

NSW INDEPENDENT SCHOOLS

2013
Year 10 Final Exam
Stage 5.3

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- A formula sheet is provided
- Board approved calculators may be used
- Write using black or blue pen
- Draw diagrams using pencil
- Write your student number and/or name at the top of every page

Total marks – 80

Section I – Pages 2–8
20 marks

- Attempt Questions 1–20
- Allow about 30 minutes for this Section

Section II – Pages 9–18
60 marks

- Attempt Questions 21–25
- Allow about 60 minutes for this section

This paper MUST NOT be removed from the examination room

- 1 Which of the following correctly shows 0.015783 with 3 significant figures?
(A) 0.02 (B) 0.016 (C) 0.0157 (D) 0.0158
-
- 2 The volume of a cube is 216 cm^3 .
What is the surface area (in cm^2) of the cube?
(A) 36 (B) 54 (C) 72 (D) 216
-
- 3 The mean test result of a group of 25 students in Class A is 72.5%.
The mean test result of a group of 20 students in Class B on the same test is 76%.
What is the mean test result of the students in the classes combined?
(A) 71% (B) 74.1% (C) 74.25% (D) 75.8%
-
- 4 At a vintage book store, for every 4 books purchased, customers receive one free book. A customer pays \$51 for books which cost \$3 each.
How many books did this customer take home
(A) 20 (B) 21 (C) 22 (D) 24
-
- 5 Each of the symbols \square and \blacklozenge represent different integers.
- | | | |
|---------------------------|---------------------------|-------------------------------|
| $\square + \blacklozenge$ | $\square - \blacklozenge$ | $\square^2 - \blacklozenge^2$ |
| 7 | 11 | ? |
- What number is missing from the table?
(A) 72 (B) 76 (C) 77 (D) 85
-
- 6 The point $M(2, 4)$ is the midpoint of the interval PQ where P is $(-2, 6)$.
What are the co-ordinates of Q ?
(A) $(6, 2)$ (B) $(2, 6)$ (C) $(2, 1)$ (D) $(0, 5)$

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7 Which of these equations is equivalent to $y = \frac{x}{5} - 3$?

- (A) $x + 5y - 3 = 0$ (B) $5x - y + 3 = 0$
 (C) $x - 5y - 15 = 0$ (D) $x - 5y + 15 = 0$

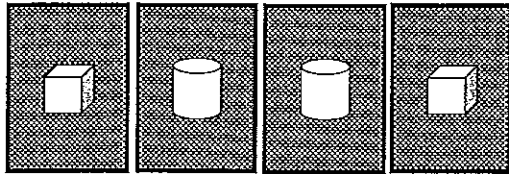
8 If $\cos A = 0.6$, and A is acute, what is the value of $\tan A$?

- (A) $\frac{5}{3}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{5}{4}$

9 There are 20 cards in a game of shape recognition for young children.

5 cards have a picture of a cube, 5 have cylinders, 5 have pyramids and 5 have cones.

Sally has been given the following four cards at random from the set of 20 and are kept out on the table.

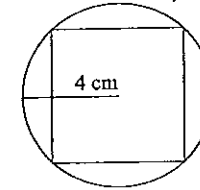


What is the probability that Sally's next (5th) card randomly drawn from the set, will give her "three of a kind"?

- (A) $\frac{2}{3}$ (B) $\frac{4}{5}$ (C) $\frac{1}{8}$ (D) $\frac{3}{8}$

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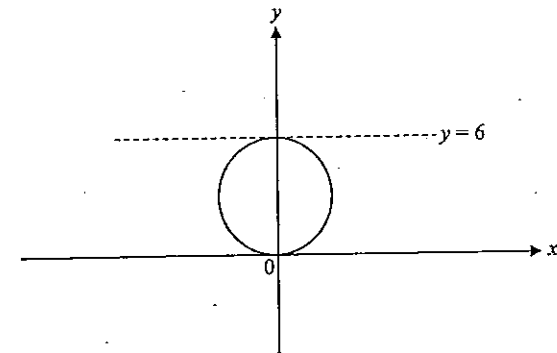
10 The vertices of the square touch the circumference of the circle which has a radius of 4 cm, as shown.



What is the area (in cm^2) of the square?

- (A) 16 (B) 32 (C) 48 (D) 64

11



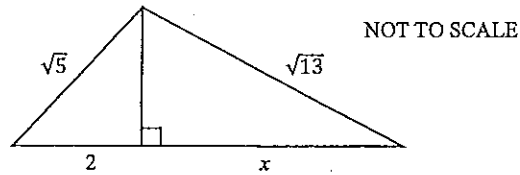
The points $(0, 0)$ and $(0, 6)$ lie on the circumference of the circle and the y -axis passes through its centre.

What is the equation of the circle?

- (A) $x^2 + y^2 = 36$ (B) $x^2 + (y + 3)^2 = 36$
 (C) $(x - 3)^2 + y^2 = 9$ (D) $x^2 + (y - 3)^2 = 9$

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12

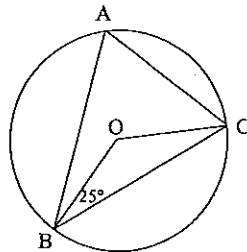


What is the value of x in the diagram?

- (A) 2 (B) $3\sqrt{2}$ (C) $2\sqrt{3}$ (D) 6

13 A, B and C are three points on the circumference of the circle, centre O .

Angle $OBC = 25^\circ$.



What is the size of angle BAC ?

- (A) 30° (B) 50° (C) 65° (D) 70°

14 P and Q are related by the equation $P = 4Q^2$ where P and Q are positive integers.

How is this equation written with Q as the subject?

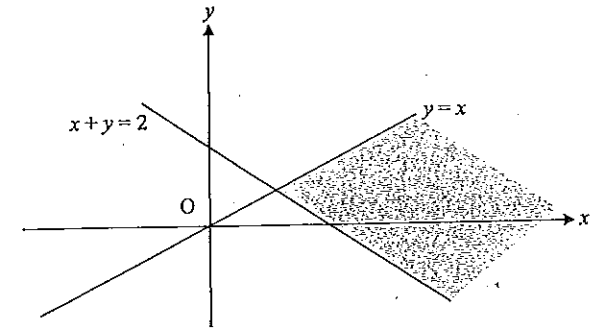
- (A) $Q = \frac{\sqrt{P}}{2}$ (B) $\sqrt{\frac{P}{2}}$ (C) $2P^2$ (D) $\frac{\sqrt{P}}{4}$

15 $(2 - 3\sqrt{3})^2 =$

- (A) $13 - 6\sqrt{3}$ (B) $31 - 12\sqrt{3}$ (C) $31 - 6\sqrt{3}$ (D) $12\sqrt{3} - 23$

STUDENT NAME/NUMBER:

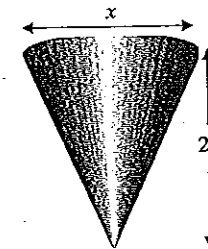
16



Which pair of inequalities satisfies the shaded region?

- (A) $y \geq x$ and $x + y \leq 2$ (B) $y \leq x$ and $x + y \geq 2$
 (C) $y \geq x$ and $x + y \geq 2$ (D) $y \leq x$ and $x + y \leq 2$

17 The height of this cone is twice the length of its diameter.



What expression gives the volume of the cone?

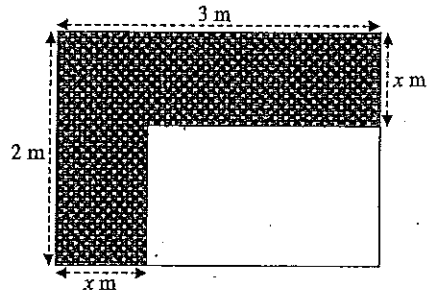
- (A) $\frac{\pi x^2}{12}$ (B) $\frac{2\pi x^3}{3}$ (C) $\frac{\pi x^3}{6}$ (D) $\frac{2\pi x^4}{3}$

18 Water is being removed from a dam at a constant rate of 2.5% of the water remaining, every 3 days.

By what percentage will the water level have dropped in the dam at the end of 9 days, if no water is added during this period?

- (A) 7.31 (B) 7.5 (C) 9.63 (D) 15.63

19 What is the area (in square metres) of the shaded section of the rectangle?



- (A) $x(5-x)$ (B) $3+2x-x^2$ (C) $x(1-x)$ (D) $1+5x-x^2$

20 If $x^2 + 8x + a = (x + b)^2$ what are the values of a and b ?

- (A) $a = 4$ and $b = 4$ (B) $a = 16$ and $b = 4$
 (C) $a = 64$ and $b = 8$ (D) $a = 16$ and $b = 8$

Section II

60 marks

Attempt Questions 21–25

Allow about 1 hour for this section

Answer the questions in the spaces provided.

All necessary working should be shown in every question (except Question 21).

Question 21 (12 marks)

Marks

Working is not required to be shown in this question.

(a) Simplify $(3a^2)^3$. 1

.....

(b) Factorise fully $px + pq - 7x - 7q$. 1

.....

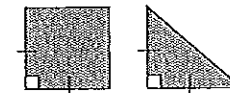
(c) Evaluate $\frac{2^{n+1}}{2^{n-1}}$. 1

.....

(d) Solve $(3x - 2)(x + 4) = 0$. 1

.....

(e) The total area of the square and the triangle is 24 cm^2 . 1



What is the area of the triangle?

.....

Question 21 continues on the next page

STUDENT NAME/NUMBER:

Question 21 (continued)

Marks

(f) Write as a decimal, the reciprocal of 1.25.

1

(g) By rationalising the denominator, simplify $\frac{2}{3-\sqrt{5}}$.

1

(h) Simplify $\frac{3m}{5} - \frac{m-2}{2}$.

1

(i) J is 20% of K .

What percentage is K of J ?

1

(j) A positive integer less than 30 is selected.

What is the probability that the selected integer is not a multiple of 3 or 5?

1

(k) The lines with equations $2x - 4y + 1 = 0$ and $y = 2mx$ are perpendicular.

What is the value of m ?

1

(l) The sum of the first n even integers is given by the formula $S_1 = n^2 + n$.

Write a simplified formula for the sum S_2 of the first $n - 1$ even integers.

1

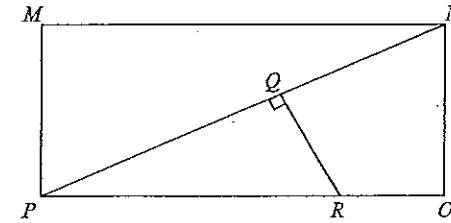
End of Question 21

STUDENT NAME/NUMBER:

Question 22 (12 marks)

Marks

(a)



PN is a diagonal of the rectangle $MNOP$ and R is on PO such that RQ is perpendicular to PN at Q .

(i) Prove that $\Delta PQR \parallel \Delta NMP$.

2

(ii) Given that $MP = 5$ cm, $MN = 10$ cm and $QR = 2$ cm, find the length of PQ .

2

(iii) Determine the area of the quadrilateral $QRON$.

2

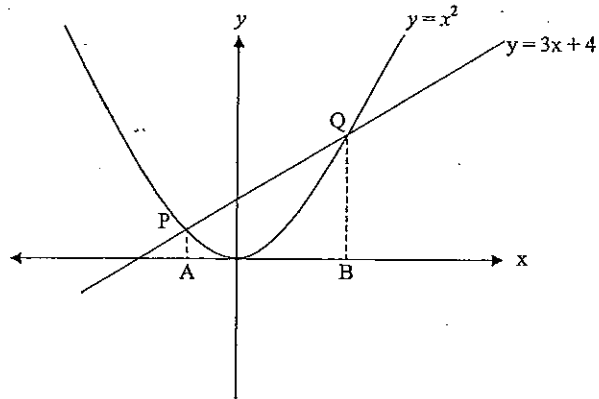
Question 22 continues on the next page

Question 22 (continued)

Marks

- (b) The diagram shows the line $y = 3x + 4$ intersecting the parabola $y = x^2$ at points P and Q .

The points A and B are the x co-ordinates of P and Q respectively.



- (i) Solve the equation $x^2 - 3x - 4 = 0$. 2

.....

- (ii) Use your answer in (i) to calculate the co-ordinates of P and Q . 2

P :

Q :

- (iii) Determine the area of the trapezium $APQB$. 2

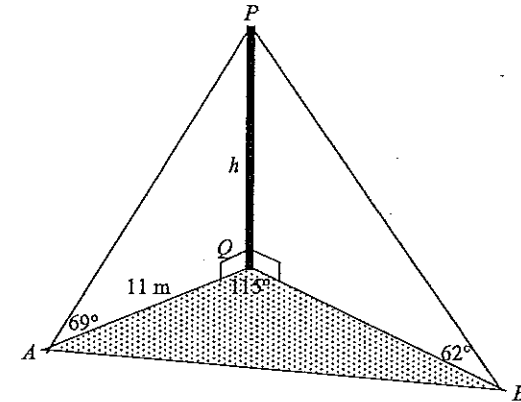
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End of Question 22

Question 23 (12 marks)

Marks

- (a) The diagram shows a pole PQ of height h metres, standing perpendicular to the ground.



The points A and B are at ground level with angles of elevation of 69° and 62° respectively, to the top of the pole P .

The point A is 11 metres from the base Q of the pole, and $\angle AQB$ is 115° at ground level.

- (i) Calculate the height of the pole to the nearest metre. 2

.....

- (ii) Determine the distance of B from the foot (Q) of the pole to the nearest metre. 2

.....

- (iii) Use the Cosine rule to calculate the distance, to the nearest metre, between A and B . 3

.....

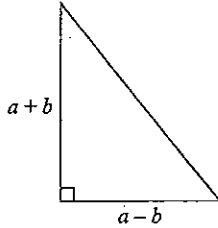
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STUDENT NAME/NUMBER:

Question 23 (continued)

Marks

- (b) The right-angled triangle has base length $(a - b)$ cm and height $(a + b)$ cm where $a > b$.



- (i) Show that the length of the hypotenuse can be given by $\sqrt{2(a^2 + b^2)}$. 2

.....

- (ii) The area of the triangle is 24 cm^2 .
 If $(a + b) = 8$, determine the value of $(a - b)$. 1

.....

- (iii) Using the answer from part (i), show that $(a^2 + b^2) = 50$. 2

.....

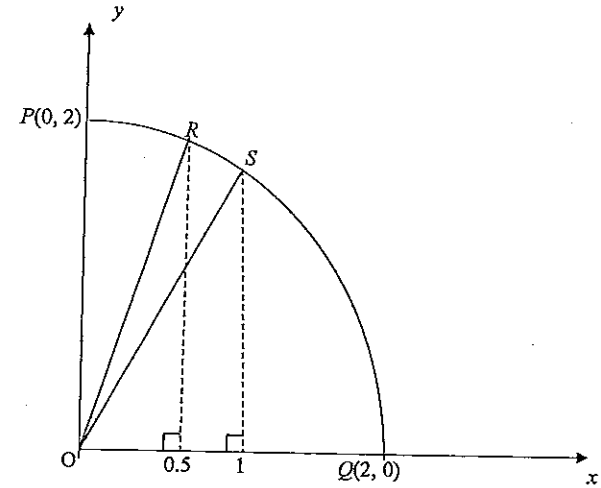
End of Question 23

STUDENT NAME/NUMBER:

Question 24 (12 marks)

Marks

- (a) The diagram shows an arc PQ , part of the circumference of a circle (centre O), drawn on the co-ordinate axes.



The points R and S are also on the circumference of the circle, and have horizontal distances from the origin, as shown.

- (i) Write down the equation of the circle, of which PQ is a part. 1

.....

- (ii) What is the length of OS ? 1

.....

- (iii) Calculate the size of angle SOQ . 2

.....

- (iv) Calculate the distance (to 2 decimal places) between R and S , along the arc PQ . 3

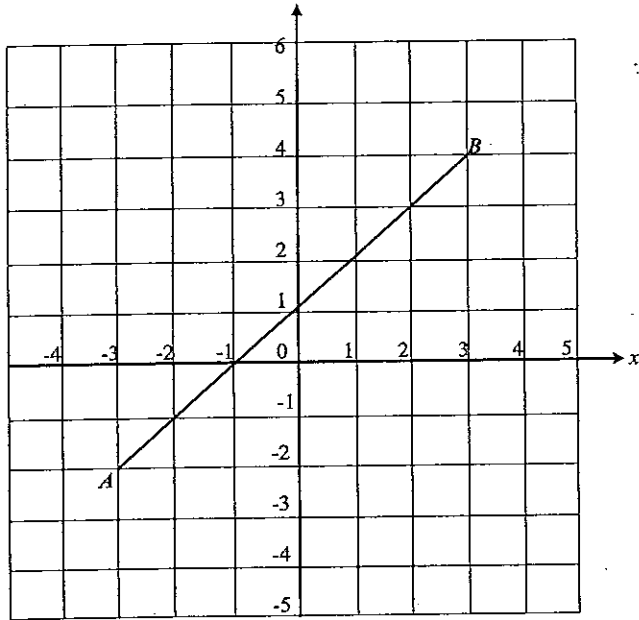
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Question 24 continues on the next page

Question 24 (continued)

Marks

- (b) The diagram shows an interval AB with respective co-ordinates $(-3, -2)$ and $(3, 4)$.



- (i) Determine the length of AB as a surd in simplest form. 2
-
-
- (ii) Write down the equation of AB . 2
-
-
- (iii) Write down the equation of the line parallel to AB and passing through $(3, 1)$. 1
-
-

End of Question 24

Question 25 (12 marks)

Marks

- (a) Jason made a deposit of $\$P$ in an investment account 5 years ago.

Jason's investment earned compound interest at the rate of 4.76% p.a paid quarterly over the first 2 years and then increased to 4.84% p.a paid quarterly for the remaining 3 years.

At the end of the first 2 years, Jason had $\$11850$ in his investment account.

- (i) Calculate the amount of Jason's initial investment ($\$P$) to the nearest dollar. 3

.....

.....

.....

.....

- (ii) Calculate the amount (to the nearest dollar) Jason will have in his account at the end of the 5 year period. 3

.....

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.....

- (iii) Determine the total amount of interest Jason earned on his investment. 1

.....

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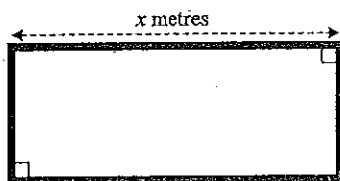
Question 25 continues on the next page

STUDENT NAME/NUMBER:

Question 25 (continued)

Marks

- (b) A rectangular enclosure is to be built for chickens using 120 metres of specialised wire fencing.



The length of the enclosure is x metres, as shown.

- (i) Write a simplified algebraic expression (in terms of x) for the width of the enclosure.

2

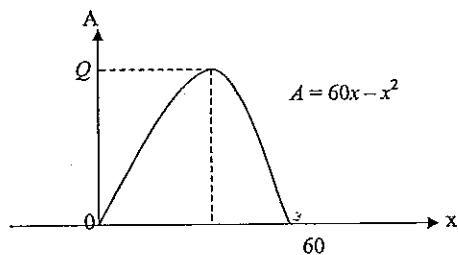
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- (ii) Show that the area (A) of the enclosure can be given by $A = 60x - x^2$.

1

.....

- (iii) A parabolic graph representing the area of the enclosure is shown below.



Explain what the value of Q represents in terms of the enclosure, and determine its value.

2

.....

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2013
 MATHEMATICS STAGE 5.3 – YEAR 10 FINAL EXAM
 MARKING GUIDELINES

SECTION 1

| | | | | | | | | | | |
|------------------|---|---|---|---|---|---|---|---|---|----|
| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Solution | D | D | B | B | C | A | C | C | D | B |
| Performance Band | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 4 |

| | | | | | | | | | | |
|------------------|----|----|----|----|----|----|----|----|----|----|
| Question | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Solution | D | C | C | A | B | B | C | A | A | B |
| Performance Band | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 4 | 5 | 5 |

SECTION 2

| Question | Marks | Performance Band | Suggested Solution |
|----------|-------|------------------|---|
| 21(a) | 1 | 2 | $27a^6$ |
| (b) | 1 | 3 | $p(x+q) - 7(x+q)$ $= (x+q)(p-7)$ |
| (c) | 1 | 3 | $2^{n+1} + 2^{n-1}$ $= 2^{n+1-n+1}$ $= 2^2$ $= 4$ |
| (d) | 1 | 3 | $(3x-2)=0$ or $(x+4)=0$ $x = \frac{2}{3}$ or $x = -4$ |
| (e) | 1 | 3 | $x^2 + \frac{1}{2}x^2 = 24$ $\frac{3}{2}x^2 = 24$ $x^2 = 16$ $x = 4$ Area of triangle = $0.5 \times 4 \times 4$ $= 8$ square cm. |
| (f) | 1 | 3 | $1.25 = \frac{5}{4}$ reciprocal is $\frac{4}{5} = 0.8$ |
| (g) | 1 | 4 | $\frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$ $= \frac{2(3+\sqrt{5})}{4}$ $= \frac{3+\sqrt{5}}{2}$ |

Question 21 continues on the next page

Question 21 continues on the next page

| Question | Marks | Performance Band | Suggested Solution |
|----------|-------|------------------|--|
| (h) | 1 | 4 | $\frac{2(3m) - 5(m-2)}{10}$ $= \frac{6m - 5m + 10}{10}$ $= \frac{m + 10}{10}$ |
| (i) | 1 | 5 | $J = 0.2k$ $= \frac{k}{5}$ $K = 5J$ K is 500% of J |
| (j) | 1 | 5 | Multiples of 3 or 5 are: 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27 There are 13 multiples Required probability is $\frac{16}{29}$ |
| (k) | 1 | 5 | Gradient of $2x - 4y + 1 = 0$ is $\frac{1}{2}$ So $2m \times \frac{1}{2} = -1$. $m = -1$ |
| (l) | 1 | 5/6 | $S_2 = (n-1)^2 + (n-1)$ $= n^2 - 2n + 1 + n - 1 = n^2 - n$ |

| Question | Marks | Performance Band | Suggested Solution |
|----------|--------|------------------|---|
| 22(a)(i) | 2 | 3 | In ΔPQR and ΔNMP <ul style="list-style-type: none"> $\angle PQR = \angle NMP = 90^\circ$ $\angle RPQ = \angle PNM$ (alternate equal angles) $\angle QRP = \angle MPN$ (3rd equal angle in both) Hence $\Delta PQR \sim \Delta NMP$ (equiangular) |
| (ii) | 1 | 4 | $\frac{PQ}{MN} = \frac{QR}{PM}$ $\frac{PQ}{10} = \frac{QR}{5}$ Hence $PQ = 4$ |
| (iii) | 1 | 4 | Area rectangle $MNOP = 10 \times 5 = 50 \text{ cm}^2$ Area $\Delta NMP = 0.5 \times 10 \times 5 = 25 \text{ cm}^2$ Area $\Delta PQR = 4 \text{ cm}^2$ Area $QRON = 50 - (25 + 4) \text{ cm}^2$ 21 cm^2 |
| (b)(i) | 1 1 | 3 | $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = 4$ or $x = -1$ |
| (ii) | 1 1 | 4 | P is where $x = -1, y = (-1)^2 = 1$ P is $(-1, 1)$ Q is where $x = 4, y = (4)^2 = 16$ Q is $(4, 16)$ |
| (iii) | 1 1 | 4/5 | Area trapezium $APQB = 0.5(AB)[(AP) + (BQ)]$ $= 0.5(5)[1 + 16]$ $= 42.5 \text{ square units}$ |

| Question | Marks | Performance Band | Suggested Solution |
|----------|-------------|------------------|--|
| 23(a)(i) | 1 1 | 3 | In ΔPQB , $\tan 69^\circ = \frac{h}{11}$ $h = 11 \times \tan 69^\circ$ $= 29 \text{ m}$ |
| (ii) | 1 1 | 4 | In ΔPQB , $\angle PQB = 28^\circ$ So $\tan 28^\circ = \frac{QB}{29}$ $QB = 29 \times \tan 28^\circ$ $= 15 \text{ m}$ |
| (iii) | 1 1 1 | 5 | $AB^2 = 11^2 + 15^2 - (2 \times 11 \times 15 \times \cos 115^\circ)$ $= 485.464$ $AB = 22 \text{ m}$ |
| (b)(i) | 1 1 | 4 | $L^2 = (a+b)^2 + (a-b)^2$ $= a^2 + 2ab + b^2 + a^2 - 2ab + b^2$ $= 2a^2 + 2b^2$ $= 2(a^2 + b^2)$ $L = \sqrt{2(a^2 + b^2)}$ |
| (ii) | 1 | 4 | $\frac{1}{2}(a-b)(a+b) = 24$ $\frac{1}{2}(a-b)(8) = 24$ $(a-b) = 6$ |
| (iii) | 1 1 | 5 | If $(a+b) = 8$, $(a-b) = 6$, then $L = 10$ using Pythagoras' theorem. Since $L = \sqrt{2(a^2 + b^2)}$ Then $2(a^2 + b^2) = 100$, hence $a^2 + b^2 = 50$ |

| Question | Marks | Performance Band | Suggested Solution |
|----------|-------------|------------------|---|
| 24(a)(i) | 1 | 3 | $x^2 + y^2 = 2^2$ $x^2 + y^2 = 4$ |
| (ii) | 1 | 3 | OS is a radius so $OS = 2$ |
| (iii) | 1 1 | 4 | $\cos \angle SOQ = \frac{OS}{OQ} = \frac{1}{2}$ $\angle SOQ = 60^\circ$ |
| (iv) | 1 1 1 | 6 | $\cos \angle ROQ = \frac{OR}{OQ} = \frac{0.5}{2} = 0.25$ $\angle ROQ = 76^\circ$ (nearest degree) Hence $\angle ROS = 16^\circ$ Length of arc $RS = \frac{16}{360} \times 2 \times \pi \times 2$ $= 0.56$ units |
| (b)(i) | 1 1 | 4 | $AB^2 = 6^2 + 6^2$ $= 72$ $AB = \sqrt{72}$ $= \sqrt{36 \times 2}$ $= 6\sqrt{2}$ |
| (ii) | 1 1 | 4 | $y = mx + b_1$ with $m = \frac{6}{6} = 1$ Let $(x, y) = (3, 4)$ So $4 = 3 + b_1$ $b_1 = 1$ And $y = x + 1$ |
| (iii) | 1 | 5 | A line parallel to $y = x + 1$ has $m = 1$ Let $(x, y) = (3, 1)$ in $y = x + b_2$ So $1 = 3 + b_2$ $b_2 = -2$ Hence, $y = x - 2$ |

| Question | Marks | Performance Band | Suggested Solution |
|----------|-------------|------------------|--|
| 25(a)(i) | 1 1 1 | 5 | $A = P(1 + r)^n$ $A = \$11850, r = 4.76 \div 4 = 1.19, n = 4 \times 2 = 8$ $11850 = P(1.0119)^8$ $P = \frac{11850}{(1.0119)^8}$ $= \$10\ 780$ |
| (ii) | 1 1 1 | 5 | $P = \$11850, r = 4.84 \div 4 = 1.21$ and $n = 3 \times 4 = 12$ $A = 11850(1.0121)^{12}$ $= \$13\ 690$ |
| (iii) | 1 | 4 | Interest earned = $\$13690 - \10780 $= \$2910$ |
| (b)(i) | 1 1 | 5 | Width (w) = $\frac{120 - 2x}{2}$ $= 60 - x$ |
| (ii) | 1 | 3 | Area (A) = $x(60 - x)$ $= 60x - x^2$ |
| (iii) | 1 1 | 6 | <p>The value of Q represents the maximum area of the enclosure.</p> <p>Since the parabolic graph is symmetrical about the line $x = 30$, the maximum value (Q) is: $60(30) - (30)^2$ $= 900$</p> <p>So the maximum area of the enclosure is 900 m^2</p> <p>Note the enclosure would be square with sides 30 m</p> |