NSW INDEPENDENT SCHOOLS

MATHEMATICS

2/3 UNIT (COMMON)

PRELIMINARY EXAMINATION

1998

Time Allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be handed in separately clearly marked Question 1, Question 2.....etc..
- ***The question paper must be handed to the supervisor at the end of the examination.***
- Write your Student Number/Name on every page.

STUDENT NUMBER / NAME.....

Student Number/Name _____

Question 1

(a) Expand and simplify each of the following : **3**

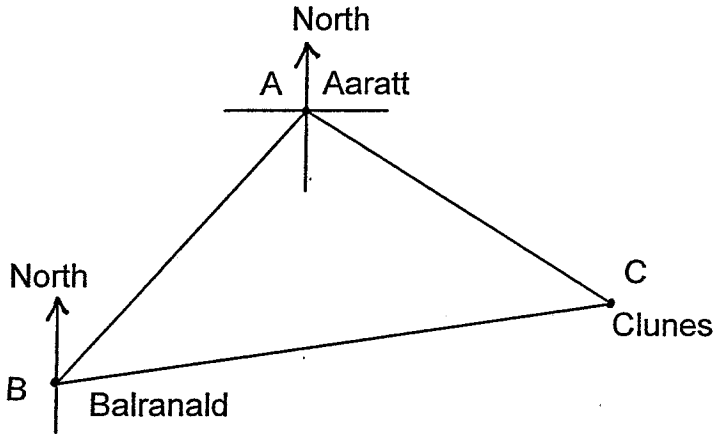
(i) $5a\left(a^2-2a\right)-3a^2(2a-3)$

(ii) $\left(x^3-5\right)\left(x^3+5\right)$

(b) Calculate correct to 3 significant figures : $\sqrt[5]{\dfrac{576+837}{(2.5)^2}}$ **2**

(c) If introduced, a 10% GST will increase the price of a Macca Burger to \$4.85. What is the pre GST price of the Macca Burger? (Round off your answer to the nearest five cents). **2**

(d) The diagram shows the path of a plane flying from Aaratt to Balranald, a distance of 340 kilometres, on a bearing of 220°T. At Balranald, the plane changed course and flew to Clunes, a distance of 620 kilometres on a bearing of 080°T. **3**



(i) Copy the diagram showing this information

(ii) Explain why $\angle ABC = 40°$.

(iii) Calculate the distance the plane has to travel to fly directly from Clunes back to Aaratt.

(e) Solve : $\dfrac{4}{x}-\dfrac{3}{2x}=9$ **2**

Student Number/Name _____

Question 2 (*Start a new page*)

(a) Given that $A=\left[\frac{9}{5}\right]^3$, $B=\left[\frac{1}{25}\right]$ and $C=81$. **3**

Find the value of x and y if $\frac{A^2}{B^5C^3}=3^x5^y$

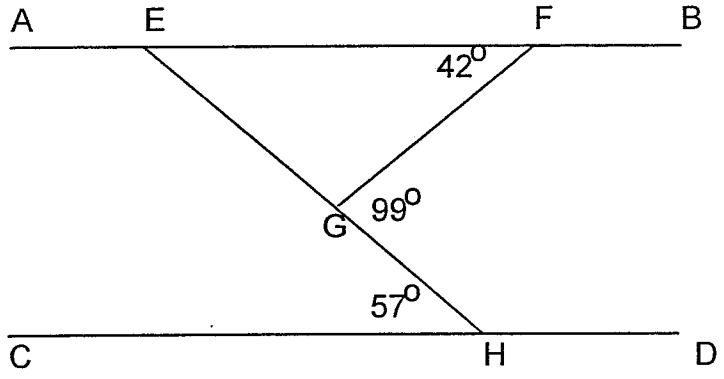
(b) Factorise completely each of the following. **4**

(i) $a^2-10a+9$

(ii) $x^2-9-xy+3y$

(iii) x^4-x

(c) Copy the diagram below : **3**



Prove that $AB \| CD$, giving reasons.

(d) Find the points of intersection of the curve $y=x^2-3$ and the straight line $x-y=1$. **2**

Student Number/Name _____

Question 3 *(Start a new page)*

(a) Solve $|2-x|=2x+1$ **3**

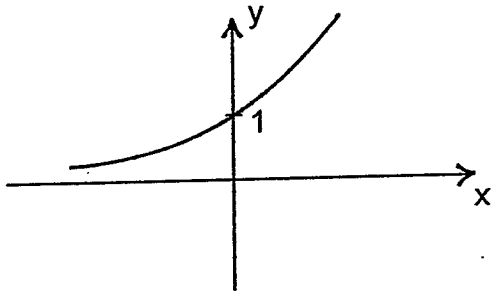
(b) Determine whether the function $f(x)=\dfrac{x^2}{x^2+4}$ is ODD or EVEN. Justify your answer. **2**

(c) Find the exact value of : $sin240^\circ \cdot tan\left(-30^\circ\right)$ **2**

(d) A helicopter left Bankstown Airport and flew 20 kilometres due South. It then turned West and flew a distance of 16 kilometres. Find the bearing of the helicopter from Bankstown Airport. **2**

(e) Which graph A, B, C or D would represent $y=a^{x+1}$ **1**

A.



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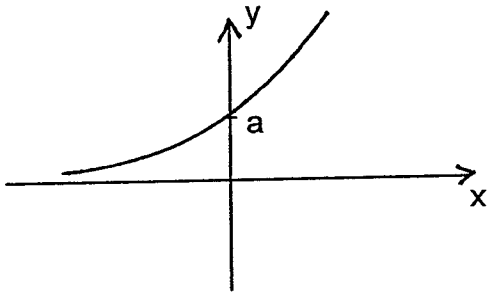

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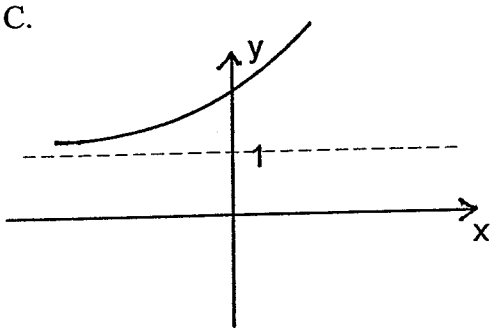
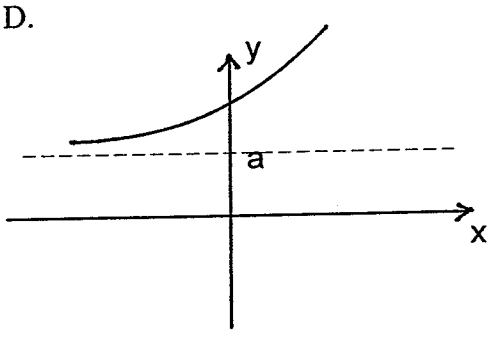


D.

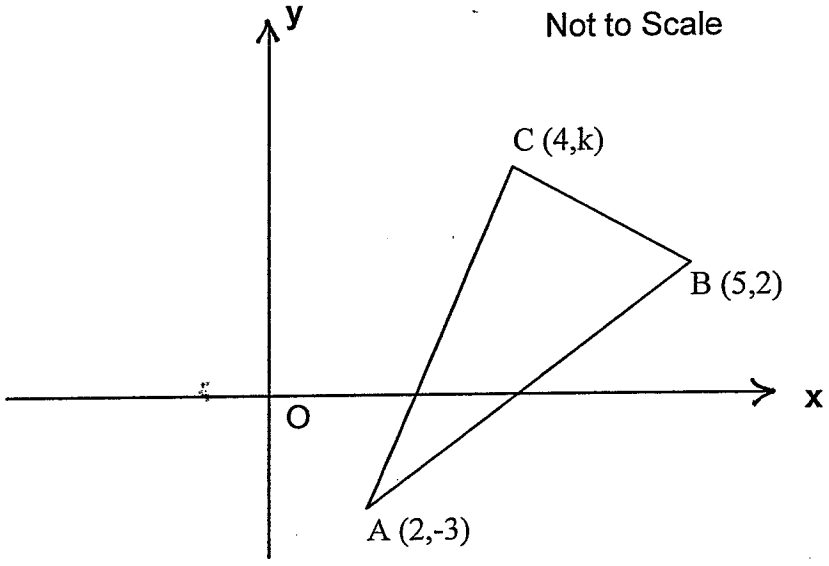


(f) Give the domain and range for : $y=\sqrt{8-x^2}$ **2**

Student Number/Name _____

Question 4 (*Start a new page*)

(a) A (2,-3), B (5,2) and C (4,k) are three points on the number plane, as shown in the diagram below. ($k > 0$). **6**



(i) Show that the equation of AB is $5x - 3y = 19$.

(ii) Show that the length of AB is $\sqrt{34}$ units.

(iii) Write down an expression, in terms of k, for the perpendicular distance from C to AB.

(iv) Given that the area of ΔABC is 17 units^2, find the value of k.

(b) Sketch the region given by the following inequalities **3**

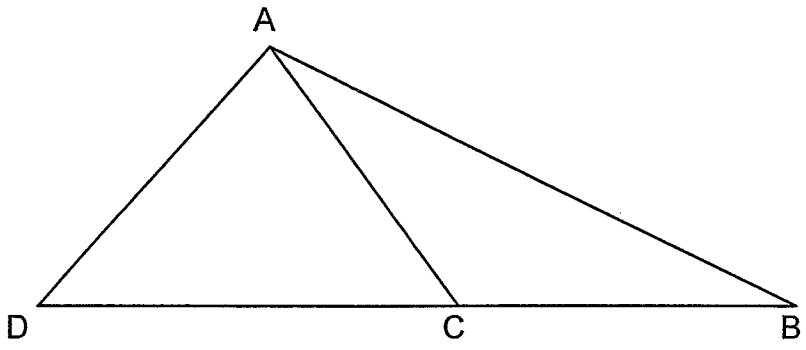
$$x + y - 2 > 0,\ y \le x^2,\ x \ge 0,\ y \ge 0.$$

(c) For what values of k will $3x^2 - kx + 12$ be always positive? **3**

Student Number/Name _____

Question 5 *(Start a new page)*

(a) In the diagram below, $AC = DC = CB$. **3**



Show that $\angle DAB = 90^o$, giving reasons. (Hint : Let $\angle ADC = x^o$)

(b) Express in simplest surd form : **4**

(i) $\sqrt{75} - \sqrt{45} + \sqrt{5}$

(ii) $\left(3\sqrt{2}\right)^5$

(iii) $\left(\sqrt{3} - 2\right)\left(2 + \sqrt{3}\right)$

(c) Prove that $\dfrac{1}{cosec\theta - cot\,\theta} - \dfrac{1}{cosec\theta + cot\,\theta} = 2\,cot\,\theta$ **3**

(d) Convert $0.3\dot{4}$ to a fraction in its simplest form. **2**

Student Number/Name _____

Question 6 (*Start a new page*)

(a) Find the derivative of : **6**

(i) $\dfrac{2x-3}{3x+5}$

(ii) $\dfrac{1}{\sqrt{2x-7}}$

(iii) $\left(3x^2-2\right)^6$

(b) For the curve represented by the equation $y=x-x^3$: **4**

(i) Find $\dfrac{dy}{dx}$

(ii) Find the equation of the tangent to the curve at the point (1,0).

(iii) State the gradient of the normal at this point

(c) By using the substitution $X=x^2-2x$, solve the equation : **2**

$$\left(x^2-2x\right)^2-7\left(x^2-2x\right)-8=0$$

Student Number/Name _____

Question 7 (*Start a new page*)

(a) If $x^2 - 4px + 3p - 2 = 0$: **3**

(i) Write down an expression, containing p, for the product of the roots of the above equation.

(ii) Find the value of p given that the product of the roots is three times the sum.

(b) Find the equation of the parabola with focus (3,1) and vertex (5,1) **3**

(c) (i) Sketch the graph of $y = cos 2\theta$ for $0^o \leq \theta \leq 360^o$ **4**

(ii) On the same set of axes sketch $y = \frac{1}{2}$

(iii) Hence state the **number** of solutions for the equation $cos 2\theta = \frac{1}{2}$

(d) Find $\lim_{x \to \infty} \left[\frac{4x^2 - 2x^3 + 1}{7x^3 - \frac{2}{x}} \right]$ **2**

INDEPENDENT TRIAL EXAMS –1998

2UNIT PRELIMINARY MATHEMATICS

1998

Question 1.

(a)(i) $5a^3 - 10a^2 - 6a^3 + 9a^2$

$= \underline{-a^3 - a^2}$

(ii) $\underline{x^6 - 25}$

(b) 2.957007519

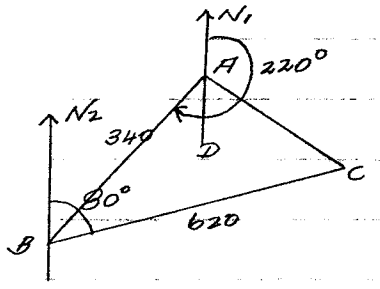
$= \underline{2.96 \text{ (to 3 o.s.f)}}$

(c) $110\%\ P = \$4.85$

$\therefore 100\%\ P = 4.40909\ldots$

$= \underline{\$4.40}$

(d)



$N_2\hat{B}A = B\hat{A}D = 40°$

alt $\angle$'s $N_1D \parallel N_2B$

$\therefore A\hat{B}C = 80° - 40°$

$= 40°$

$AC^2 = 340^2 + 620^2 - 2 \times 340 \times 620 \times \cos 40°$

$AC = 420.756\ldots$

$\doteq \underline{420 \text{ km}}$

(e) $\frac{4}{5c} - \frac{3}{2c} = 9$

$(\times 2c)\quad 8 - 3 = 18c$

$\underline{c = \frac{5}{18}}$

Question 2

(a) $\left(\frac{3^2}{5}\right)^6 \Big/ \left(\frac{1}{5^2}\right)^5 \times (3^4)^3$

$= \frac{3^{12}}{5^6} \times \frac{5^{10}}{3^{12}}$

$= 5^4$

$\underline{x = 0 \quad y = 4}$

(b)(i) $\underline{(a-1)(a-9)}$

(ii) $(x+3)(x-3) - y(x-3)$

$= \underline{(x-3)(x+3-y)}$

(iii) $x(x^3 - 1)$

$= \underline{x(x-1)(x^2+x+1)}$

(c) $F\hat{G}H = E\hat{F}G + F\hat{E}G$

(Ext $\angle$ = sum of 2 int opp)

$99° = 42° + F\hat{E}G$

$\therefore F\hat{E}C = 57°$

$E\hat{H}C = 57°$ (data)

$\therefore AB \parallel CD$ (pair of alt $\angle$'s are equal)

(d) $y = x - 1$

$y = x^2 - 3$

$x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$y = 1, -2$

$\underline{(2,1)\ (-1,-2)}$ are points of intersection.

Question 3

(a)

	Test
$2 - x = 2x + 1$	$\cdot$ Test
$3x = +1$	$\lvert 2 - \frac{1}{3}\rvert = 2(\frac{1}{3}) + 1$
$x = \frac{1}{3}$	✓
$\therefore -2 + x = 2x + 1$	$\lvert 2 - -3\rvert = 2(-3) +$
$x = -3$	×

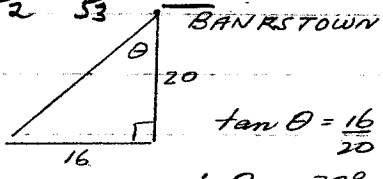
$\therefore \underline{x = \frac{1}{3} \text{ is the solution}}$

(b) $f(-x) = \frac{(-x)^2}{(-x)^2 + 4} = f(x)$

$\therefore \underline{f(x) \text{ is EVEN}}$

(c) $-\frac{\sqrt{3}}{2} \times \frac{-1}{\sqrt{3}} = \underline{\frac{1}{2}}$

(d)



$\tan\theta = \frac{16}{20}$

$\therefore \theta = 38°40'$

$\therefore$ Bearing $\underline{\text{is } 218°40'}$

(e) $\underline{B}$

(f) domain $\underline{-2\sqrt{2} \le x \le 2\sqrt{2}}$

range $\underline{0 \le y \le 2\sqrt{2}}$

Question 4

(a)(i) $\frac{y+3}{x-2} = \frac{2+3}{5-2} = \frac{5}{3}$

$\therefore 3y + 9 = 5x - 10$

$\therefore \underline{5x - 3y = 19}$

(ii) $AB = \sqrt{(5-2)^2 + (2+3)^2}$

$= \underline{\sqrt{34} \text{ units}}$

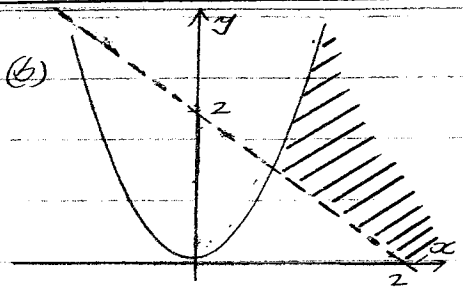
(iii) $P = \underline{\left|\frac{20 - 3k - 19}{\sqrt{34}}\right|}$

(iv) $17 = \frac{1}{2} \cdot \sqrt{34}\ \frac{|1 - 3k|}{\sqrt{34}}$

$\therefore \underline{k = 11\frac{2}{3} \quad (k \neq -11)}$

These suggested answers/marking schemes are issued as a guide only -offerred as an assistance in constructing your own marking format (individual teachers/schools find many other acceptable responses)

(b)



(c) $3x^2 - kx + 12$

+ if $\Delta < 0$

$\therefore (-k)^2 - 4 \times 3 \times 12 < 0$

$k^2 < 144$

$-12 < k < 12$

(c) LHS =

$$\frac{(\operatorname{cosec}\theta + \cot\theta)\cdot(\operatorname{cosec}\theta - \cot\theta)}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{2\cot\theta}{1} = RHS$$

(d) Let $x = 0.34\dot{}$ — ①

$(\times 10)$ $10x = 3.4\dot{}$ — ②

$(\times 10)$ $100x = 34.4\dot{}$ — ③

③ − ② $90x = 31$

$$x = \frac{31}{90}$$

(c) $x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = 8, -1$

$x^2 - 2x = 8$ $\quad x^2 - 2x = -1$

$x^2 - 2x - 8 = 0$ $\quad x^2 - 2x + 1 = 0$

$(x-4)(x+2) = 0$ $\quad (x-1)^2 = 0$

$x = -2, 4, 1$

Question 5

(a) Let $A\hat{D}C = x°$

ΔACD is isos $(AC = DC)$

$\therefore D\hat{A}C = x°$ (base $\angle$s of isos Δ)

$x° + x° + A\hat{C}D = 180°$

($\angle$ sum of ΔACD)

$\therefore A\hat{C}D = (180 - 2x)°$

ΔACB is isos $(AC = CB)$

$\therefore B\hat{A}C = A\hat{B}C$ (base $\angle$s of isos Δ)

$A\hat{C}D = B\hat{A}C + A\hat{B}C$

(Ext $\angle$ = sum of 2 int opp $\angle$s)

$\therefore B\hat{A}C = \frac{1}{2}(180 - 2x)°$

$= (90 - x)°$

$D\hat{A}B = x° + (90 - x)°$

$= 90°$

(b)(i) $5\sqrt{5} - 3\sqrt{5} + \sqrt{5}$

$= 3\sqrt{5}$

(ii) $972\sqrt{2}$

(iii) $(\sqrt{3})^2 - 4 = -1$

Question 6

(a)

(i) $$\frac{(3x+5)2 - (2x-3)3}{(3x+5)^2}$$

$$= \frac{19}{(3x+5)^2}$$

(ii) $-\frac{1}{2}(2x-7)^{-3/2} \times 2$

$$= \frac{-1}{\sqrt{(2x-7)^3}}$$

(iii) $6(3x^2-6)^5 6x$

$= 36x(3x^2-6)^5$

(b) $\frac{dy}{dx} = 1 - 3x^2$

Tangent

(ii) $m = 1 - 3(1)^2 = -2$

$\therefore y - 0 = -2(x-1)$

$y = -2x + 2$

$2x + y - 2 = 0$

(iii) Normal $\quad m = +\frac{1}{2}$

Question 7

(a) (i) $\alpha\beta = \frac{3k-2}{4k}$

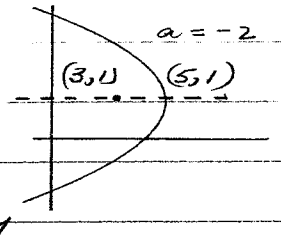
(ii) $3k - 2 = 3(4k)$

$-2 = 9k$

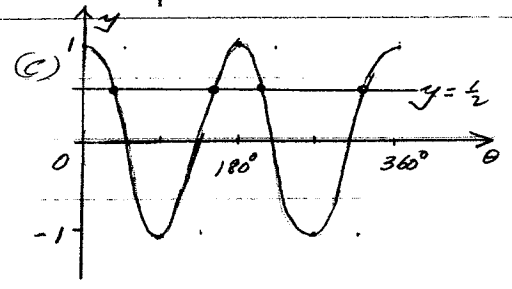
$k = -2/9$

(b) $(y-1)^2 = 4(-2)(x-5)$

$y^2 - 2y + 8x - 39 = 0$



(c)



(iii) 4 solutions

(d) $\lim_{x \to \infty} \left\{ \frac{\frac{4}{x} - 2 + \frac{1}{x^3}}{7 - \frac{2}{x^4}} \right\}$

$= -\frac{2}{7}$