)(4+2t+t^2)$

(ii) $x^2(x-1) - 1(x-1)$

$= (x^2-1)(x-1)$

$= (x-1)^2(x+1)$

(e) $(x-9)(x+4) = 0$

$x = 9, -4$

Question 2

(a) $C\hat{G}F = 70^\circ$ (alt ∠'s $BG \parallel EF$)

$\triangle CGE$ is isosceles ($CE = EG$)

$\therefore G\hat{C}E = 70^\circ$ (base ∠'s of an isos △)

$\therefore A\hat{C}B = 70^\circ$ (Vert. opp.)

Question 3

(a) $x + 2y - 1 = 0$

$y = 3x + 4$

$\therefore x + 2(3x+4) - 1 = 0$

$7x + 7 = 0$

$x = -1$

$y = 3(-1) + 4 = 1.$

$\therefore (-1, 1)$ is pt of intersection

(b) $2x + 3y - 6 = 0 \Rightarrow m = -\frac{2}{3}$

$y + 2 = -\frac{2}{3}(x - 1)$

$3y + 6 = -2x + 2$

$2x + 3y + 4 = 0$

Q3(c) $\frac{x}{2} + \frac{y}{-3} = 1$

$3x - 2y = 6$

(d)(i) $2(1) + (-2) - 5 \neq 0$

$\therefore$ P does not lie on l

(ii) $d = \left| \frac{2(1) + (-2) - 5}{\sqrt{2^2 + 1^2}} \right|$

$= \frac{5}{\sqrt{5}} = \sqrt{5}$

(e)(i) $AB = \sqrt{(1 - -2)^2 + (-3 - 0)^2}$

$= \sqrt{18}$ OR $3\sqrt{2}$

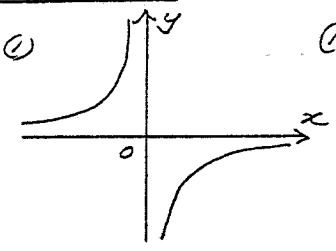
(ii) $m_{AB} = \frac{-3 - 0}{1 - -2} = -1$

$\therefore m_1 = +1$ $(m_1 m_2 = -1)$

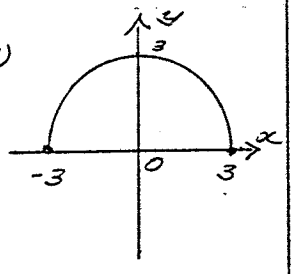
(f) $-\frac{1}{2} \times \sqrt{2} = -\frac{\sqrt{2}}{2}$

Question 4

(a)(i)



(ii)



(b)(i) $y' = \frac{(x-1) \cdot 1 - (x+1) 1}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$

(ii) $y' = \frac{1}{2}(4x^3 - 12x^2)$

$= 2x^3 - 6x^2$

(iii) $y = (x+1)^{1/2}$ $y' = \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1$

$= \frac{1}{2\sqrt{x+1}}$

(iv) $y' = 4(x^2 + 2x)^3 (2x + 2)$

(c) $\alpha + \beta = 2$ $\frac{1}{\alpha} + \frac{1}{\beta}$

$\alpha\beta = -3$ $= \frac{\alpha + \beta}{\alpha\beta}$

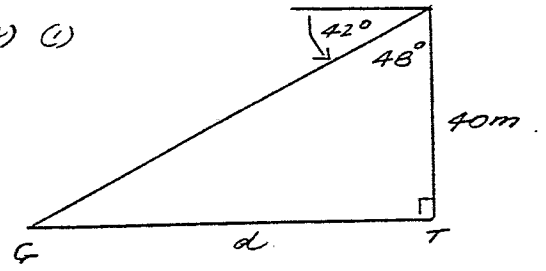
$= \frac{-2}{3}$

(d) $\tan(180 - \theta) = -\tan\theta$

$= -T$

Question 5

(a)(i)



(ii) $\tan 42° = d/40$

$d = 40 \tan 48°$

$= 44.4$ m

(iii) $\tan\theta = \frac{40}{22.2}$

$\theta = 60°58'$

(b) $\Delta < 0$ for + def $(a > 0)$

$(-2)^2 - 4(1)(-2m + 3) < 0$

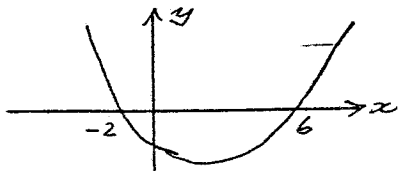
$4 + 8m - 12 < 0$

$8m < 8$

$m < 1$

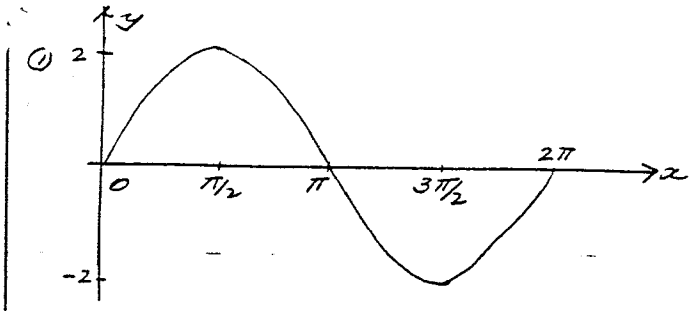
(c) $x^2 - 4x - 12 \leq 0$

$(x - 6)(x + 2) \leq 0$



$-2 \leq x \leq 6$

(i)



(ii) Range $-2 \leq y \leq 2$

(iii) $y = -1$

(e) $x + 3 = 2x - 1$

$4 = x$

Check $|4+3| = 2(4) - 1$ T.

$-x - 3 = 2x - 1$

$-2 = +3x$

$x = -2/3$

Check $|-2/3 + 3| = 2(-2/3) - 1$ F.

$\therefore$ $x = 4$

Question 6

(a) y-axis $\quad x = 0$

$y = -5$

$y' = x - 5$

$m = -5$

$y + 5 = -5(x - 0)$

$5x + y + 5 = 0$

(b) $A\hat{C}B = 20° + 36° = 56°$.

$AB^2 = 180^2 + 212^2 - 2 \cdot 180 \cdot 212 \cos 56°$

$= 34666.39761\ldots$

$AB \doteq 186$ m

(c) $2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$ { Quad 2, 3; Acute $\angle$ 30° }

$\theta = 150°, 210°$

(d) $AP = BP$

$\therefore AP^2 = BP^2$.

$(x-2)^2 + (y-1)^2 = (x+1)^2 + (y+4)^2$

$-4x + 4 - 2y + 1 = 2x + 1 + 8y + 16$

$6x + 10y + 12 = 0$.

$3x + 5y + 6 = 0$

Question 7

(a) $\lim_{x \to 4} \left\{ \frac{(x-4)(x+3)}{(x-4)} \right\} = 7$

(b) LHS $= \sin^2\alpha(1 - \sin^2\beta) - (1 - \sin^2\alpha)\sin^2\beta$

$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta$

$= \sin^2\alpha - \sin^2\beta$ = RHS

(c) (i) $f(\frac{1}{2}) = \frac{1}{2} + 1 = 1\frac{1}{2}$ $\quad f(-\frac{1}{2}) = 1 - (-\frac{1}{2}) = 1\frac{1}{2}$.

(ii) $f(-x) = 1 - -x \quad 0 \leq x \leq 2$.

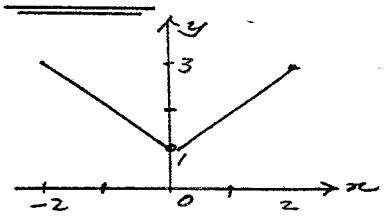
$= 1 + x = f(x)$

$f(-x) = -x + 1. \quad -2 \leq x < 0$

$= 1 - x = f(x)$

EVEN

(iii)



(d) In $\triangle ABC$.

$AX : XB = AY : YC$.

a line // one side of a $\triangle$ divides the other two sides in proportion

Sim in $\triangle ASC$.

$AR : RS = AY : YC$

$\therefore$ $AX : XB = AR : RS$