INDEPENDENT

2002
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5minutes
- Working time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided with this paper
- All necessary working should be shown in every question

Total marks - 120

Attempt Questions 1 – 10

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

STUDENT NAME / NUMBER ....

Marks

Question 1. (12 marks)

(a) Evaluate correct to 3 significant figures : $\sqrt{\dfrac{\pi\times(5.34)^2}{25.74-7.29}}$ **2**

(b) Graph on a number line the values of x for which $|x-2|\geq 1$ **2**

(c) Simplify : $\dfrac{x^2-4}{xy}\times\dfrac{2x}{2x-4}$ **2**

(d) Show that $\dfrac{1}{2-\sqrt{3}}+\dfrac{1}{2+\sqrt{3}}$ is rational. **2**

(e) Solve : $x^2-4x=0$. **2**

(f) Simplify : $\dfrac{2^{n+1}-2^n}{2^{2n+1}-2^{2n}}$ **2**

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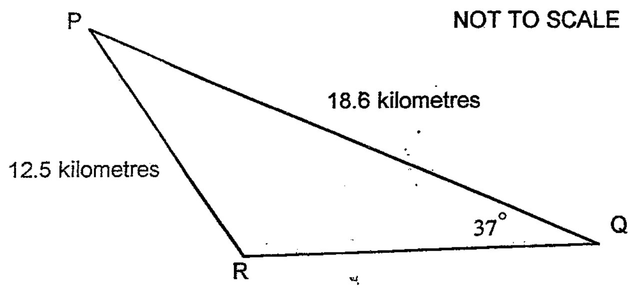
Marks

Question 2 (12 marks) ***Start a new page***

(a) Evaluate : $\int_0^2 e^{5x}\,dx$. **2**

(b) Find : $\int \frac{dx}{2x-3}$ **2**

(c) **3**



In the diagram above, PQ = 18.6 kilometres, PR = 12.5 kilometres and $\angle PQR = 37°$. $\angle PRQ$ is obtuse. Find the size of $\angle PRQ$ correct to the nearest minute.

(d) Differentiate :

(i) $\frac{\tan x}{x}$ **2**

(ii) $e^x \cos x$ **2**

(iii) $\log_e(2x-5)$. **1**

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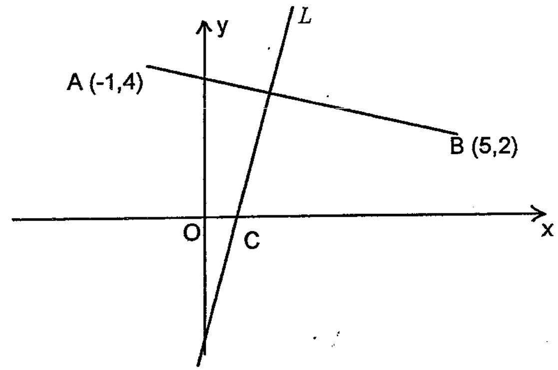
Marks

Question 3 (12 marks) ***Start a new page***

(a) The diagram below shows the points A (-1,4) and B (5,2). The line L has equation $3x - y - 3 = 0$ and cuts the x-axis at C.



(i) Show that the length of AB is $2\sqrt{10}$ units. **1**

(ii) Find the coordinates of M, the midpoint of AB. **1**

(iii) Find the gradient of AB. **1**

(iv) Show that the equation of AB is $x + 3y - 11 = 0$ **1**

(v) Prove that L is the perpendicular bisector of AB. **2**

(vi) Find the coordinates of C. **1**

(vii) Write down the equation of the circle with AB as diameter. **1**

(b) α and β are the roots of the equation $x^2 - 6x + 10 = 0$ Find the values of :

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **2**

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Marks

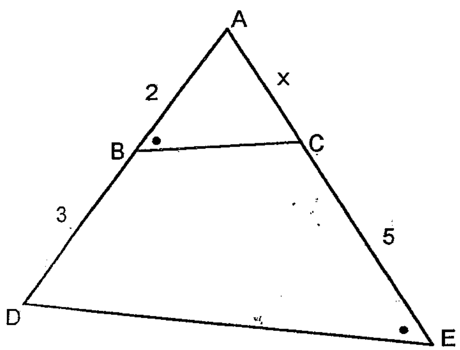
Question 4 (12 marks) ***Start a new page***

(a) Given that $\log_a b = 3.5$ and $\log_a c = 0.35$, find the value of :

(i) $\log_a\left(\dfrac{c}{b}\right)$ **1**

(ii) $\log_a(bc)^2$ **2**

(b)



In the diagram above, $\angle ABC = \angle AED$, $AE = 2$, $BD = 3$, $AC = 5$ and $AC = x$.

Copy the diagram onto your worksheet.

(i) Prove that triangle ABC is similar to triangle AED. **3**

(ii) Hence find the value of x. **2**

(c) In a local Primary School the student population is 46% male and 54% female. Two students are selected at random to perform office duties. Find, correct to two decimal places, the probability that :

(i) Both are female. **1**

(ii) One is female and the other male **2**

(iii) Neither student is female. **1**

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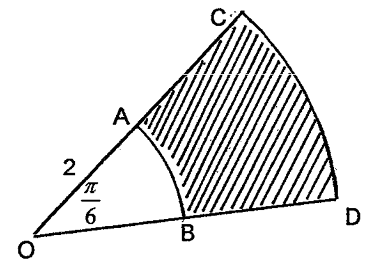
Marks

Question 5 (12 marks) ***Start a new page***

(a) Find the equation of the normal to the curve $y = e^{3x} + 1$ at the point on the curve where $x = 0$. **3**

(b) **3**



AB and CD are arcs of concentric circles with centre O.

$\angle AOB = \dfrac{\pi}{6}$ radians. OA = 2 centimetres. The shaded section has area 5π centimetres^2.

Calculate the length of AC.

(c) Consider the curve given by the equation : $y = 2x(x-3)^2$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of any points of inflexion. **1**

(iii) Sketch the graph of $y = 2x(x-3)^2$ showing the above information. **2**

Marks

Question 6 (12 marks) ***Start a new page***

(a) The population P of a certain bacteria is falling according to the formula :

$$P = 3000e^{-kt}, \text{ where } t \text{ is in days.}$$

(i) What is the initial population of the bacteria? **1**

(ii) Show that $\frac{dP}{dt} = -kP$. **1**

(iii) If it takes 4 days for the number of bacteria to fall to 2 000, what is the value of k? **2**

(iv) How long will it take for the number of bacteria to fall to 10% of the initial number and find the rate of change at this time. **2**

(b) A cylindrical container closed at both ends is made from a sheet of thin plastic. The surface area of the cylinder is 600π centimetres^2.

(i) Show that the height h of the cylinder is given by the expression : **2**

$$h = \frac{300}{r} - r, \text{ where } r \text{ is the radius.}$$

(ii) Find an expression for the volume V in terms of r. **1**

(iii) Find the height of the container if the volume is to be a maximum. **3**

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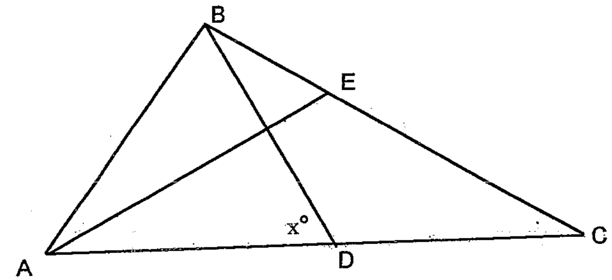
Marks

Question 7 (12 marks) ***Start a new page***

(a)



Triangle ABC is a right angled triangle with $\angle ABC = 90°$. D is a point on AC such that AB = BD = DC. E lies on BC such that AE bisects $\angle BAD$. Let $\angle ADB = x°$.

Copy the diagram onto your worksheet showing this information.

(i) Show that $\angle DBC = (2x - 90)°$. **2**

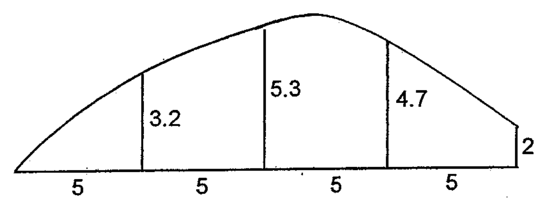
(ii) Hence find the value of x. **1**

(iii) Show that triangle AEC is isosceles. **2**

(b) (i) Show that : $(\tan\alpha + \sin\alpha)(\cot\alpha + \cos\alpha) = (1 + \cos\alpha)(1 + \sin\alpha)$ **2**

(ii) Hence or otherwise solve : $(\tan\alpha + \sin\alpha)(\cot\alpha + \cos\alpha) = 0, 0 \le \alpha \le 2\pi$ **2**

(c)



The diagram above shows the Herb garden in Don's backyard. All measurements are in metres.

(i) Using the Trapezoidal Rule with 4 intervals to find an approximate value for the area of the garden. **2**

(ii) If 21 millimetres of rain fell overnight, given that 1metre^3 = 1 000 litres, how many litres of rain fell on Don's Herb garden? **1**

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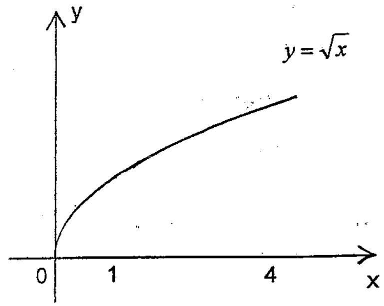
Marks

Question 8 (12 marks) ***Start a new page***

(a) **3**



The graph of $y=\sqrt{x}$ is shown in the diagram above. The arc of the curve between $x=1$ and $x=4$ is rotated about the y-axis.

Calculate the volume thus formed.

(b) A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x=t+\cos t$, where t is the time measured in seconds.

(i) Where is the particle initially? **1**

(ii) When, and where, does the particle first come to rest? **3**

(iii) When does the particle next come to rest? **1**

(iv) What is the acceleration of the particle after $\frac{\pi}{6}$ seconds? **2**

(c) For what values of k does the quadratic equation $2x^2+3x+k=0$ have real roots? **2**

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Marks

Question 9 (12 marks) ***Start a new page***

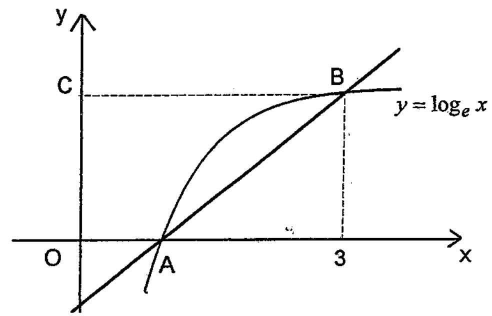
(a) A gardener plants a bed of roses. The bed is planted so that the first row has 24 rose plants. The second row has 29 rose plants. Each succeeding row has 5 more rose plants than the previous row.

(i) Calculate the number of roses in the 8^{th} row. **1**

(ii) Which row would be the first to contain more than 150 rose plants? **2**

(iii) The gardener has planted 2 895 roses. Assuming that the above pattern has been continued, how many rows were planted? **2**

(b)



The diagram shows the graphs of $y=\log_e x$ and a straight line which cuts $y=\log_e x$ at the points A and B (A being on the x-axis). C lies on the y-axis such that BC is parallel to the x-axis.

(i) Find the coordinates of A and B. **2**

(ii) Show that the equation of AB is $y=\frac{\log_e 3}{2}(x-1)$. **2**

(iii) Calculate the area enclosed by the curve $y=\log_e x$, the line BC and the x and y-axes. **2**

(iv) Hence find the area between $y=\log_e x$ and $y=\frac{\log_e 3}{2}(x-1)$. **1**

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Question 10 (12 marks) *Start a new page*

(a) Ricardo has set up his retirement fund and after 10 years he has accumulated $67 000. Due to an accident, he is no longer able to work and makes no further contributions to the fund. He is leaving the money in the retirement fund to accumulate interest at 8% p.a. compounded annually, but needs to withdraw $12 000 at the end of each year for normal living expenses

(i) Show that at the end of the first year he will have $\$(67000 \times 1.08 - 12000)$ in the fund. **1**

(ii) Find a similar expression for the amount in the fund after 3 years **2**

(iii) Hence find how many years the fund will last before there is no money in it. **2**

(b) (i) On the same set of axes, sketch the graphs of the functions **2**

$$y = 2\cos x$$
$$y + 1 = 0 \qquad \text{where } 0 \le x \le 2\pi$$

(ii) Hence find the number of solutions for $\cos x = -\frac{1}{2}$ in the same domain. **1**

(c) A bottle had 500 millilitres of water in it. More water was poured into the bottle for 10 seconds until it was full. During this time the volume flow rate of water, in millilitres per second, was given by the formula

$$\frac{dV}{dt} = 2(10 - t)$$

(i) Find a formula for the volume of water V in the bottle after t seconds where $t \le 10$. **2**

(ii) How many millilitres of water were in the bottle when it was full? **1**

(iii) What was the initial flow rate? **1**

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2002

Maths 2Unit –HSC Trial Exam

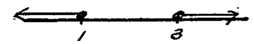
Suggested Answers & Mapping Grid

Q1 (a) $2.2035 \Rightarrow \underline{2.20}$

(b) $x - 2 \ge 1$ $\quad -x + 2 \ge 1$

$\underline{x \ge 3}$ $\quad -x \ge -1$

$\underline{\text{OR } x \le 1}$



(c) $\frac{(x+2)(x-2)}{xy} \times \frac{x}{(x-2)}$

$= \frac{x+2}{y}$

(d) $\frac{1(2+\sqrt{3}) + 1(2-\sqrt{3})}{2^2 - (\sqrt{3})^2}$

$= \underline{4}$

(e) $x(x-4) = 0$

$\underline{x = 0, 4}$

(f) $\frac{2^n(2-1)}{2^{2n}(2-1)}$

$= 2^{n-2n}$

$= \underline{2^{-n}}$

Q2 (a) $\left[\frac{1}{5} e^{5x}\right]_0^2$

$= \frac{1}{5}(e^{10} - e^0)$

$= \underline{\frac{1}{5}(e^{10} - 1)}$

(b) $\underline{\frac{1}{2} \ln(3x - 3) + C}$

(c) $\frac{\sin \hat{R}}{18.6} = \frac{\sin 37}{12.5}$

$\sin \hat{R} = 0.895$

$\hat{R} = 180° - 63°34'$

$= \underline{116°26'}$

(d) (i) $\underline{\frac{x \sec^2 x - \tan x}{x^2}}$

(ii) $e^x(-\sin x) + \cos x \cdot e^x$

$= \underline{e^x(\cos x - \sin x)}$

(iii) $\underline{\frac{2}{2x - 5}}$

Q3 (a)(i) $AB = \sqrt{(5 - -1)^2 + (2-4)^2}$

$= \sqrt{40}$

$= \underline{2\sqrt{10}}$

(ii) $M = \left(\frac{-1+5}{2}, \frac{4+2}{2}\right)$

$= \underline{(2, 3)}$

(iii) $m_{AB} = \frac{2-4}{5 - -1}$

$= \underline{\frac{-1}{3}}$

(iv) $y - 4 = -\frac{1}{3}(x + 1)$

$3y - 12 = -x - 1$

$\underline{x + 3y - 11 = 0}$

(v) $L: y = 3x - 3$

$\therefore m_L = 3$

$m_{AB} \cdot m_L = -\frac{1}{3} \cdot 3$

$= -1$

$\therefore L \perp AB$

Also $3(2) - 3 - 3 = 0$

$\therefore L$ passes through M

$\therefore \underline{L \text{ is } \perp \text{ bisector } AB}$

(vi) At C $y = 0$

$3x - 0 - 3 = 0$

$x = 1$

$\underline{C \text{ is } (1, 0)}$

(vii) $\underline{(x-2)^2 + (y-3)^2 = 10}$

(b)(i) $\alpha + \beta = -\frac{b}{a} = 6$

(ii) $\alpha\beta = c/a = 10$

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

Q4 (a)(i) $\log_a\left(\frac{c}{b}\right) = \log_a c - \log_a b$

$= 0.35 - 3.5$

$= \underline{-3.15}$

(ii) $\log_a(bc)^2 = 2(\log_a b + \log_a c)$

$= 2(3.5 + 0.35)$

$= \underline{7.7}$

(b)(i) In Δ's ABC, AED

$\hat{A}$ is common

$A\hat{B}C = A\hat{E}D$ (data)

$\therefore \Delta ABC \parallel\!| \Delta AED$

(equiangular)

(ii) $\frac{AB}{AE} = \frac{AC}{AD}$ (corr. sides in sim Δ's)

$\therefore \frac{2}{x+5} = \frac{x}{5}$

$x^2 + 5x - 10 = 0$

$x = \frac{-5 \pm \sqrt{25 + 40}}{2}$

As $x > 0$

$\underline{x = \frac{-5 + \sqrt{65}}{2}}$

(c) (i) $P(FF) = .54 \times .54$

$= \underline{0.29}$

(ii) $P(MF) = 0.46 \times 0.54 + 0.54 \times 0.46$

$= \underline{0.50}$

(iii) P(Neither F)

$= 1 - (P(FF) + P(MF))$

$= 1 - (0.29 + 0.50)$

$= \underline{0.21}$

(OR $P(MM)$)

Q5(a) $y' = 3e^{3x}$

$x=0 \quad y = e^0 + 1 = 2$

$m_1 = 3e^0 = 3$

Normal $m_2 = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x - 0)$

$3y - 6 = -x$

$\underline{x + 3y - 6 = 0}$

(b) $5\pi = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot\frac{\pi}{6}(R^2 - 2^2)$

$60 = R^2 - 4$

$R = 8$

$\therefore \underline{AC = 6}$

(c) $y = 2x(x-3)^2$

$y' = 2(x-3)^2 + 2x \cdot 2(x-3)$

$= (x-3)(6x-6)$

$y' = 0 \quad x = 3, 1$

$y' = 6(x^2 - 4x + 1)$

$y'' = 6(2x - 4)$

$y'' = 0 \quad x = 2$

(i) $x = 1 \quad y = 8 \quad y'' < 0$

$\therefore \underline{(1,8)\ \text{Max TP}}$

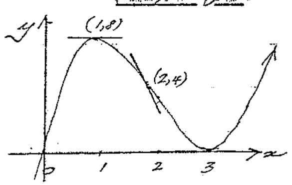
$x = 3 \quad y = 0 \quad y'' > 0$

$\underline{(3,0)\ \text{Min TP}}$

(ii) $x = 2 \quad y = 4$

y'' changes sign

$\therefore \underline{(2,4)\ \text{P of I}}$



Q6.(a)(i) $t = 0 \quad \underline{P = 3000}$

(ii) $\frac{dP}{dt} = 3000 \cdot -k \cdot e^{-kt}$

$= -k(3000e^{-kt})$

$= \underline{-kP}$

(iii) $t = 4 \quad P = 2000$

$2000 = 3000e^{-4k}$

$e^{-4k} = 2/3$

$\therefore 4k = \ln(3/2)$

$\underline{k = 0.10137}$

(iv) $P = 300$

$300 = 3000e^{-kt}$

$e^{-kt} = \frac{1}{10}$

$kt = \ln(10/1)$

$t = \frac{\ln 10}{0.10137}$

$= \underline{22.7\ \text{days}}$

(b)(i) $SA = 2\pi r^2 + 2\pi rh$

$600\pi = 2\pi r^2 + 2\pi rh$

$r^2 + rh = 300$

$h = \frac{300 - r^2}{r}$

$= \underline{\frac{300}{r} - r}$

(ii) $V = \pi r^2 h$

$= \pi r^2\left(\frac{300}{r} - r\right)$

$= \underline{\pi(300r - r^3)}$

(iii) $\frac{dV}{dr} = \pi(300 - 3r^2) = 0$

$r = 10 \quad (r \neq -10)$

$\frac{d^2V}{dr^2} = -6\pi r < 0$

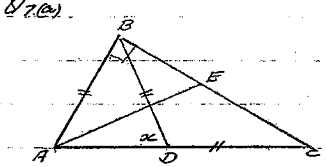
$\therefore$ Max V.

$h = \frac{300}{10} - 10$

$h = \underline{20\ \text{cm}}$

when V is max

Q7(a).



(i).

$x = A\hat{D}B = D\hat{A}B$ (opp = sides ΔABD)

$\therefore A\hat{B}D = 180 - 2x°$ ($\angle$ sum ΔABD)

$\therefore D\hat{B}C = 90 - (180 - 2x)$

$= \underline{(2x - 90)°}$

(ii) $D\hat{B}C = B\hat{C}D = (2x - 90°)$

(opp = sides ΔBDC)

$\therefore x = 2(2x - 90)$ (ext. $\angle$ of Δ)

$\underline{x = 60°}$

$B\hat{A}D = 60°$ (see (i))

$\therefore E\hat{A}D = 30°$ (AE bisector)

$D\hat{B}C = D\hat{C}B = 2x - 90$

$= 30°$

$\therefore \underline{\Delta AEC\ \text{isosceles}}$

(b)(i) $\left(\frac{\sin\alpha}{\cos\alpha} + \sin\alpha\right)\left(\frac{\cos\alpha}{\sin\alpha} + \cos\alpha\right)$

$= 1 + \sin\alpha + \cos\alpha + \cos\alpha\sin\alpha$

$= \underline{(1 + \cos\alpha)(1 + \sin\alpha)}$

(ii) $\cos\alpha = -1$ or $\sin\alpha = -1$

$\underline{\alpha = \pi \text{ or } 3\pi/2}$

(c)(i)

$A = \frac{1}{3}\cdot 5\cdot(0 + 2 + 2(3.2 + 5.3 + 4.7))$

$= \underline{710\ m^2}$

(ii) $V = 710 \times 0.21$

$= 149.1\ m^3$

$= \underline{149\,100\ \text{litres}}$

Q8 (a) $x = 1 \quad y = 1$

$x = 4 \quad y = 2$

$V = \pi\int_1^2 x^2 \cdot dy$

$= \pi\int_1^2 y^4 \cdot dy$

$= \frac{\pi}{5}\left[y^5\right]_1^2$

$= \frac{\pi}{5} \times (32 - 1)$

$= \underline{\frac{31\pi}{5}\ \text{units}^3}$

(b) $x = t + \cos t$

(i) $t = 0 \quad \underline{x = 1}$

(ii) $v = 1 - \sin t = 0$

$\sin t = 1$

$\underline{t = \pi/2\ s}$

$x = \pi/2 + \cos\pi/2$

$\underline{x = \pi/2\ m}$

(iii) $t = \underline{5\pi/2\ s}$

(iv) $\ddot{x} = a = -\cos t$

$t = \frac{\pi}{6} \quad a = -\cos\pi/6$

$= \underline{-\sqrt{3}/2\ ms^{-2}}$

(c) $\Delta = 9 - 8k \geq 0$

$\underline{k \leq 1\frac{1}{8}}$

Q9 (a) $a = 24 \quad d = 5$ AP.

(i) $T_8 = 24 + 7\times5$

$= \underline{59}$

(ii) $24 + (n-1)5 > 150$

$19 + 5n > 150$

$n > 26\frac{1}{5}$

$\underline{n = 27}$

(iii) $2895 = \frac{n}{2}(48 + (n-1)5)$

$5790 = n(5n + 43)$

$5n^2 + 43n - 5790 = 0$

$(n - 30)(5n + 193) = 0$

$\therefore \underline{n = 30}$ as $n \neq \frac{193}{5}$

(Formula easier)

(b)(i) A $y = 0 \quad x = 1 \quad \underline{(1,0)}$

B $x = 3 \quad y = \log_e 3$

$\underline{(3, \log_e 3)}$

(ii) $\frac{y - 0}{x - 1} = \frac{3\ln 3 - 0}{3 - 1}$

$\therefore \underline{y = \frac{3\ln 3}{2}(x - 1)}$

(iii) $y = \log_e x$

$x = e^y$

$\therefore A = \int_0^{\log 3} e^y \cdot dy$

$= e^y\Big]_0^{\log 3}$

$= e^{\log 3} - e^0$

$= 3 - 1$

$= \underline{2\ \text{units}^2}$

(iv) $\Delta ABC = \frac{1}{2}\cdot\log_e 3(1 + 3)$

$= 2\log_e 3$

Required A $= \underline{(2\log_e 3 - 2)}$

Q10(a)

(i) $A_1 = 67000 + 8\%$ of $67000 - 12000$

$= 67000 \times 1.08 - 12000$

(ii) $A_2 = (67000 \times 1.08 - 12000) \times 1.08 - 12000$

$= 67000 \times 1.08^2 - 12000(1.08 + 1)$

$A_3 = 67000 \times (1.08)^3 - \underline{12000(1.08^2 + 1.08 + 1)}$

$A_n = 67000(1.08)^n - 12000(1.08^{n-1} + \ldots + 1.08 + 1)$ GP

$A_n = 0$.

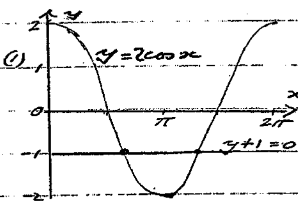
$\therefore 67000(1.08)^n = 12000 \times 1\frac{(1.08^n - 1)}{1.08 - 1}$

$6640 \times 1.08^n = 12000$

$n \log 1.08 = \log\frac{12000}{6640}$

$\underline{n \doteq 7.69\ \text{years}}$

(b)(i)



(ii) $2\cos x = -1$

$\cos x = -\frac{1}{2}$

2 solutions

(c)(i) $V = 20t - t^2 + c$

$t = 0 \quad V = 500 = c$

$\therefore \underline{V = 20t - t^2 + 500}$

(ii) $t = 10$

$V = 20(10) - 10^2 + 500$

$= \underline{600\ ml}$

(iii) $t = 0$

$\frac{dV}{dt} = 2(10)$

$= \underline{20\ ml\,s^{-1}}$