

2002 Higher School Certificate Trial Examination

Mathematics Extension 1

General Instructions

- Reading time 5minutes
- Working time 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided with this paper
- All necessary working should be shown in every question

Total marks - 84

Attempt Questions 1 – 7

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME:

Student Name/Number:

Qı	nestion 1	Begin a new page	Mark
a.	i. Sketch the graph of	$f y = x^3 - 4x$	1
	ii. Hence or otherwis	se, solve $x^3 - 4x > 0$	2
Ъ.	Find the coordinates (-1, -2) externally in	of the point which divides the interval joining (2, 4) and a the ratio 2:1.	3
c.	Find the exact value of	of cos 15°	2
đ.	i. Sketch the graph o	of $y = \tan x$ for $0 \le x \le \pi$	1
	ii. Hence or otherwis	se, find values of $x (0 \le x \le \pi)$ such that the series	3
	$1 + \sqrt{3} \tan x$	+ $3\tan^2 x + 3\sqrt{3}\tan^3 x +$ has a limiting sum	
Qu	estion 2	Begin a new page	
a.	Use the identity sin 2:	$x = 2\sin x \cos x \text{ to find } \int_0^{\frac{\pi}{6}} \sin^2 x \cos^2 x dx$	3
b.	Evaluate $\int_0^1 \frac{4x}{(4x + x)^2}$	$\frac{dx}{1)^2}$, using the substitution $u = 4x + 1$	3
C.		ints on the circumference of a circle. AB produced ed at a point P. AB = 13 cm, BP = 3 cm and CD = 8 cm	-
	i. Draw a clear sketch	h showing the above information	1
	ii. Find the length of	DP	2
d.	Given $P(x) = x^3 - a$	$x^2 + 4$,	
	i. find a if $x + 1$ is a fa	actor of $P(x)$	1
	ii. Hence write $P(x)$ in	n terms of its linear factors	2

4.

3

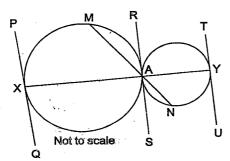
5

1

3

- a. Two circles touch at the point A. Lines through A meet the circles at X and Y and at M and N respectively, as shown RS, the tangent at A is shown.
 - Copy the diagram into your workbook.

Prove that the tangents at X and Y are parallel.



b. In how many ways can a jury of 7 people reach a majority decision?

(A majority decision is one to which the majority agree.)

c. Use the Principle of Mathematical Induction to prove that

 $5^n > 3^n + 2^n$ for integers n > 1

Question 4

Begin a new page

a. i. Show that $x^3 - 3x + 1 = 0$ has a root between x = 1 and x = 2

ii. Using x = 1.5 as a first approximation, obtain a better approximation of the root using Newton's Method once. [Answer to 2 decimal places]

- b. Use $(1 + x)^6 = (1 + x)(1 + x)^5$ to show that $\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$
- c. $P(2ap, ap^2)$ is any point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus, S, of the parabola.
 - i. Find the equation of the line k

ii. The line k intersects the x-axis at the point Q. Find the coordinates of the midpoint, M, of the interval QS

iii What is the equation of the locus of M?

/1,

2

2

Student Name/Number:

Question 5

Begin a new page

Marks

3

a. The acceleration, $a \cdot ms^2$, of a particle moving in a straight line is given by the equation, $a = \frac{x^3}{8} + \frac{x}{8}$, where x is the displacement in metres of the particle from the origin. v is the velocity of the particle at any time, t.

i. If
$$v = \frac{1}{4}$$
 when $x = 0$, show that $v^2 = \frac{(1 + x^2)^2}{16}$

ii. If x = 1 when t = 0, find an expression for the displacement of the particle in terms of t.

b. A population of marsupials has an initial population of 500. Factors which influence the population include birthrates, the number of marsupials killed by feral animals, the amount of feed and so on.

The change in population, N, is given by the formula

$$N = \frac{500}{1 + ke^{-1.5t}}, \text{ where } k \text{ is a constant and } t \text{ is in months}$$

i. Explain why the population will eventually die out

1

3

ii. If at t = 0, the change in population is 1, use the formula to find how long it will take for only 100 marsupials to remain. Give your answer to the nearest month.

iii. Show that
$$\frac{dN}{dt} = \frac{3N}{1000} (500 - N)$$

Student Name/Number:	

Q	uestion 6 Begin a new page	Marks
a.	In how many different ways can 3 black and 3 white tiles be placed in the following grid:	2
b .	The velocity, $v \text{ ms}^{-1}$, of a particle moving along the x-axis in simple harmonic motion is given by $v^2 = 21 - 4x - x^2$, where x is the position of the particle.	
	i. Between which two points on the x-axis does the particle oscillate?	1
	ii. Find an expression for the acceleration, $a \text{ ms}^{-2}$, in terms of x	2
	iii. What is the maximum velocity of the particle?	1
C.	An object is projected at an initial velocity, $V \text{ms}^{-1}$, from ground level at an angle of θ to the horizontal. Use $g=10 \text{ms}^{-2}$	
	i Show that the horizontal and vertical components of the position of the particle are given by $x = Vt\cos\theta$ and $y = -5t^2 + Vt\sin\theta$	2
	ii. Derive an expression for the Cartesian equation for the motion (i.e. find y in terms of x)	2
	iii. A particle is projected from ground level with an initial velocity of 80 ms ⁻¹ . It just clears a 2 metre high wall 25 metres from the point of projection. The base of the wall is at the same level as the point of projection.	2
	Calculate the angle(s) of projection to the nearest minute.	

Student Name/Number:

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Questi	on 7		1	Begin a ı	new pag	8				W.	Marks
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2002 HSC INDEPENDENT TRIAL EXAMINATIONS: MATHEMATICS EXTENSION 1 SAMPLE SOLUTIONS AND SUGGESTED MARKING SCHEME

Ouestion 1

a. Outcomes Assessed:

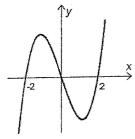
i. PE3

ii. PE3

Marking	Guidelines

		Criteria	Marks
i.	*	correct graph with intercepts clearly marked. finds x > 2	(1) 1
H.		finds $-2 < x < 0$	1





ii. -2 < x < 0 and x > 2

b. Outcomes assessed:

	Criteria		Mařks
*	states correct formula	1.00	1
	recognises division is external by using -2:1 or equivalent		1
*	correct answer	<u> </u>	1

$$\left(\frac{kx_2 + kx_1}{k + l}, \frac{ky_2 + ky_1}{k + l}\right) = \left(\frac{-2 \times -1 + 1 \times 2}{-2 + 1}, \frac{-2 \times -2 + 1 \times 4}{-2 + 1}\right) = (-4, -8)$$

c. Outcomes assessed:

H5

Criteria	21 17 1	34 L 34	Marks
 knows and uses expansion for cos(A - B) knows exact values and obtains correct answer 	r or equivalent		4 4

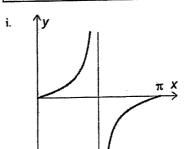
$$\cos 10^{\circ} = \cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

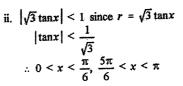
d. Outcomes assessed:

i. H5

i. HE1. H5

		Criteria	Marks
i.		sketches correct graph with appropriate scales	1
ii.	*	sets up correct inequality finds first quadrant solution	1
		finds second quadrant solution	11





Question 2

a. Outcomes assessed:

HE6

HE6

1	Criteria	1 8	å, jara - 6 i a	Marks
	• uses the identity for sin ² 0 to change the integral			1
1	obtains correct integral			
١	substitutes and obtains correct answer	. 18 16	<u> </u>	

$$\int_0^{\frac{\pi}{6}} \sin^2 x \cos^2 x dx = \int_0^{\frac{\pi}{6}} \frac{1}{4} \sin^2 2x dx = \frac{1}{4} \int_0^{\frac{\pi}{6}} \frac{1}{2} (1 - \cos 4x) dx$$
$$= \frac{1}{8} \left[x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{6}} = \frac{1}{8} \left[\frac{\pi}{6} - \frac{\sin \frac{4\pi}{6}}{4} - 0 \right] = \frac{1}{8} \left[\frac{\pi}{6} - \frac{\sqrt{3}}{8} \right]$$

b. Outcomes assessed:

Ť.	Criteria	Marks
•	evaluates limits and expression for dx	1
5 .	correctly substitutes to obtain integral in u	1
•	evaluates integral	4

Question 2 (continued)

b.
$$u = 4x + 1;$$
 $4x = u - 1$

$$\frac{du}{dx} = 4;$$
 $x = 0 \Rightarrow u = 1$

$$= \frac{1}{4} \int_{1}^{5} \frac{1}{u} - \frac{1}{u^{2}} dt$$

$$\frac{du}{dx} = dx;$$
 $x = 1 \Rightarrow u = 5$

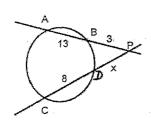
$$= \frac{1}{4} \left[\log u + \frac{1}{u} \right]_{1}^{5}$$

$$= \frac{1}{4} \left[\log 5 - \frac{4}{5} \right]$$

c. Outcomes assessed: i. PE6

Criteria	Marks
 i. draws correct diagram ii. knows AP.BP = CP.DP and correctly uses it evaluates x and chooses appropriate solution 	1 1 1

ii. PE3



ü.				AP.PB	=	CP.DP
				16 × 3	=	$(x + 8) \times x$
	:.	x^2	+	8x - 48	=	0
				$\rightarrow x$	Ė	-12, 4
				so x	=	4

d. Outcomes assessed:

ii. PE3

		Criteria	Marks
i.	*	finds correct value for a	1
ii.	•	uses a correct method (implicit or explicit)	1
		obtains factorisation	

i.
$$P(-1) = (-1)^3 - a \times (-1)^2 + 4 = 0$$

 $a = 3$
ii. $P(-1) = (-1)^3 - a \times (-1)^2 + 4 = 0$

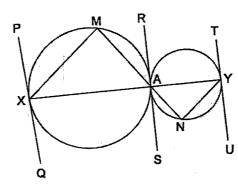
i. PE3

ii.
$$P(x) = (x + 1)(x - 2)(x - 2)$$

Question 3

a. Outcomes assessed:

Criteria	Marks
 constructs XM and YN shows that ∠UYN = ∠PXM using appropriate reasoning shows that ∠AYN = ∠MXA using appropriate reasoning concludes PQ is parallel to TU with reason 	1 1 1



PE3

∠UYN = ∠YAN (alt. segment thm) ∠YAN = ∠MAX (vert. opp) ∠MAX = ∠PXM (alt. segment thm) ∴ ∠UYN = ∠PXM

Also, ∠AYN = ∠SAN (alt. segment thm)
∠SAN = ∠RAM (vert. opp)
∠RAM = ∠MXA (alt. segment thm)
∴ ∠AYN = ∠MXA

∴ ∠UYN + ∠AYN = ∠PXM + ∠MXA i.e ∠UYA = ∠PXA and TU || PQ (alternate angles are equal)

b. Outcomes assessed: PE3

	Criteria			Marks
•	interprets what majority means writes down correct combinations	100	ĝ£ = ° .	1 1
3	calculates the answer	\$6 · \$ * 4 .		. 1

Majority decision means all 7 vote 'yes' or 6 vote 'yes' or 5 vote 'yes' or 4 vote 'yes':

No of ways =
$$\begin{pmatrix} 7 \\ 7 \end{pmatrix} + \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ 5 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix} = 6$$

c. Outcomes assessed: HE2

	Criteria	Marks
	shows true for $n = 2$	1
	writes assumption for S _k	1
	uses S _k in proof for S _{k+1}	1
•	proves S _{k+1}	1
٠	writes concluding statement	1

Question 3 c. (continued)

Let S_n be the statement $5^n > 2^n + 3^n$ for integers n > 1

$$S_2$$
: $5^2 > 2^2 + 3^2$ is true

Assume S, i.e. assume that $5^k > 2^k + 3^k$ for integers k

 $5^{k+1} = 5 \times 5^k > 5 \times (2^k + 3^k)$ since S_k is true

 $5^{k+1} > [2^{k+1} + 3^{k+1}] + 3 \times 2^k + 2 \times 3^k$

Therefore, if S_{k} is true, then S_{k+1} is true.

But S_2 is true so S_3 is true and so on for all integer values of n > 1

Question 4

a. Outcomes assessed:

i. PE3 ii. HE3

		Criteria	Marks
i.	•	substitutes $x = 1$ and $x = 2$ and draws appropriate conclusion	100
ii.	٠.	states Newton's Method	1
	٠	substitutes into the formula	1
	*	evaluates x ₁ to 2 decimal places	1

i. If
$$P(x) = x^3 - 3x + 1$$

 $P(1) = 1 - 3 + 1 = -1$

ii.
$$x_1 = x - \frac{P(x)}{P'(x)}$$

= $x - \frac{x^3 - 3x + 4}{2}$

Therefore, a root lies between x = 1 and x = 2

= 1.5 -
$$\frac{1.5^3 - 3 \times 1.5 + 1}{3 \times 1.5^2 - 3}$$
 = 1.5333333333

 $x_1 = 1.53$

b. Outcomes assessed:

HE3

Criteria	 3	Marks
expands $(1+x)^6$ and $(1+x)^5$	·	· 1
finds coefficient of x^3		1
draws conclusion		1

$$(1+x)^6 = 1 + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6$$

$$(1+x)(1+x)^5 = (1+x)(1+\binom{5}{1}x+\binom{5}{2}x^2+\binom{5}{3}x^3+\binom{5}{4}x^4+\binom{5}{5}x^5)$$

From the expansion of $(1 + x)^6 = (1 + x)(1 + x)^5$, the x^3 terms are as follows:

Question 4 b (continued)

LHS:
$$\binom{6}{3}x^3$$
 RHS: $1 \times \binom{5}{3}x^3 + x \times \binom{5}{2}x^2$ $\therefore \binom{6}{3} = \binom{5}{3} + \binom{5}{2}$

c. Outcomes assessed:

ii. H5

		Criteria	Marks
 		knows or finds the gradient of the tangent at P	1
"	•	finds the equation of k	1
l ii.	•	finds the coordinates of Q, the x-intercept	1
	•	finds the coordinates of M, the midpoint	1
iii.	٠	states the equation of the locus	<u> </u>

i. The gradient of the tangent at P is p and S(0, a) so $y - a = p(x - 0) \Rightarrow y = px + a$

ii. At Q,
$$y = 0$$
 so $x = -\frac{p}{a}$; therefore, $M\left(-\frac{p}{2a}, \frac{a}{2}\right)$ iii. The locus is $y = \frac{a}{2}$

Ouestion 5

a. Outcomes assessed: i. HE5

	į	Criteria		Marks
i.	•	knows $a = \frac{d}{dx} \left[\frac{1}{2} v^2 \right]$		1
		integrates a and includes c evaluates c and finds the appropriate expression for v^2		1 1
ii.	r •	uses the inverse of $\frac{dv}{dt}$ to obtain an appropriate integral	r.	1
	•	obtains the integral and evaluates the constant rearranges the expression to obtain x in terms of t		1 1

i.
$$\frac{d}{dx} \left[y_2 v^2 \right] = \frac{1}{8} \left(x^3 + x \right)$$

$$\frac{1}{2} v^2 = \frac{1}{8} \left(\frac{x^4}{4} + \frac{x^2}{2} \right) + c$$
when $v = \frac{1}{4}$, $x = 0 \rightarrow c = \frac{1}{32}$

$$\therefore \frac{1}{2} v^2 = \frac{x^4}{32} + \frac{x^2}{16} + \frac{1}{32}$$

$$v^2 = \frac{1}{16} \left(x^4 + 2x^2 + 1 \right)$$

$$v^2 = \frac{(x^2 + 1)^2}{16}$$

ii, Since
$$v > 0$$
 when $x = 0$

$$v = \frac{1 + x^2}{4} = \frac{dx}{dt}$$

$$\frac{dt}{dx} = \frac{4}{1 + x^2}$$

$$t = 4\tan^{-1}x + c$$
If $t = 0$, $x = 1 \rightarrow c = -\pi$

$$t = 4\tan^{-1}x - \pi$$

$$\tan^{-1}x = \frac{1}{4}(t + \pi)$$

$$x = \tan\left[\frac{1}{4}(t + \pi)\right]$$

b. Outcomes assessed:

i. HE2, HE7 ii. HE3 iii. HE3

1.	*	shows that as $t - \infty$, $N - 500$ so the number of marsupials $- 0$ substitutes $N = 1$ and $t = 0$ to find k puts $N = 400$, rearranges and takes log of both sides				1 1 1	
iii.	*	finds value for t finds correct derivative of N makes appropriate substitutions to obtain result	1	1	5112	1 1 1	

i. as
$$t \to 0$$
, $ke^{-1.5t} \to 0$, $N \to 500$
ii. if $t = 0$, $N = 1 \Rightarrow k = 499$

$$400 = \frac{500}{1 + 499e^{-1.5t}}$$

$$1 + 499e^{-1.5t} = 1.25$$

$$e^{-1.5t} = \frac{.25}{499}$$

$$t = \frac{1}{-1.5} \log_e \left(\frac{.25}{499}\right) \approx 5 \text{ months}$$
iii.
$$\frac{dN}{dt} = \frac{-(-1.5ke^{-1.5t}) \times 500}{\left(1 + ke^{-1.5t}\right)^2}$$

$$= \frac{1.5}{1 + ke^{-1.5t}} \times \frac{500ke^{-1.5t}}{1 + ke^{-1.5t}}$$

$$= \frac{3N}{1000} \times \left(500 - \frac{500}{1 + ke^{-1.5t}}\right)$$

$$= \frac{3N}{1000} (500 - N)$$

Question 6

a. Outcomes assessed:

PE3

Criteria	Marks
 considers different possibilities adjusts for repeated colours 	1

If the top row is all the same, then there are 2 possibilities, all black above all white and vice versa; If the top row contains two of one colour and one of the other, the total number of possibilities

(including having them around the other way) is given by $\frac{{}^{3}P_{3}}{2!} \times \frac{{}^{3}P_{3}}{2!} \times 2 = 18$

Total = 20

b. Outcomes assessed:

i. HE3

ii. HE3

iii. HE3

	l ; ż '	Criteria		Marks
i.	•	puts $y = 0$ and solves $y = y + y + y + y + y + y + y + y + y + $		1
ii.	÷	uses $a = \frac{d}{dx} \left[\frac{1}{2} v^2 \right]$ to differentiate	ri Turk di kananan di kanan	1
iii.	•	finds the correct expression for α puts $x = -2$ (centre of motion) and evaluates ν		1

i.
$$v = 0 \Rightarrow 21 - 4x - x^2 = 0 \Rightarrow x = -7$$
, 3 ii. $\frac{1}{2}v^2 = \frac{1}{2}(21 - 4x - x^2) \Rightarrow \frac{d}{dx}(\frac{1}{2}v^2) = -2 - x$
iii. $x = -2 \Rightarrow v^2 = 21 - 4 \times -2 - (-2)^2 = 25 \Rightarrow v = \pm 5 \therefore v = 5$

Question 6 (continued)

c. Outcomes assessed

i. HE3 ii. HE3 iii. HE3

i -		Criteria	Marks
i.	•	sets up acceleration equations, integrates and finds constants of	1
		integration integrates velocity and finds constants of integration	1
ii.	•	finds t in terms of x and substitutes into y	1
	٠	simplifies to resultant formula substitutes $x = 25$, $y = 2$ and $V = 80$ and simplifies	1
iii.	•	solves for both values	11

i. Bookwork

ii. Bookwork to
$$y = x \tan \theta - \frac{5x^2}{V^2} (1 + \tan^2 \theta)$$

iii.
$$2 = 25 \tan \theta - \frac{5 \times 25^2}{80^2} (1 + \tan^2 \theta)$$

$$125 \tan^2 \theta - 1600 \tan \theta + 253 = 0$$

$$3200 = 40000 \tan \theta - 3125(1 + \tan^2 \theta)$$

$$128 = 1600 \tan \theta - 125 - 125 \tan^2 \theta$$

$$\tan\theta = \frac{1000 \pm \sqrt{243073}}{350}$$

$$\theta = 85^{\circ}30', 4^{\circ}33'$$

Question 7

a. Outcomes assessed:

HE4

	Criteria	1	Marks
•	finds primitive correctly substitutes limits and evaluates		1

$$\int_0^{\frac{1}{4}} \frac{4dx}{\sqrt{1-4x^2}} = 4\left[\frac{1}{2}\sin^{-1}2x\right]_0^{\frac{1}{4}} = 2\sin^{-1}\frac{2}{4} - 2\sin^{-1}0 = \frac{\pi}{3}$$

b. Outcomes assessed

i. HE3

: 41.	Criteria 1974		Marks
i. •	uses binomial probability formula with appropriate values and then	. (8:11)	1
ii. 🖸	evaluates it understands the possibilities are for 0, 1 or 2 faults and adds results		. 1
	from formula for each obtains correct answer		1

ii. HE3

$$i.\binom{20}{1} \times 0.08^1 \times 0.92^{19} = 0.328$$

ii.
$$\binom{20}{0} \times .08^0 \times .92^{20} + \binom{20}{1} \times .08^1 \times .92^{19} + \binom{20}{2} \times .08^2 \times .92^{18} = 0.788$$

Question 7 (continued)

c. Outcomes assessed:

i. HE

ii. H

iel iii. Hel, He7

	Criteria	Marks
i.	solves equations to show result	1
ü.	 uses expansions for cos (A - B) and cos (A + B) adds these and obtains result as required 	1
iii.	 reduces two pairs of terms of the equation to two terms 	1
	 reduces this pair of terms to a single term breaks equation into simple equations involving cosines and solves 	1 1
	$\cos \frac{1}{2}\theta = 0$ to obtain $\theta = \pi$	
	finds remaining values	1. 1.

i. Adding gives
$$x + y = 2a$$
 and hence $a = \frac{x + y}{2}$, subtracting gives $x - y = 2b$ and $b = \frac{x - y}{2}$

ii.
$$\cos x + \cos y$$

= $\cos(a + b) + \cos(a - b)$
= $\cos a \cos b - \sin a \sin b + \cos a \cos b + \sin a \sin b$
= $2\cos a \cos b$
= $2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$

iii.
$$\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos \theta$$

$$= 2\cos \frac{7\theta}{2}\cos \frac{\theta}{2} + 2\cos \frac{3\theta}{2}\cos \frac{\theta}{2}$$

$$= 2\cos \frac{\theta}{2}\left(\cos \frac{7\theta}{2} + \cos \frac{3\theta}{2}\right)$$

$$= 4\cos \frac{\theta}{2}\cos 5\frac{\theta}{2}\cos \theta = 0$$

$$\therefore \cos \frac{\theta}{2} = 0 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2} \Rightarrow \theta = \pi$$

and
$$\cos \frac{3\pi}{2} = 0 \Rightarrow \frac{3\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{2}$$

hence $\theta = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}$

and $\cos \frac{5\theta}{2} = 0 \Rightarrow \frac{5\theta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$ and $\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

2002 HSC Independent Trial Examinations: Mathematics Extension 1 Mapping Grid

Question	Syllabus Topic	Outcomes
la.i.	Polynomials (16.1)	PE3
la.ii.	Basic Arithmetic and Algebra (1.4)	PE3
1b.	Linear Functions and Lines (6.7)	PE3
1c.	Trigonometric Ratios (5.7)	H5!
1d.i.	Trigonometric Functions (13.2)	H5
Id.ii.	Series and Applications (7.3), Trigonometric Ratios (5.9)	HE1, H5
2a.	Trigonometric Functions (13.6)	HE6
2b.	Integration (11.5)	HE6
2c.i.	Plane Geometry (2.1)	PE6
2c.ii.	Plane Geometry (2.9, 2.10)	PE3
2d.į. & ii.	Polynomials (16.2)	PE3
3a.	Plane Geometry (2.9, 2.10)	PE3
3b.	Permutations and Combinations (18.1)	PE3
3c.	Series and Applications (7.4)	HE2
4a.i. & ii.	Polynomials (16.4)	i. PE3; ii. HE3
4b.	Binomial Theorem (17.3)	нез
4c.	Quadratic Polynomial and Quadratic (9.6)	i. PE4; ii. H5; iii. PE4
.5a.	Applications of Calculus to the Physical World (14.3)	i. HE5; ii. HE4, HE5
5b.	Applications of Calculus to the Physical World (14.2)	i. HE2, HE7; ii. HE3; iii. HE3
ба.	Permutations and Combinations (18.1)	PE3
6b.	Applications of Calculus to the Physical World (14.4)	i. HE3; ii. HE3; iii. HE3
6с.	Applications of Calculus to the Physical World (14.3)	i. HE3; ii. HE3; iii. HE3
7a.	Inverse Trigonometric Functions (15.5)	HE4
7b.	Further Probability (18.2)	i. HE3; ii. HE3
7c.i & ii.	Trigonometric Ratios (5.7)	i. HE1; ii. HE1
7c.iii.	Trigonometric Ratios (5.9)	iii. HE1, HE7