NSW INDEPENDENT SCHOOLS

2016
Higher School Certificate
Trial Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A reference sheet is provided
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 2 – 5
10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Section II - Pages 6 – 9
60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

Student name / number ______________

Marks

Section I

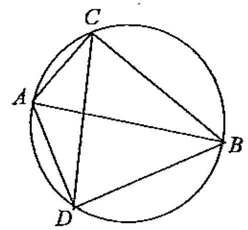
10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

1 What is the value of $\lim_{x\to 0}\frac{\sin\left(\frac{1}{2}x\right)}{2x}$? 1

(A) 0
(B) $\frac{1}{4}$
(C) 1
(D) 4

2 In the diagram below, AB is a diameter of the circle. C is a point on the circle such that $AC=\frac{1}{2}AB$. D is a point on the circle. What is the size of $\angle ADC$? 1



NOT TO SCALE

(A) 15°
(B) 30°
(C) 45°
(D) 60°

3 The equation $x^3+bx^2+cx+d=0$ has roots α, β and γ. 1

What is the value of $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$?

(A) $-b$
(B) $-\frac{b}{d}$
(C) $\frac{b}{d}$
(D) b

Marks

4 Which of the following is equal to $\log_{\frac{1}{a}} x$? **1**

(A) $-\log_a x$

(B) $\dfrac{-1}{\log_a x}$

(C) $\dfrac{1}{\log_a x}$

(D) $\log_a x$

5 Four different coloured, fair dice are rolled together. In how many ways can exactly two 'sixes' occur? **1**

(A) 25

(B) 100

(C) 150

(D) 250

6 Which of the following is a simplification of $\cot 2x + \tan x$? **1**

(A) $\sec 2x$

(B) $\sec x$

(C) $\operatorname{cosec} x$

(D) $\operatorname{cosec} 2x$

7 What is the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^6$? **1**

(A) −20

(B) −15

(C) 15

(D) 20

Student name / number ______________

Marks

8 Which of the following is an expression for $\dfrac{d}{dx}\sin^{-1}(2x-1)$? **1**

(A) $\dfrac{-1}{\sqrt{x(x-1)}}$

(B) $\dfrac{-1}{2\sqrt{x(x-1)}}$

(C) $\dfrac{1}{2\sqrt{x(1-x)}}$

(D) $\dfrac{1}{\sqrt{x(1-x)}}$

9 The side x cm of a cube is decreasing in such a way that the volume V cm^3 is decreasing at a constant rate of 6 cm^3 per minute. What is the rate at which the side of the cube is decreasing when the side is 4 cm ? **1**

(A) $\frac{1}{8}$ cm / min

(B) $\frac{1}{6}$ cm / min

(C) $\frac{1}{4}$ cm / min

(D) $\frac{1}{2}$ cm / min

10 A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line where x is given by $x = 4\sin^2 t - 1$. Where is the centre of motion? **1**

(A) $x = -1$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

Student name / number ____________________

Marks

Section II

60 Marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section.

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a separate writing booklet.**

(a) Solve the inequality $\frac{x-1}{x+2}>0$. 2

(b) $A(-3,1)$ and $B(1,-2)$ are two points. Find the coordinates of the point P that divides the interval AB externally in the ratio 3:1. 2

(c) Find $\int \frac{1+2x}{1+x^2}\,dx$. 2

(d)(i) Find the tangent of the acute angle between the lines $y=x$ and $y=2x$. 1

(ii) Hence show that the line $y=2x$ bisects the acute angle between the lines $y=x$ and $y=7x$. 2

(e) Use Mathematical Induction to show that for all positive integers n 3

$1\times4\ +\ 2\times5\ +\ 3\times6\ +\ldots+n(n+3)\ =\ \tfrac{1}{3}n(n+1)(n+5)$.

(f) Use the substitution $x=u-2$ to evaluate $\int_{-1}^{2}\frac{3x+5}{\sqrt{x+2}}\,dx$. 3

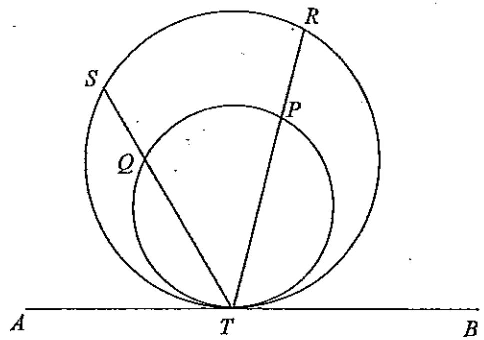
Student name / number ____________________

Marks

Question 12 (15 marks) **Use a separate writing booklet.**

(a) The three numbers $a,\ b,\ c$ are consecutive terms in an arithmetic progression. Show that the three numbers $e^a,\ e^b,\ e^c$ are consecutive terms in a geometric progression. 2

(b) In the diagram the two circles touch internally at T. ATB is the common tangent to the two circles at T. P and Q are points on the smaller circle and R and S are points on the larger circle such that TPR and TQS are straight lines. 3
Copy the diagram and show that $PQ \parallel RS$.



(c) The polynomials $P(x)$ and $Q(x)$ are such that $P(x)=x(x-1)Q(x)+ax+b$ for some constants a and b. $(x-1)$ is a factor of $P(x)$ and when $P(x)$ is divided by x the remainder is 2. Find the remainder when $P(x)$ is divided by $x(x-1)$. 3

(d) The region bounded by the curve $y=\cos^{-1}x$ and the y axis between $y=\frac{\pi}{12}$ and $y=\frac{\pi}{4}$ is rotated through one complete revolution about the y axis. Find the exact volume of the solid formed. 3

(e) Consider the function $f(x)=\sin^{-1}(1-x)+\frac{\pi}{2}$.

(i) Find the domain and range of the function. 2

(ii) Sketch the graph of the function showing clearly the shape of the curve and the coordinates of the endpoints. 2

Student name / number ____________________

Marks

Question 13 (15 marks) **Use a separate writing booklet.**

(a) $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0, a)$. The tangent to the parabola at T has equation $tx - y - at^2 = 0$. Find in simplest form in terms of t:

(i) The perpendicular distance d from F to the tangent at T. **1**

(ii) The ratio $\dfrac{d}{FT}$. **2**

(b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line and velocity $v\ \text{ms}^{-1}$ given by $v = (1-x)^2$. Initially the particle is at O. Find an expression for x as a function of t. **3**

(c) In each game of chess that Bobby plays against Boris there is a probability of $\frac{1}{3}$ that Bobby wins the game, a probability of $\frac{1}{6}$ that Boris wins and a probability of $\frac{1}{2}$ that the game is drawn. They play 4 games of chess against each other.

(i) Find the probability that Bobby wins 2 games and Boris wins 2 games. **2**

(ii) Find the probability that Bobby wins 1 game, Boris wins 1 game and the other 2 games are drawn. **2**

(d) Consider the equation $x^3 + 2x - 7 = 0$.

(i) Show that the equation has a root α such that $1 < \alpha < 2$. **2**

(ii) Show that α is the only real root of the equation. **1**

(iii) Use one application of Newton's method with an initial approximation $\alpha_0 = 1\cdot5$ to find the next approximation for α correct to 1 decimal place. **2**

Student name / number ____________________

Marks

Question 14 (15 marks) **Use a separate writing booklet.**

(a) At time t years the number N of individuals in a population is such that $\dfrac{dN}{dt} = -0\cdot1(N - P)$ for some constant P.

(i) Show that $N = P + Ae^{-0\cdot1t}$, where A is constant, satisfies the given differential equation. **1**

(ii) If the initial population size is 500 and the limiting population size is 100, find the values of P and A. **2**

(b) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity $v\ \text{ms}^{-1}$ and acceleration in ms^{-2} given by $\ddot{x} = -4(x-1)$. When the particle is at the centre of its motion it has speed $6\ \text{ms}^{-1}$.

(i) Show that $v^2 = -4x^2 + 8x + 32$. **2**

(ii) Find the period and amplitude of the motion. **2**

(c) In the expansion of $(1+ax)^n$ in ascending powers of x, the first three terms are $1 + 6x + 16x^2 + \dots$.

(i) Write down two equations in a and n. **2**

(ii) Hence find the values of a and n. **2**

(d) A particle is projected from a point O on the top of a vertical cliff of height h metres above horizontal ground with speed of projection $V = 20\sqrt{2}\ \text{ms}^{-1}$ at an angle $\alpha = 45°$ above the horizontal. It moves in a vertical plane under gravity where the acceleration due to gravity is $10\ \text{ms}^{-2}$. At time t seconds its horizontal and vertical displacements from O, x metres and y metres respectively, are given by

$x = Vt\cos\alpha$ and $y = Vt\sin\alpha - 5t^2$. (DO NOT PROVE THESE RESULTS)

The particle hits the ground with speed $52\ \text{ms}^{-1}$.

(i) Find the time at which the particle hits the ground. **3**

(ii) Hence find the height of the cliff. **1**

Section I **Questions 1-10 (1 mark each)**

Question	Answer	Solution	Outcomes
1	B	$\lim_{x\to 0}\frac{\sin\left(\frac{1}{2}x\right)}{2x}=\frac{1}{4}\cdot\lim_{x\to 0}\frac{\sin\left(\frac{1}{2}x\right)}{\left(\frac{1}{2}x\right)}=\frac{1}{4}\times 1=\frac{1}{4}$	H5
2	B	$\angle ACB=90^\circ$ (*$\angle$ in a semicircle is a right angle*). Then $\angle ABC=30^\circ$ since $AC=\frac{1}{2}AB \Rightarrow \sin\angle ABC=\frac{1}{2}$. But $\angle ADC$, $\angle ABC$ both stand on same arc AC. $\therefore \angle ADC=\angle ABC=30^\circ$	PE3
3	C	$\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}=\frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}=\frac{-b}{-d}=\frac{b}{d}$	PE3
4	A	If $y=\log_{\frac{1}{a}}x$, $x=\left(\frac{1}{a}\right)^y=a^{-y} \Rightarrow -y=\log_a x$. $\therefore \log_{\frac{1}{a}}x=-\log_a x$	H3
5	C	${}^4C_2$ ways of choosing the two dice showing 'six', then 5 possible numbers on each of the remaining two dice. $\therefore {}^4C_2\times 5\times 5=150$ ways	PE3
6	D	$\cot 2x+\tan x=\frac{\cos 2x}{\sin 2x}+\frac{\sin x}{\cos x}=\frac{\cos 2x+2\sin^2 x}{2\sin x\cos x}=\frac{\cos 2x+1-\cos 2x}{\sin 2x}=\operatorname{cosec} 2x$	H5
7	A	$\left(x-\frac{1}{x}\right)^6$ has general term ${}^6C_r\left(-\frac{1}{x}\right)^r x^{6-r}={}^6C_r(-1)^r x^{6-2r}$, $r=0,1,\ldots,6$. For term independent of x, $6-2r=0$ $\therefore r=3$ and term is ${}^6C_3(-1)^3=-20$	HE3
8	D	$\frac{d}{dx}\sin^{-1}(2x-1)=\frac{2}{\sqrt{1-(2x-1)^2}}=\frac{2}{\sqrt{4x-4x^2}}=\frac{1}{\sqrt{x(1-x)}}$	HE4
9	A	$V=x^3$ $\therefore \frac{dV}{dt}=3x^2\frac{dx}{dt}$. $\therefore$ *for* $x=4$, $-6=3\times 16\frac{dx}{dt}$ $\therefore \frac{dx}{dt}=-\frac{1}{8}$	HE5
10	C	$x=4\sin^2 t-1=2(1-\cos 2t)-1=1-2\cos 2t$ $\therefore$ Centre is at $x=1$	HE3

Section II

Question 11

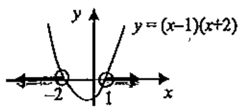
a. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• includes $x>1$ in solution	1
• combines this correctly with the second inequality for x	1

Answer

$\frac{x-1}{x+2}>0 \Leftrightarrow (x-1)(x+2)>0$.



$\therefore x<-2$ *or* $x>1$

Q 11 (cont)

b. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• finds one coordinate of P	1
• finds second coordinate of P	1

Answer

$(-3, 1) \quad (1, -2)$

$3 : -1$

$P\left(\frac{3+3}{3-1}, \frac{-6-1}{3-1}\right)$ $\qquad$ P has coordinates $\left(3, -\frac{7}{2}\right)$

c. Outcomes assessed: H8, HE4

Marking Guidelines

Criteria	Marks
• rearranges integrand and finds primitive of one term	1
• completes primitive	1

Answer

$$\int\frac{1+2x}{1+x^2}\,dx=\int\left(\frac{1}{1+x^2}+\frac{2x}{1+x^2}\right)dx=\tan^{-1}x+\ln\left(1+x^2\right)+c$$

d. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • finds the value of the required tangent ratio	1
ii • finds the tangent of the acute angle between $y=2x$ and $y=7x$	1
• considers the relative positions of the three lines and the equal angles to make the deduction	1

Answer

Let α, β be the acute angles between lines $y=x$ and $y=2x$, $y=2x$ and $y=7x$ respectively.

i. $\tan\alpha=\frac{2-1}{1+2\times 1}=\frac{1}{3}$ $\qquad$ ii. $\tan\beta=\frac{7-2}{1+7\times 2}=\frac{1}{3}$

Hence $\alpha=\beta$ and considering the relative position of the three lines, with $y=2x$ an anticlockwise turn α of $y=x$, then $y=7x$ an anticlockwise turn β of $y=2x$, the line $y=2x$ bisects angle between $y=x$ and $y=7x$.

Q11 (cont)

e. Outcomes assessed: HE2

Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements and verifies that the first is true	1
• writes an expression for the *LHS* of the $(k+1)$st statement conditional on the truth of the kth	1
• rearranges to produce RHS of $(k+1)$st and completes the Mathematical Induction process	1

Answer

Let $S(n)$, $n=1,2,3,\ldots$ be the sequence of statements defined by

$$S(n):\ 1\times4+2\times5+3\times6+\ldots+n(n+3)=\tfrac{1}{3}n(n+1)(n+5)$$

Consider $S(1)$: $LHS=1\times4=4,\quad RHS=\tfrac{1}{3}\times1\times2\times6=4$. Hence $S(1)$ is true.

If $S(k)$ *is true*: $1\times4+2\times5+3\times6+\ldots+k(k+3)=\tfrac{1}{3}k(k+1)(k+5)$ **

Consider $S(k+1)$:

$$\begin{aligned} LHS &= \{1\times4+2\times5+3\times6+\ldots+k(k+3)\}+(k+1)(k+4)\\ &=\tfrac{1}{3}k(k+1)(k+5)+(k+1)(k+4) \qquad \textit{if } S(k) \textit{ is true, using } {**}\\ &=\tfrac{1}{3}(k+1)\{k(k+5)+3(k+4)\}\\ &=\tfrac{1}{3}(k+1)\{k^2+8k+12\}\\ &=\tfrac{1}{3}(k+1)(k+2)(k+6)\\ &=\tfrac{1}{3}(k+1)\{(k+1)+1\}\{(k+1)+5\}\\ &=RHS \end{aligned}$$

Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(1)$ is true, hence by Mathematical Induction, $S(n)$ is true for all integers $n\geq1$.

f. Outcomes assessed: HE6

Marking Guidelines

Criteria	Marks
• converts integral into a definite integral in terms of u	1
• finds the primitive	1
• evaluates	1

Answer

$x=u-2$

$dx=du$

$\begin{cases} x=-1 \Rightarrow u=1 \\ x=2 \Rightarrow u=4 \end{cases}$

$\dfrac{3x+5}{\sqrt{x+2}}=\dfrac{3u-1}{\sqrt{u}}$

$$\begin{aligned} \int_{-1}^{2}\frac{3x+5}{\sqrt{x+2}}\,dx &= \int_1^4\left(3u^{\frac{1}{2}}-u^{-\frac{1}{2}}\right)du\\ &=2\left[u^{\frac{3}{2}}-u^{\frac{1}{2}}\right]_1^4\\ &=2\{(8-1)-(2-1)\}\\ &=12 \end{aligned}$$

Question 12

a. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• use the condition for three terms in AP to relate $a-b$, $b-c$.	1
• apply the common ratio test for three terms in GP	1

Answer

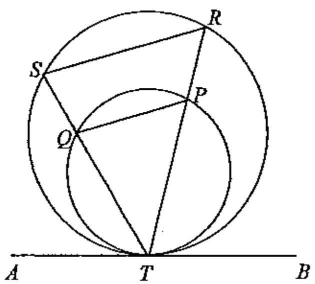
a, b, c in AP. $\therefore a-b=c-b$ Then $\dfrac{e^b}{e^a}=e^{b-a}=e^{c-b}=\dfrac{e^c}{e^b}$. $\therefore e^a, e^b, e^c$ in GP.

b. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• invokes alternate segment theorem in either of the two circles	1
• applies same theorem to second circle to identify equal corresponding angles for PQ, RS	1
• deduces $PQ \parallel RS$, applying test for parallel lines	1

Answer



For circle TPQ,

$\angle BTP=\angle TQP$ (*$\angle$ between tangent and chord drawn to point of contact is equal to $\angle$ subtended by that chord in the alternate segment*)

Similarly, for circle TRS,

$\angle BTR=\angle TSR$

$\therefore \angle TQP=\angle TSR$ (*$\angle BTP$, $\angle BTR$ same as T, P, R collinear*)

$\therefore PQ \parallel RS$ (*equal corresponding $\angle$'s on transversal TS*)

c. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
• uses factor theorem to find one equation for a and b	1
• uses remainder theorem to find b and hence a	1
• deduces the required remainder	1

Answer

$P(x)=x(x-1)Q(x)+ax+b$ $\quad(x-1)$ is a factor of $P(x) \Rightarrow P(1)=0 \quad \therefore a+b=0$

Division of $P(x)$ by x leaves remainder 2 $\Rightarrow P(0)=2 \quad \therefore b=2$ Then $a=-2$.

Using given division transformation, when $P(x)$ is divided by $x(x-1)$ remainder is $ax+b=-2x+2$

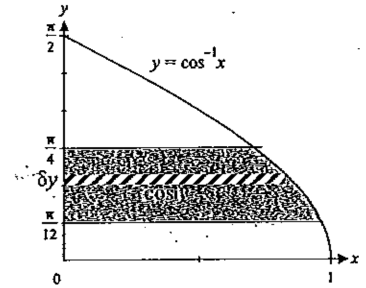
Q 12 (cont)

d. Outcomes assessed: H5, H8

Marking Guidelines

Criteria	Marks
• expresses volume as a definite integral in terms of $\cos 2y$	1
• finds the primitive function	1
• evaluates	1

Answer



Element of volume obtain by rotation of strip around y axis is

$\delta V = \pi \cos^2 y \; \delta y$

$$V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos^2 y \, dy$$

$$= \tfrac{\pi}{2} \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (1+\cos 2y)\, dy$$

$$= \tfrac{\pi}{2}\left[y + \tfrac{1}{2}\sin 2y\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$$

$$= \tfrac{\pi}{2}\left\{\left(\tfrac{\pi}{4} - \tfrac{\pi}{12}\right) + \tfrac{1}{2}\left(\sin\tfrac{\pi}{2} - \sin\tfrac{\pi}{6}\right)\right\}$$

$$= \tfrac{\pi}{2}\left(\tfrac{\pi}{6} + \tfrac{1}{4}\right)$$

Volume is $\frac{1}{24}\pi(2\pi+3)$ cu. units

e. Outcomes assessed: HE4

Marking Guidelines

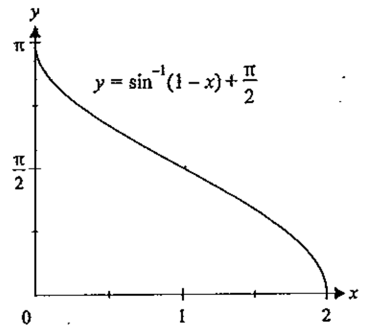
Criteria	Marks
i • states the domain	1
• states the range	1
ii • curve of correct shape in first quadrant	1
• endpoints correct	1

Answer

i. Domain: $-1 \le 1-x \le 1 \;\Rightarrow\; -1 \le x-1 \le 1$ $\quad \therefore 0 \le x \le 2$

Range : $-\frac{\pi}{2} \le \sin^{-1}(1-x) \le \frac{\pi}{2} \Rightarrow 0 \le \sin^{-1}(1-x) + \frac{\pi}{2} \le \pi$ $\quad \therefore 0 \le f(x) \le \pi$

ii.



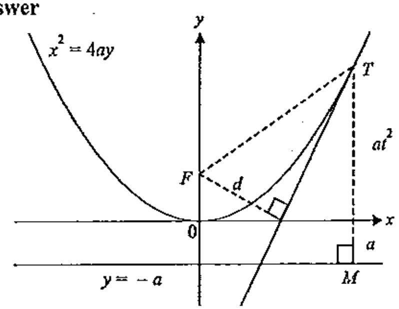
Question 13

a. Outcomes assessed: PE3

Marking Guidelines

Criteria	Marks
i • finds d in terms of t	1
ii • finds FT in terms of t	1
• finds the required ratio in terms of t	1

Answer



i. $F(0, a)$, equation of tangent $tx - y - at^2 = 0$

$$d = \frac{\left|0 - a - at^2\right|}{\sqrt{t^2 + (-1)^2}} = \frac{a(t^2+1)}{\sqrt{t^2+1}} = a\sqrt{t^2+1}$$

ii. $FT = TM = at^2 + a = a(t^2+1)$

$$\frac{d}{FT} = \frac{1}{\sqrt{t^2+1}}$$

b. Outcomes assessed: HE5

Marking Guidelines

Criteria	Marks
• finds $\frac{dt}{dx}$ as function of x	1
• finds t as a function of x, using initial conditions to evaluate the constant of integration	1
• rearranges to find x as a function of t	1

Answer

$v = (1-x)^2$

$\frac{dx}{dt} = (1-x)^2$

$\frac{dt}{dx} = (1-x)^{-2}$

$t = (1-x)^{-1} + c$

$\left.\begin{matrix} t=0 \\ x=0 \end{matrix}\right\} \Rightarrow \begin{matrix} 0 = 1+c \\ \therefore c = -1 \end{matrix}$

$\therefore t = \frac{1}{1-x} - 1$

$1 - x = \frac{1}{t+1}$

$\therefore x = 1 - \frac{1}{t+1}$

$= \frac{t}{t+1}$

c. Outcomes assessed: HE3

Marking Guidelines

Criteria	Marks
i • writes expression using powers of $\frac{1}{3}, \frac{1}{6}$, realising that there is a multiplier for the orders	1
• counts the orders correctly to evaluate the probability	1
ii • writes expression using powers of $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$, realising that there is a multiplier for the orders	1
• counts the orders correctly to evaluate the probability	1

Answer

i. ${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{6}\right)^2 = \frac{1}{54}$ $\qquad$ ii. ${}^4C_2 \times 2 \times \left(\frac{1}{2}\right)^2\left(\frac{1}{3}\right)\left(\frac{1}{6}\right) = \frac{1}{6}$

Q13 (cont)

d. Outcomes assessed: PE3, HE7

Marking Guidelines

Criteria	Marks
i • verifies that the cubic expression changes sign over the given interval	1
• notes the continuity of the function to justify the deduction	1
ii • explains why the graph of this cubic function crosses the x axis once, giving one real root	1
iii • writes a numerical expression for the next approximation	1
• evaluates this approximation to the required accuracy	1

Answer

i. Let $f(x)=x^3+2x-7$. Then $f(x)$ is a continuous function and $f(1)=-4<0$, $f(2)=5>0$.

Hence $f(\alpha)=0$ for some α such that $1<\alpha<2$.

ii. $f'(x)=3x^2+2>0$ for all real x. $\therefore f(x)$ is monotonic increasing, and its graph cannot cross the x axis more than once. Hence α is the only real root of the equation.

iii. $\alpha_1=1\cdot5-\dfrac{f(1\cdot5)}{f'(1\cdot5)}=1\cdot5-\dfrac{1\cdot5^3+2\times1\cdot5-7}{3\times1\cdot5^2+2}\approx1\cdot6$ (*to 1 dec. pl.*)

Question 14

a. Outcomes assessed: HE3

Marking Guidelines

Criteria	Marks
i • verifies by differentiation that the given expression for N satisfies the differential equation	1
ii • uses the initial population size to find $P+A$	1
• uses the limiting population size to find P, then states the value of A	1

Answer

i. $N=P+Ae^{-0\cdot1t}$

$\dfrac{dN}{dt}=-0\cdot1\,Ae^{-0\cdot1t}$

$=-0\cdot1(N-P)$

ii. $t=0,\ N=500 \Rightarrow P+A=500$

$t\to\infty,\ N\to100 \Rightarrow P=100$

$\therefore A=400$

b. Outcomes assessed: HE3

Marking Guidelines

Criteria	Marks
i • finds an expression for v^2 in terms of x by integration	1
• uses given information to evaluate the constant of integration	1
ii • finds n to determine the period	1
• finds the possible values of x, or uses $v_{max}=nA$, to determine the amplitude	1

Answer

i. $\ddot{x}=-4(x-1)$

$\frac{1}{2}\dfrac{dv^2}{dx}=-4(x-1)$

$v^2=-4(x-1)^2+c$

$x=1,\ v=6 \Rightarrow c=36$

$\therefore v^2=-4(x-1)^2+36$

$=-4x^2+8x+32$

ii. $v^2\ge0 \Rightarrow (x-1)^2\le9$

$-3\le x-1\le3$

$\therefore -2\le x\le4$

$n=2 \Rightarrow T=\dfrac{2\pi}{n}=\pi$

Amplitude is 3 m, period is π s.

Q14 (cont)

c. Outcomes assessed: HE3

Marking Guidelines

Criteria	Marks
i • writes an equation using the coefficient of x	1
• writes an equation using the coefficient of x^2	1
ii • writes an equation in a single pronumeral	1
• states both values	1

Answer

i. $(1+ax)^n=1+6x+16x^2+\ldots$

${}^nC_1a=6 \Rightarrow na=6$ (1)

${}^nC_2a^2=16 \Rightarrow n(n-1)a^2=32$ (2)

ii. $(2)\div(1) \Rightarrow (n-1)a=\frac{16}{3}$ (3)

$(3)\div(1) \Rightarrow \dfrac{n-1}{n}=\dfrac{8}{9}\quad \therefore n=9,\ a=\frac{2}{3}$

d. Outcomes assessed: HE3

Marking Guidelines

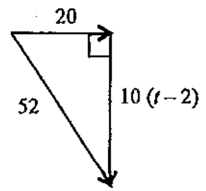
Criteria	Marks
i • finds $\dot{x}$ and $\dot{y}$ in terms of t	1
• uses given magnitude of vector sum at impact to find equation for t when particle hits ground	1
• solves equation to find time to impact	1
ii • uses expression for y and this value of t to find h	1

Answer

i. $x=20t \Rightarrow \dot{x}=20$

$y=20t-5t^2 \Rightarrow \dot{y}=20-10t$

$=-10(t-2)$



The vector sum of $\vec{\dot{x}}$ and $\vec{\dot{y}}$ has magnitude 52

$\therefore 10^2\left\{2^2+(t-2)^2\right\}=52^2$

$(t-2)^2=\dfrac{52^2}{10^2}-2^2$

$=\dfrac{2304}{10^2}$

$t=2+\dfrac{48}{10}$

Hence particle hits the ground after $6\cdot8$ s

ii. Particle hits ground when $y=-h$

$-h=20t-5t^2$

$h=5(t-2)^2-20$

$h=5\times\dfrac{2304}{100}-20$

$=95\cdot2$

Hence height of cliff is $95\cdot2$ m