NSW INDEPENDENT SCHOOLS

2015
Higher School Certificate
Preliminary Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 3 – 5

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 6 – 9

60 marks

Attempt Questions 11 – 14

Allow about 1 hour 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ……………………..

Student name / number ____________________

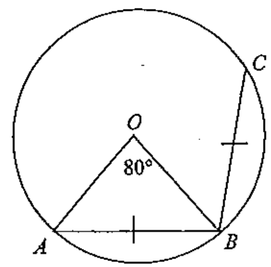
Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

1



In the diagram, AB is a chord of a circle with centre O such that $\angle AOB = 80°$. 1
C is a point on the circle such that $BC = BA$. What is the size of $\angle ABC$?

(A) 90°

(B) 100°

(C) 110°

(D) 120°

2 $A(-2,5)$ and $B(4,-1)$ are two points. What are the coordinates of the point $P(x,y)$ that divides AB internally in the ratio 2 : 1 ? 1

(A) $(-5,8)$

(B) $(0,3)$

(C) $(2,1)$

(D) $(7,-4)$

3 If $y=(x^3+1)^5$, which of the following is an expression for $\frac{dy}{dx}$? 1

(A) $3x^2$

(B) $5(3x^2)^4$

(C) $5(x^3+1)^4$

(D) $15x^2(x^3+1)^4$

Marks

4 What is the size of the acute angle between the lines $2x-y=0$ and $x+y=0$, correct to the nearest degree? **1**

(A) 18°
(B) 19°
(C) 71°
(D) 72°

5 Which of the following is an expression for $\cos(A-B)-\cos(A+B)$? **1**

(A) $2\sin A\sin B$
(B) $2\cos A\cos B$
(C) $-2\cos A\cos B$
(D) $-2\sin A\sin B$

6 If $f(x)=\dfrac{x}{2x-1}$, which of the following is an expression for $f'(x)$? **1**

(A) $\dfrac{-2}{(2x-1)^2}$
(B) $\dfrac{-1}{(2x-1)^2}$
(C) $\dfrac{1}{(2x-1)^2}$
(D) $\dfrac{2}{(2x-1)^2}$

7 The equation $x^3-3x^2-2x+1=0$ has roots α, β and γ. What is the value of $\alpha^2+\beta^2+\gamma^2$? **1**

(A) 3
(B) 5
(C) 9
(D) 13

Marks

8 Which of the following is an expression for $\dfrac{(n+2)!-n!}{(n+1)!}$? **1**

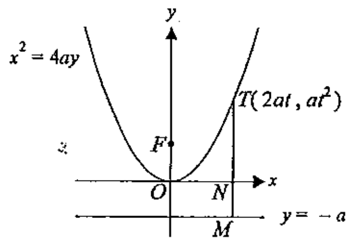
(A) $\dfrac{n^2+3n+1}{n+1}$
(B) $\dfrac{n^2+3n+2}{n+1}$
(C) $\dfrac{n^2+2n+2}{n+1}$
(D) $\dfrac{n^2+2n+3}{n+1}$

9 The equation $\tan^2\theta+b\tan\theta+c=0$ has roots $\tan\alpha$ and $\tan\beta$. Which of the following is an expression for $\tan(\alpha+\beta)$? **1**

(A) $\dfrac{-b}{1+c}$
(B) $\dfrac{-b}{1-c}$
(C) $\dfrac{b}{1-c}$
(D) $\dfrac{b}{1+c}$

10 **1**

In the diagram, $T(2at, at^2)$, where $a>0$, is a point on the parabola $x^2=4ay$ with focus $F(0,a)$ and directrix $y=-a$.
M is the foot of the perpendicular from T to the directrix and TM cuts the x axis at N.



Which of the following statements is NOT correct ?

(A) $TM=TF$
(B) $TN+FO=TF$
(C) $TM=a(t+1)^2$
(D) $TF+NO=a(t+1)^2$

Student name / number ____________________

Marks

Section II

60 Marks
Attempt Questions 11-14.
Allow about 1 hour and 45 minutes for this section.

Answer the questions in writing booklets if provided, or on your own paper.
Start each question in a new booklet, or on a new page.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a separate writing booklet.**

(a) $P(x)$ is an odd polynomial function whose graph passes through the point $(1,3)$. Find the remainder when $P(x)$ is divided by $(x+1)$. **2**

(b) $A(-3,6)$ and $B(1,2)$ are two points. Find the coordinates of the point $P(x,y)$ that divides AB externally in the ratio 3 : 1. **2**

(c) Find the number of ways in which 3 boys and 3 girls can line up in a queue so that the first and last in the queue are both boys. **2**

(d) Solve the equation $\tan 2x + \tan x = 0$ for $0° < x < 180°$. **3**

(e) The Great Pyramid of Egypt has a horizontal square base $ABCD$ of side 230 m. The diagonals of its base intersect at O. The top T of the pyramid is 147 m vertically above O. Find the angle of elevation of T from A, giving the answer correct to the nearest degree. **3**

(f) The normal to the parabola $x^2 = 4ay$ at the point $T(2at, at^2)$ on the parabola has equation $x + ty - 2at - at^3 = 0$. (Do NOT prove this result.) The normal cuts the x axis at X and the y axis at Y. If T is the midpoint of XY, find the coordinates of T. **3**

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Marks

Question 12 (15 marks) **Use a separate writing booklet.**

(a) The polynomial $P(x)$ is such that $P(x) = (x-1)(x-2)Q(x) + 3x + k$ for some polynomial $Q(x)$ and some constant k. If the remainder is -1 when $P(x)$ is divided by $(x-1)$, find the remainder when $P(x)$ is divided by $(x-2)$. **2**

(b) Find the value of m, where $m > 0$, such that the acute angle between the lines $y = 2x$ and $y = mx$ is $45°$. **2**

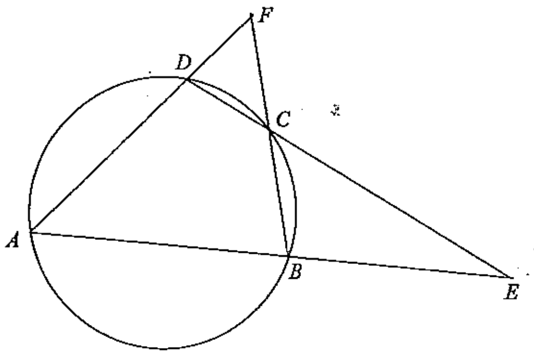
(c) A multiple choice test contains 5 questions. In each question there are 4 answers, one of which is correct, the other three being incorrect. If all 5 questions are attempted, find the number of ways in which exactly 3 questions can be answered correctly. **2**

(d) Giving your answers correct to the nearest degree, use the substitution $t = \tan\frac{x}{2}$ to solve the equation $\cos x + 2\sin x = -1$ for $0° \le x \le 360°$. **3**

(e)(i) $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$. Use differentiation to show that the tangent to the parabola at T has gradient t. **1**

(ii) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The angles of inclination to the positive x axis of the tangents to the parabola at P and Q are θ and 2θ respectively. Express q in terms of p. **2**

(f) **3**



In the diagram, $ABCD$ is a cyclic quadrilateral. AB produced and DC produced meet at E. AD produced and BC produced meet at F. $\angle BEC = x°$, $\angle CFD = 2x°$ and $\angle DAB = 3x°$. Copy the diagram. Find the value of x, giving reasons.

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Marks

Question 13 (15 marks) **Use a separate writing booklet.**

(a) Solve the equation $2\sin(x-30°)=\sqrt{2}$ for $0° \le x \le 360°$. 2

(b) Consider the polynomial $P(x)=2x^3-5x^2-4x+3$.

(i) Show that $(x+1)$ is a factor of $P(x)$. 1

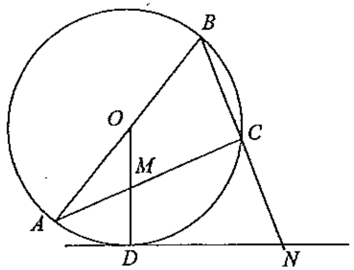
(ii) Solve the equation $P(x)=0$. 2

(c) Find the number of ways in which the letters of the word FORMULA can be arranged in a line

(i) without restriction. 1

(ii) so that the 4 consonants are all next to each other and the 3 vowels are all next to each other. 2

(d)



In the diagram, AB is a diameter of a circle with centre O. C and D are points on the circle. OD cuts AC at M. The tangent to the circle at D cuts BC produced at N. Copy the diagram. Show $MCND$ is a cyclic quadrilateral. 3

(e) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2=4ay$.

(i) Show that the chord PQ has equation $2y-(p+q)x+2apq=0$. 2

(ii) If P and Q move on the parabola such that the chord PQ always passes through the point $(0, 4a)$ on the y axis, show that OP and OQ are always perpendicular to each other. 2

Student name / number ____________________

Marks

Question 14 (15 marks) **Use a separate writing booklet.**

(a) Show that $\dfrac{1+\tan x}{1-\tan x}=\dfrac{1+\sin 2x}{\cos 2x}$. 2

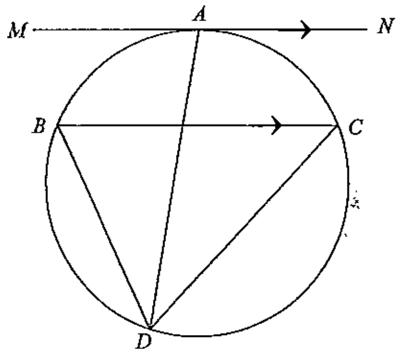
(b) The equation $x^3+bx^2+cx+d=0$ has roots α, α^3 and $\dfrac{1}{\alpha}$, where $\alpha \ne 0$. 3

Show that $b^3d-c^3=0$.

(c) Solve the inequality $\dfrac{5x}{x^2+1} \le x$. 3

(d) 6 boys and 5 girls are available for selection in a mixed team of 7 players. Find the number of ways that the team can be chosen if it has to contain at least 3 girls and at least 3 boys. 3

(e)



In the diagram, MAN is tangent to the circle at A. BC is a chord of the circle such that $BC \parallel MAN$. D is a point on the circle. 4

Copy the diagram. Show that AD bisects $\angle BDC$.

Section 1 **Questions 1-10 (1 mark each)**

Question	Answer	Solution	Outcomes
1	B	In ΔAOB, $AO=BO$ (radii) $\therefore \angle OAB=\angle OBA=50^\circ$ ($\angle$ sum Δ is 180°) But $\Delta COB\equiv\Delta AOB$ (SSS) $\therefore \angle OBC=\angle OBA=50^\circ$ $\therefore \angle ABC=\angle OBA+\angle OBC=100^\circ$	PE3
2	C	$x=\frac{2\times4+1\times(-2)}{2+1}=2$ $\quad y=\frac{2\times(-1)+1\times5}{2+1}=1$	P4
3	D	Using the chain rule, $\frac{dy}{dx}=5\left(x^3+1\right)^4.3x^2=15x^2\left(x^3+1\right)^4$	PE5
4	D	$\tan\theta=\left\lvert\frac{2-(-1)}{1+2\times(-1)}\right\rvert=3$ $\quad \therefore \theta\approx72^\circ$ (to the nearest degree)	P4
5	A	$\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\therefore \cos(A-B)-\cos(A+B)=\quad 2\sin A\sin B$	P4
6	B	Using the quotient rule, $f'(x)=\frac{1.(2x-1)-x.2}{(2x-1)^2}=\frac{-1}{(2x-1)^2}$	PE5
7	D	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=3^2-2\times(-2)=13$	PE3
8	A	$\frac{(n+2)!-n!}{(n+1)!}=\frac{n!\{(n+1)(n+2)-1\}}{n!(n+1)}=\frac{n^2+3n+1}{n+1}$	PE3
9	B	$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-b}{1-c}$	P4, PE3
10	C	$TM=TF$ (locus definition of parabola) A correct $FO=NM=a$ $\therefore TN+FO=TN+NM=TM=TF$ B correct $TM=at^2+a=a(t^2+1)\neq a(t+1)^2$ since $t\neq0$ C incorrect $TF+NO=TM+NO=at^2+a+2at=a(t^2+2t+1)=a(t+1)^2$ D correct	PE3

Section II

Question 11

a. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• applies the remainder theorem	1
• deduces the value of the remainder using the given coordinates and property of an odd function	1

Answer

$(1,3)$ lies on the graph of $P(x)$ $\therefore P(1)=3$. Since $P(x)$ is odd, remainder is $P(-1)=-P(1)=-3$.

Q11 (cont)

b. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• finds x coordinate of P	1
• finds y coordinate of P	1

Answer

$x=\frac{3\times1+(-1)\times(-3)}{3+(-1)}=3$ $\qquad y=\frac{3\times2+(-1)\times6}{3+(-1)}=0$ $\qquad$ Point is $P(3,0)$

c. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• counts the arrangements of the first and last	1
• multiplies by the arrangements of the remaining 4	1

Answer

The first and last in the queue can be selected from the boys in 3×2 ways
The remaining positions can be filled in $4!$ ways.
Hence the number of arrangements is $3\times2\times4!=144$

d. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• writes the equation in terms of $\tan x$	1
• solves for $\tan x$	1
• solves for x	1

Answer

$\tan2x+\tan x=0,\ 0^\circ<x<180^\circ$

$\frac{2\tan x}{1-\tan^2x}+\tan x=0$ $\qquad \tan x=0 \quad or \quad \tan^2x=3$

$\qquad \tan x=\pm\sqrt{3}$

$\tan x\left\{2+\left(1-\tan^2x\right)\right\}=0$ $\qquad 0^\circ<x<180^\circ \quad \therefore x=60^\circ,\ 120^\circ$

e. Outcomes assessed: P4, PE6

Marking Guidelines

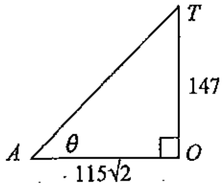
Criteria	Marks
• finds the distance AO	1
• writes an expression for the tangent of the angle of elevation using right triangle AOT	1
• calculates the angle of elevation	1

Answer

The diagonal of the square base has length $230\sqrt{2}$ metres.

$\therefore AO=115\sqrt{2}$. (Diagonals of a square bisect each other)

$\tan\theta=\frac{147}{115\sqrt{2}}$ $\qquad \therefore \theta\approx42^\circ$ is angle of elevation of T from A.



Q11(cont)

f. Outcomes assessed: PE4

Marking Guidelines

Criteria	Marks
• finds the coordinates of either X or Y	1
• uses the fact that T is the midpoint of XY to write an equation for t	1
• solves for t and states the coordinates of T in terms of a.	1

Answer

$x+ty-2at-at^3=0 \qquad y=0 \Rightarrow x=a t\left(2+t^2\right) \qquad \therefore\ X\left(at(2+t^2),0\right)$

$x=0 \Rightarrow y=a\left(2+t^2\right) \qquad Y\left(0,a(2+t^2)\right)$

$T\left(2at, at^2\right)$ is the midpoint of XY.

$$\therefore at^2=\tfrac{1}{2}a\left(2+t^2\right)$$
$$2t^2=2+t^2$$
$$t^2=2$$

$$\therefore T\left(2a\sqrt{2},2a\right) \text{ or } T\left(-2a\sqrt{2},2a\right)$$

Question 12

a. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• applies the remainder theorem to evaluate k	1
• applies the remainder theorem after substituting for k to find the required the remainder	1

Answer

$P(x)=(x-1)(x-2)Q(x)+3x+k \qquad P(1)=-1 \Rightarrow 3+k=-1 \qquad \therefore k=-4$

Then $P(x)=(x-1)(x-2)Q(x)+3x-4 \qquad \therefore P(2)=6-4=2$ and required remainder is 2.

b. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• writes an equation for m in terms of absolute values	1
• solves this equation for positive m	1

Answer

$\tan 45° = \left|\dfrac{m-2}{1+2m}\right| \qquad \therefore |2m+1|=|m-2|$

$2m+1=m-2 \quad or \quad 2m+1=-(m-2) \qquad \therefore m=\tfrac{1}{3}$ since $m>0$

$m=-3 \quad or \quad 3m=1$

c. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• counts the number of combinations of three questions answered correctly from five	1
• for each such combination, counts the number of ways of answering those remaining incorrectly	1

Answer

$${}^5C_3\times 3\times 3=90$$

Q12 (cont)

d. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• for angles other than 180°, writes the equation in terms of t and solves	1
• finds corresponding solution for x	1
• tests 180° and finds it is also a solution	1

Answer

$\cos x+2\sin x=-1, \quad 0°\le x\le 360°. \quad$ For $x\ne 180°$, let $t=\tan\frac{x}{2}$.

$$\frac{1-t^2}{1+t^2}+\frac{4t}{1+t^2}=-1$$
$$1-t^2+4t=-1-t^2$$
$$t=-\tfrac{1}{2}$$

$$\tan\tfrac{x}{2}=-\tfrac{1}{2}, \quad 0°\le\tfrac{x}{2}\le 180°$$
$$\tfrac{x}{2}\approx 180°-26\cdot565°$$
$$x\approx 306°52'$$

For $x=180°$:
$$\cos 180°+2\sin 180°=-1+0$$
$$=-1$$

Hence solutions are $x\approx 306°52'$ or $x=180°$

e. Outcomes assessed: PE4

Marking Guidelines

Criteria	Marks
i • proves the result using differentiation	1
ii • writes $\tan\theta$, $\tan 2\theta$ in terms of p and q	1
• expresses q in terms of p	1

Answer

i. $x=2at \qquad \dfrac{dx}{dt}=2a \qquad \therefore \dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}=\dfrac{2at}{2a}=t$

$y=at^2 \qquad \dfrac{dy}{dt}=2at \qquad$ Gradient of tangent at T is t

ii. $\tan\theta=p$ and $\tan 2\theta=q \qquad \therefore q=\dfrac{2p}{1-p^2}$

f. Outcomes assessed: PE2, PE3

Marking Guidelines

Criteria	Marks
• finds either $\angle FDC$ or $\angle ADC$ in terms of x	1
• finds $\angle ABC$ in terms of x	1
• writes and solves an equation for x	1

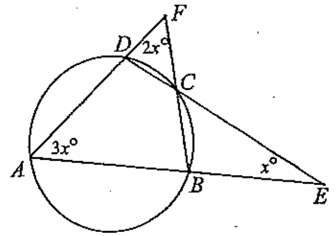
Answer

$\angle FDC=4x°$ *(Exterior $\angle$ of ΔADE is equal to sum of interior opposite $\angle$'s)*

$\therefore \angle ABC=4x°$ *(Exterior $\angle$ of cyclic quad. ABCD is equal to interior opposite $\angle$)*

$\therefore 9x=180$ *($\angle$ sum of ΔABF is 180°)*

$\therefore x=20$



Question 13

a. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• rearranges equation and recognises exact trigonometric ratio, stating one value for $x-30°$	1
• states both solutions for x	1

Answer

$$2\sin(x-30°)=\sqrt{2}\ ,\ 0°\le x\le 360° \qquad x-30°=45°,\ 135°$$
$$\sin(x-30°)=\tfrac{1}{\sqrt{2}} \qquad x=75°,\ 165°$$

b. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
i • applies the factor theorem	1
ii • completes the factorisation of $P(x)$	1
• writes down the three solutions	1

Answer

i. $P(x)=2x^3-5x^2-4x+3$ $\quad\therefore P(-1)=-2-5+4+3=0$ $\quad\therefore (x+1)$ is a factor of $P(x)$.

ii. $P(x)=(x+1)(2x^2-7x+3)$

$=(x+1)(2x-1)(x-3)$ $\quad\therefore P(x)=0$ has solutions $x=-1,\ \tfrac{1}{2},\ 3$

c. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
i • calculates the number of arrangements	1
ii • writes numerical expression with factors 4! and 3!	1
• includes the factor 2 and calculates the product	1

Answer

i. $7!=5040$ $\qquad$ ii. $2\times 4!\times 3!=288$

d. Outcomes assessed: PE2, PE3

Marking Guidelines

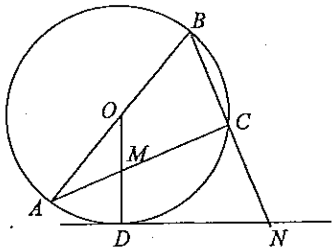
Criteria	Marks
• explains right angle at C	1
• explains right angle at D	1
• applies an appropriate test for a cyclic quadrilateral	1

Answer

$\angle ACB=90°$ $\quad$ (*$\angle$ in semi-circle is a right angle*)

$\angle ODN=90°$ $\quad$ (*tangent $\perp$ to radius drawn to point of contact*)

$\therefore MCND$ is cyclic $\quad$ (*exterior $\angle$ equal to interior opposite $\angle$*)



Q13 (cont)

e. Outcomes assessed: PE4

Marking Guidelines

Criteria	Marks
i • finds gradient of PQ	1
• finds equation of PQ	1
ii • finds product pq if PQ passes through given point	1
• finds product of gradients of OP and OQ in terms of p and q to deduce result	1

Answer

i. $m_{PQ}=\dfrac{a(p^2-q^2)}{2a(p-q)}=\tfrac{1}{2}(p+q)$ $\qquad \therefore PQ$ has equation $\quad y-ap^2=\tfrac{1}{2}(p+q)(x-2ap)$

$2y-(p+q)x+2apq=0$

ii. $(0,4a)$ lies on $PQ \Rightarrow 8a+2apq=0 \Rightarrow pq=-4$

$m_{OP}=\dfrac{ap^2}{2ap}=\tfrac{1}{2}p$ $\qquad \therefore m_{OP}\,.\,m_{OQ}=\tfrac{1}{4}pq=-1$ and $OP\perp OQ$ if $(0,4a)$ lies on PQ.

Question 14

a. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• uses expressions for $\cos 2x$ and $\sin 2x$ in terms of $\cos x$ and $\sin x$	1
• completes the proof using appropriate trigonometric identities	1

Answer

$$\frac{1+\sin 2x}{\cos 2x}=\frac{(\cos x+\sin x)^2}{\cos^2 x-\sin^2 x}=\frac{\cos x+\sin x}{\cos x-\sin x}=\frac{1+\tan x}{1-\tan x}$$

b. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• writes expressions for the coefficients in terms of α	1
• writes the quotient of c and b in terms of α	1
• completes proof by eliminating α	1

Answer

$x^3+bx^2+cx+d=0$ has roots $\alpha,\ \alpha^3,\ \dfrac{1}{\alpha}\ (\alpha\ne 0)$

$d=-\alpha\,.\,\alpha^3\,.\,\dfrac{1}{\alpha}$ $\qquad c=\alpha\,.\,\alpha^3+\alpha^3\,.\,\dfrac{1}{\alpha}+\dfrac{1}{\alpha}\,.\,\alpha$ $\qquad b=-\left(\alpha+\alpha^3+\dfrac{1}{\alpha}\right)$

$d=-\alpha^3$ $\qquad c=\alpha^4+\alpha^2+1$ $\qquad b=-\dfrac{1}{\alpha}(\alpha^4+\alpha^2+1)$

$\therefore\left(\dfrac{c}{b}\right)^3=(-\alpha)^3=-\alpha^3=d$. $\quad$ Hence $c^3=b^3d$. $\quad \therefore b^3d-c^3=0$

Q14 (cont)

c. Outcomes assessed: PE3, PE6

Marking Guidelines

Criteria	Marks
• rearranges to form a polynomial inequality	1
• writes one inequality for x	1
• writes the second inequality and indicates how they are to be combined	1

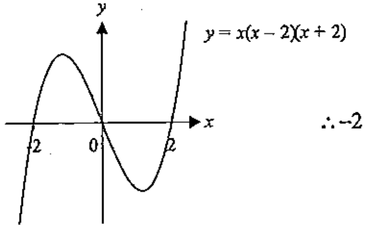
Answer

$$\frac{5x}{x^2+1} \le x$$

$$5x \le x\left(x^2+1\right)$$

$$0 \le x\left\{\left(x^2+1\right)-5\right\}$$

$$0 \le x(x-2)(x+2)$$



$\therefore -2 \le x \le 0 \quad or \quad x \ge 2$

d. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• writes an expression for the number of ways of selecting 4 boys and 3 girls	1
• writes an expression for the number of ways of selecting 3 boys and 4 girls	1
• adds these expressions and calculates the possible number of combinations	1

Answer

$^6C_4 \times {}^5C_3 + {}^6C_3 \times {}^5C_4 = 15\times 10 + 20 \times 5 = 250$

e. Outcomes assessed: PE2, PE3

Marking Guidelines

Criteria	Marks
• deduces $\angle CDA = \angle CBA$	1
• deduces $\angle CBA = \angle MAB$	1
• deduces $\angle MAB = \angle ADB$	1
• completes the proof	1

Answer

Join AB and AC.

$\angle CDA = \angle CBA$ (*$\angle$'s subtended at the circumference by the same arc AC are equal*)

$\angle CBA = \angle MAB$ (*alternate $\angle$'s within parallel lines are equal*)

$\angle MAB = \angle ADB$ (*$\angle$ between tangent and chord BA drawn to point of contact is equal to $\angle$ subtended by chord BA in the alternate segment*)

$\therefore \angle CDA = \angle ADB$ and hence AD bisects $\angle BDC$