NSW INDEPENDENT SCHOOLS

PRELIMINARY EXAMINATION

1999

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

Attempt ALL questions.

ALL questions are of equal value.

All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.

Board-approved calculators may be used.

Each question attempted is to be handed in separately clearly marked Question 1, Question 2 etc..

Write your student Name/Number on every page.

This paper **must not be removed** from the examination room.

STUDENT NAME / NUMBER ....

STUDENT NUMBER/NAME _______________

Question 1

(a) Solve : $\frac{1}{3}(x-2)-\frac{1}{2}(2-x)=1$ **2**

(b) Find the value of x if $\sqrt{x}=\sqrt{50}-\sqrt{18}$ **2**

(c) A formula used in Physics to determine the acceleration, f, of a simple pulley system with masses M and m at each end is : **2**

$$f=\frac{(M-m)g}{(M+m)}$$ (where g is the acceleration due to gravity).

Find the value of m if $M=16$, $g=10$ and $f=2.5$.

(d) In a survey taken several months ago on the Republic referendum, 52% voted "YES" and 48% voted "NO". Following the decision to change the wording of the question, a second survey was taken. It was found that 10% of the "YES" voters changed to "NO", while 10% of the "NO" voters changed to "YES". What percentage of the people surveyed voted "YES" in the second survey? **2**

(e) Solve the following quadratic equation leaving your answer in surd form. **2**

$$(2x-1)^2=6$$

(f) Express $\dfrac{1}{\sqrt{3}-2}$ with rational denominator. **2**

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Question 2 *(Start a new page)*

(a) Expand and simplify **4**

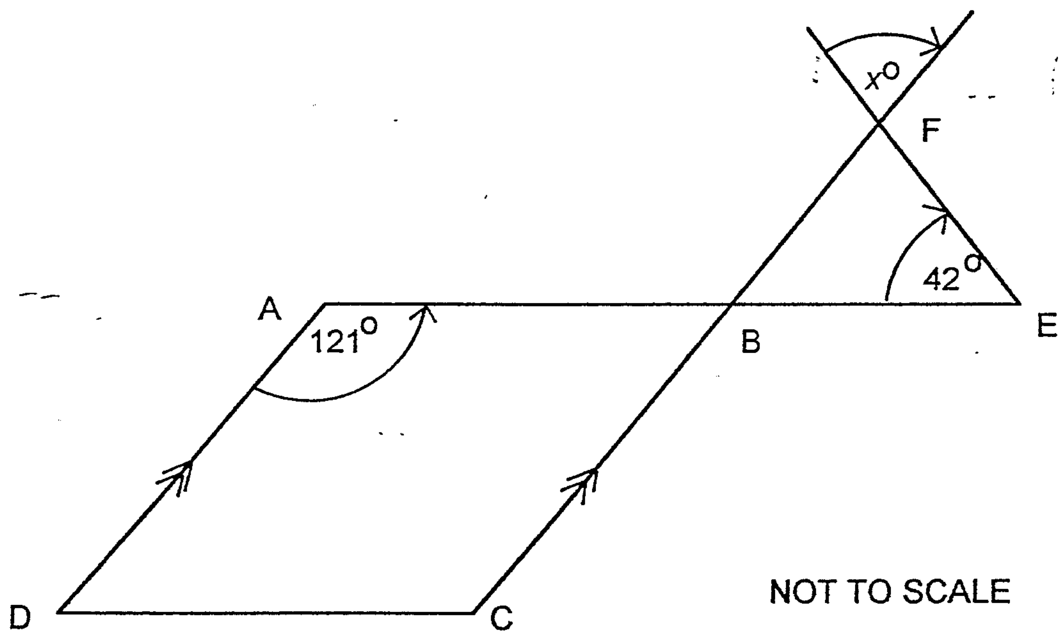
(i) $(2x-3)^2 - (2x+3)^2$

(ii) $\sqrt{(a-5)(a+5)+25}$

(b) The three legs of the triangular sailing course for next year's Olympic Games have lengths 8 kilometres, 10 kilometres and 16 kilometres. Draw a sketch showing this information and calculate the size of the smallest angle that the yachts will have to turn to negotiate the course. (answer to the nearest minute) **3**

(c) Express $1.0\dot{2}\dot{6}$ as a simple fraction. **2**

(d) In the diagram below $AD \parallel BC$. **3**



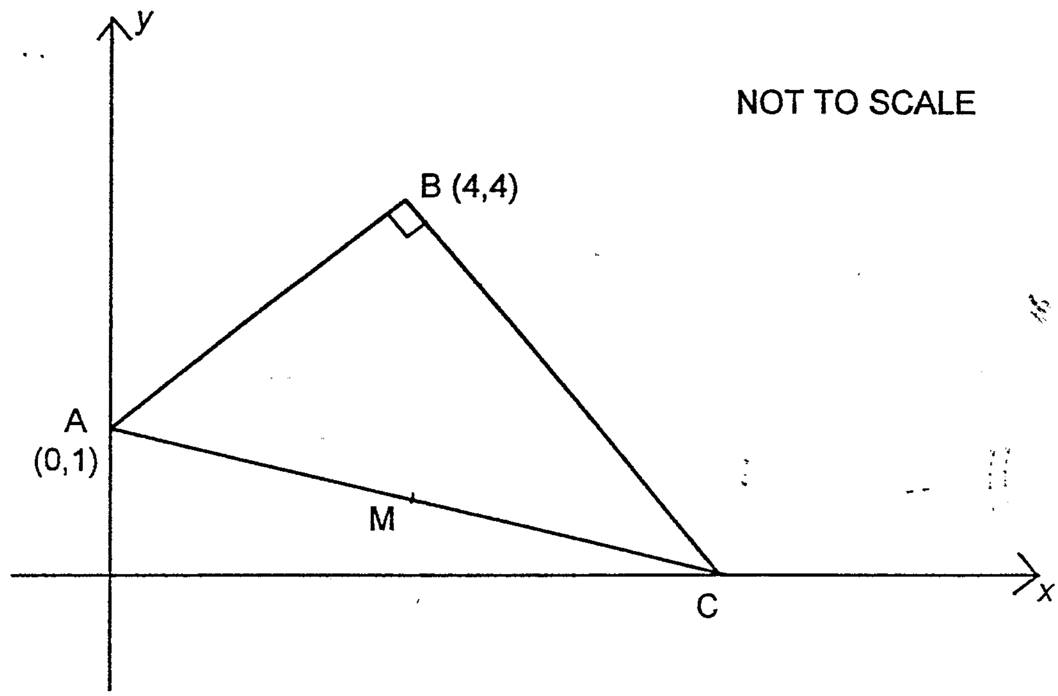
Copy the diagram onto your worksheet and find the value of x, giving reasons.

Question 3 *(Start a new page)*

(a) Find the exact value of: $\tan 120^{\circ} . \sin(-30^{\circ})$ **2**

(b) Find the shortest distance between the parallel lines, $y = 2x$ and $4x - 2y + 7 = 0$ **2**

(c) **8**



In the diagram above, C lies on the x-axis and AB is perpendicular to BC. M is the midpoint of AC. Copy the diagram onto your worksheet.

(i) Find the gradient of AB.

(ii) Show that the equation of BC is $4x + 3y - 28 = 0$.

(iii) Show that C has coordinates (7,0).

(iv) Find the coordinates of point M.

(v) A circle has centre M and radius MC. Find the radius of this circle.

(vi) Find the equation of this circle and show that it also passes through the points A and B.

Question 4 *(Start a new page)*

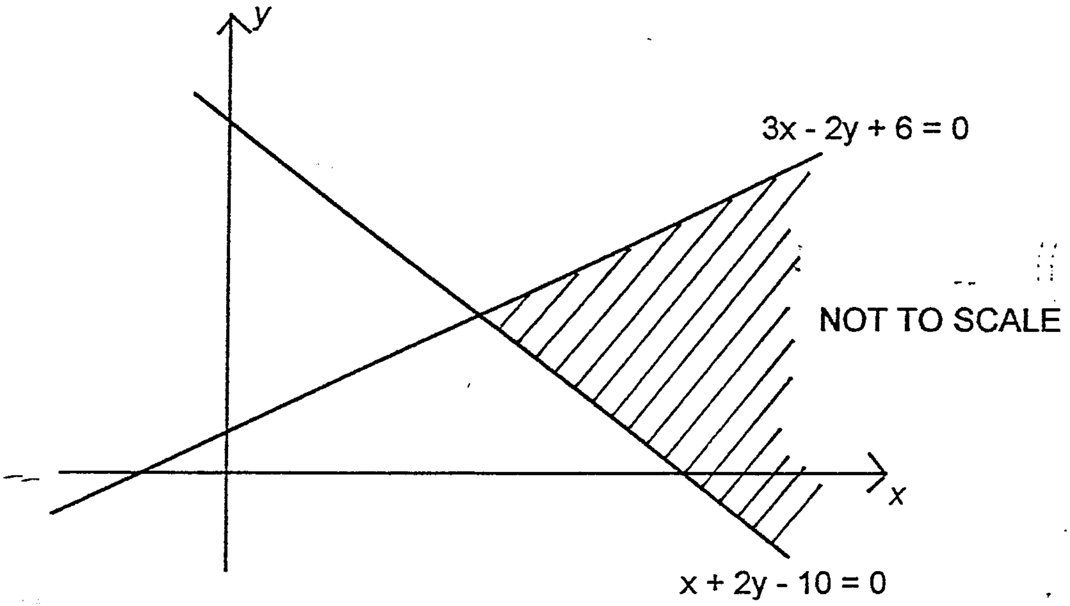
(a) Factorise completely each of the following : **3**

(i) $x^2 + 4x + 4 - y^2$.

(ii) $5x^2 - 9xy - 2y^2$

(b) Simplify : $3\sin^2 A + 4\cos^2 A - 3$ **2**

(c) The diagram below shows the graphs $3x - 2y + 6 = 0$ and $x + 2y - 10 = 0$. **2**



State the pair of inequalities which define the shaded region.

(d) Solve for p : $|p-2| = 2p-1$ **3**

(e) On the same set of axes, draw neat sketches of the graphs (for $0 \leqslant x \leqslant 360°$) of the following functions: **2**

$$y = 2\sin x$$
$$y = 2\sin x + 1$$

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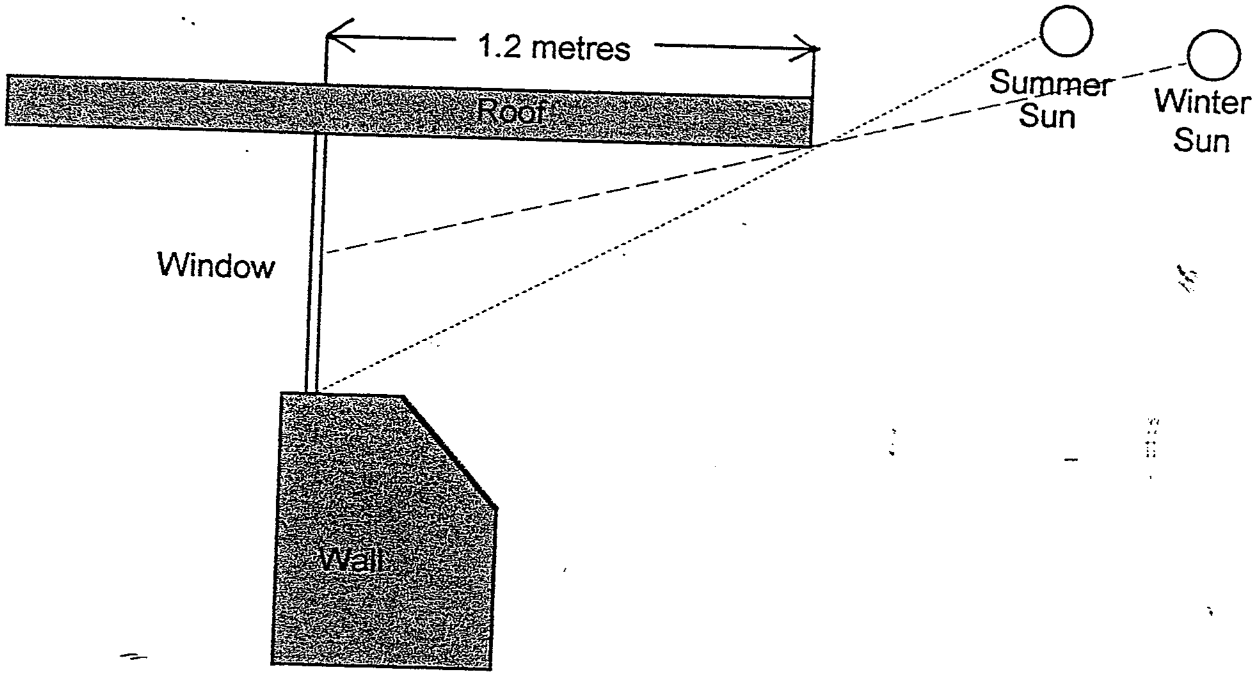
Question 5 *(Start a new page)*

(a) For what values of k will $kx^2 - x + k$ be negative definite ? **4**

(b) The diagram below shows a window of a house protected from the summer sun by an overhanging roof which is 1.2 metres wide. In the middle of summer, when the angle of elevation of the sun is 42°, the window is completely shaded as illustrated. In the middle of winter, exactly half of the window is shaded. **4**



(i) Copy the sketch and show the angle of elevation in the middle of summer.

(ii) Calculate the height of the window.

(iii) Find the angle of elevation of the sun in the middle of winter. (answer to the nearest degree)

(c) (i) Express $-x^2 + 2x + 3$ in the form $-(x - a)^2 + b$ **4**

(ii) Sketch the graph of the parabola $y = -x^2 + 2x + 3$.

(iii) Using this graph, or otherwise, find the maximum value of $-x^2 + 2x + 3$.

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Question 6 *(Start a new page)*

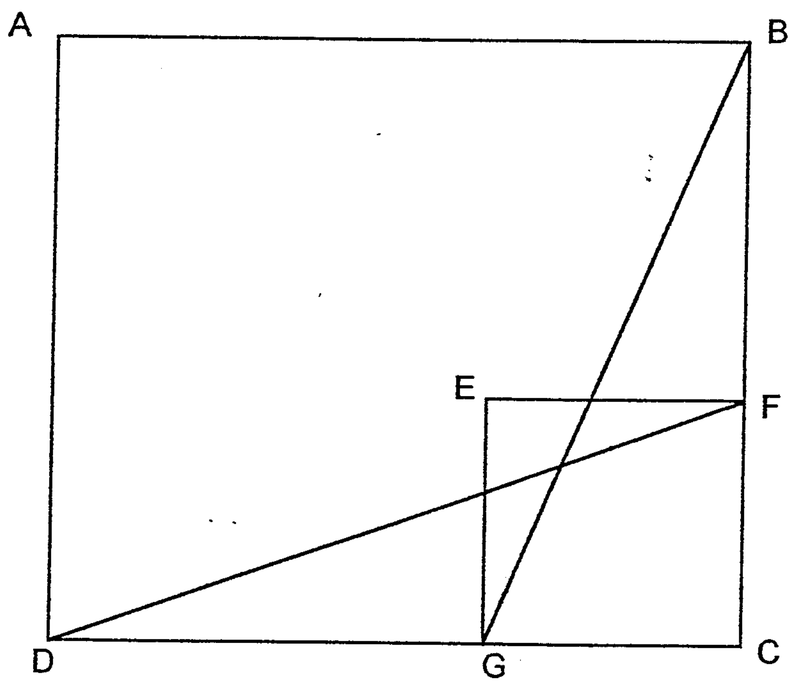
(a) Differentiate each of the following : **6**

(i) $\frac{1}{2}x^4 - 3x + 7$

(ii) $\dfrac{2x}{x^2 + 4}$

(iii) $\dfrac{x^2}{\sqrt{x}}$

(b) In the diagram below, ABCD and EFCG are squares. **4**



Copy the diagram onto your worksheet.

(i) Prove that $\Delta DFC \equiv \Delta BGC$

(ii) Hence show that $\angle FDC = \angle BGE$

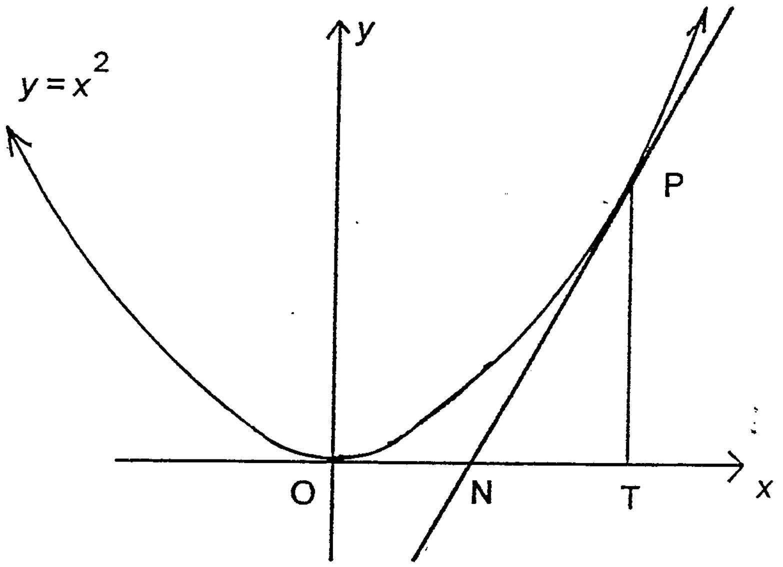
(c) Find the point(s) of intersection of the graphs $xy = -4$ and $2x + y = 2$ **2**

Question 7 *(Start a new page)*

(a) If $\tan B = -2$ and $0^\circ < B < 270^\circ$, find the value of cosec B in surd form. **2**

(b) Draw neat sketch of the graph of the function $y = \sqrt{4-x}$
State the domain of this function. **2**

(c) In the diagram below, PN is the tangent to the parabola $y = x^2$ touching it at point P. OT = 3 units. **5**



NOT TO SCALE

(i) Find the coordinates of point P.

(ii) Show that PN has equation $y = 6x - 9$.

(iii) Find the coordinates of the point N.

(iv) Hence, or otherwise, find the area of triangle OPN.

(d) Given that $f(x) = \dfrac{a^x + a^{-x}}{2}$, show that $f(2x) = 2[f(x)]^2 - 1$ **3**

NSW INDEPENDENT TRIAL EXAMS 1999 SOLUTIONS

MATHEMATICS 2UNIT PRELIMINARY 1999.

Q1

(a) $2(x-2)-3(2-x) = 6$

$2x-4-6+3x = 6$

$5x = 16$

$x = 3.2$

(b) $5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$= \sqrt{4}.\sqrt{2}$

$= \sqrt{8}$

$x = 8$

(c) $2.5 = \dfrac{(16-m)\times 10}{(16+m)}$

$40 + 2.5m = 160 - 10m$

$12.5m = 120$

$m = 9.6$

(d) $52\% - 10\% \text{ of } 62\% + 10\% \text{ of } 48\%$

$= (52 - 5.2 + 4.8)\%$

$= 51.6\%$

(e) $2x-1 = \pm\sqrt{6}$

$2x = 1 \pm \sqrt{6}$

$x = \dfrac{1\pm\sqrt{6}}{2}$

(f) $\dfrac{1}{\sqrt{3}-2} \times \dfrac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \dfrac{\sqrt{3}+2}{3-4}$

$= -\sqrt{3}-\sqrt{2}$

Q2

(i)

(a) $(4x^2-12x+9)-(4x^2+12x+9)$

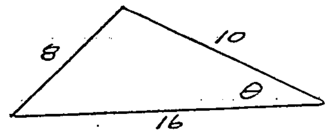
$= 4x^2-12x+9-4x^2-12x-9$

$= -24x$

(ii) $\sqrt{a^2-25+25}$

$= \sqrt{a^2} = a$

(b)



$\cos\theta = \dfrac{16^2+10^2-8^2}{2\times16\times10}$

$\theta = 24°9'$

(c) $x = 1.0\dot{2}\dot{6}$ —(1)

(1)×10) $10x = 10.\dot{2}\dot{6}$ —(2)

(2)×100) $1000x = 1026.\dot{2}\dot{6}$ —(3)

(3)−(2) $990x = 1016$

$x = \dfrac{1016}{990}$

$= 1\dfrac{13}{495}$

(d) $\angle ABC + 121° = 180°$

(Coint ∠'s, AD∥BC)

$\angle ABC = 59°$

$\angle EBF = 59°$ (vert opp)

$B\hat{F}E + 59° + 42° = 180°$

(Angle sum of $\triangle BFE$)

$\therefore B\hat{F}E = 79°$

$\therefore \alpha = 79$ (vert opp)

Q3

(a) $-\sqrt{3}\times -\frac{1}{2} = \dfrac{\sqrt{3}}{2}$

(b) (0,0) lies on $y = 2x$

$d = \left|\dfrac{4\times0-2\times0+7}{\sqrt{4^2+2^2}}\right|$

$= \dfrac{7}{\sqrt{20}}$

(c)(i) $m_{AB} = \dfrac{4-1}{4-0} = \dfrac{3}{4}$

(ii) $m_{BC} = -\dfrac{4}{3}$

Eqn BC $y-4 = -\frac{4}{3}(x-4)$

$3y-12 = -4x+16$

$4x+3y-28 = 0$

(iii) At C, $y = 0$

$\therefore 4x+3\times0-28 = 0$

$x = 7$

C is $(7,0)$

(iv) $M = \left(\dfrac{7+0}{2}, \dfrac{0+1}{2}\right)$

$= \left(3\frac{1}{2}, \frac{1}{2}\right)$

(v) $r = \sqrt{(7-3\frac{1}{2})^2+(0-\frac{1}{2})^2}$

$= \sqrt{12\frac{1}{4}+\frac{1}{4}}$

$= \sqrt{12\frac{1}{2}}$

$= \dfrac{5}{\sqrt{2}}$

(vi) $(x-3\frac{1}{2})^2+(y-\frac{1}{2})^2 = \dfrac{25}{2}$

sub (0,1) $(-3\frac{1}{2})^2+(\frac{1}{2})^2$

$= 12\frac{1}{2}$

sub (4,4) $(\frac{1}{2})^2+(3\frac{1}{2})^2$

$= 12\frac{1}{2}$

$\therefore$ The circle passes through A and B

Q4

(a)

(i) $(x+2)^2-y^2$

$= (x+2+y)(x+2-y)$

(ii) $(5x+y)(x-2y)$

(b) $3(\sin^2A+\cos^2A) + \cos^2A - 3$

$= 3\times1+\cos^2A-3$

$= \cos^2A$

(c) $3x-2y+6 > 0$

$x+2y-10 > 0$

(d) $p-2 = 2p-1$ OR $p-2 = 1-2p$

$-1 = p$ ∴ $p = 1$

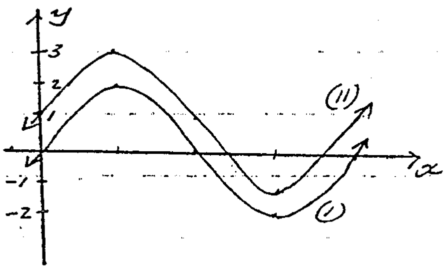
CHECK $|-1-2| = 2\times-1-1$ $|1-2| = 2\times1-1$

$3 = -3$ × $1 = 1$ ✓

$\therefore p = 1$

These suggested answers/marking schemes are issued as a guide only
-offerred as an assistance in constructing your own marking format
(individual teachers/schools find many other acceptable responses)

Q4(e)



Q5

(a) $\Delta = (-1)^2 - 4 \cdot k \cdot k$

$= 1 - 4k^2 < 0$

$4k^2 > 1$

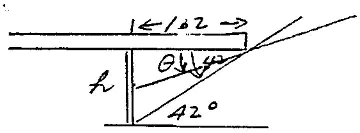
$k^2 > \frac{1}{4}$

$k > \frac{1}{2}$ or $k < -\frac{1}{2}$

For $-$ve, $a < 0$

$\therefore \underline{k < -\frac{1}{2}}$

(b)



(i) $\tan 42° = h/1.2$

$h = 1.2 \tan 42°$

$= \underline{1.08\text{m}}$

(iii) $\tan\theta = \frac{0.54}{1.2}$

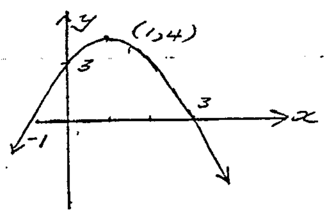
$\theta = \underline{24°}$

(c)(i) $-(x^2 - 2x) + 3$

$= -(x^2 - 2x + 1) + 4$

$= \underline{-(x-1)^2 + 4}$

(ii)



(iii) Max val is $\underline{4}$

Q6

(a)(i) $\frac{1}{2} \cdot 4x^3 - 3$

$= \underline{2x^3 - 3}$

(ii) $\frac{(x^2+4)\cdot 2 - 2x \cdot 2x \cdot 1}{(x^2+4)^2}$

$= \underline{\frac{8 - 2x^2}{(x^2+4)^2}}$

(iii) $y = x^{1\frac{1}{2}} = x^{3/2}$

$y' = \frac{3}{2} x^{1/2}$

$= \underline{\frac{3\sqrt{x}}{2}}$

(b)(i) In Δ's DFC, BGC

DC = BC (sides of square ABCD)

FC = GC (sides of square EFCG)

$\angle C$ is common.

$\therefore \underline{\Delta DFC \equiv \Delta BGC \text{ (SAS)}}$

(ii) $\angle FDC = \angle GBC$

(corr angles in cong Δ's)

EG // BC (EG, FC opp sides of square EFGC)

$\therefore \angle GBC = \angle BGE$ (alt $\angle$'s EG // BC)

$\therefore \underline{\angle FDC = \angle BGE}$

(c) $y = 2 - 2x$.

$x(2 - 2x) = -4$

$-2x^2 + 2x = -4$

$-2x^2 + 2x + 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

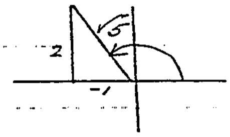
$x = 2, -1$

$y = -2, 4$

$\underline{(2,-2), (-1,4)}$ are the points of int.

Q7

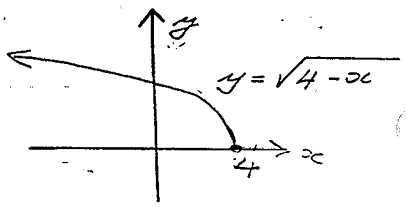
(a) B is in Quad 2



$\underline{\text{cosec } B = +\frac{\sqrt{5}}{2}}$

(b) $4 - x \geq 0$

$\underline{\text{Domain } x \leq 4}$



(c)(i) P $x = 3$ $\underline{\text{P is } (3,9)}$

(ii) $y' = 2x$

$m = 6$

$y - 9 = 6(x - 3)$

$\underline{y = 6x - 9}$

(iii) N, $y = 0$ $x = 1\frac{1}{2}$

$\underline{(1\frac{1}{2}, 0)}$

(iv) base $= 1\frac{1}{2}$

height $= 9$

$A = (3 \times 9) - (\frac{1}{2} \times 1\frac{1}{2} \times 9)$

$= \underline{20\frac{1}{4} \text{ units}^2}$

(d) $f(2x) = \frac{a^{2x} + a^{-2x}}{2}$

$2[f(x)]^2 - 1$

$= 2\left[\frac{a^{2x} + 2 + a^{-2x}}{4}\right] - 1$

$= \frac{a^{2x} + 2 + a^{-2x}}{2} - 1$

$= \frac{a^{2x} + 2 + a^{-2x} - 2}{2}$

$= \frac{a^{2x} + a^{-2x}}{2}$

$= f(2x)$