NSW INDEPENDENT TRIAL EXAMS – 2008

2008
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- **Reading time – 5 minutes**
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ……………………..

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$

Total marks – 120 marks
Attempt Questions 1 – 10
All questions are of equal value

Answer the questions on your own paper or writing booklet, if provided. Start each question on a new page.

Question 1 (12 marks) Use a SEPARATE page or writing booklet **Marks**

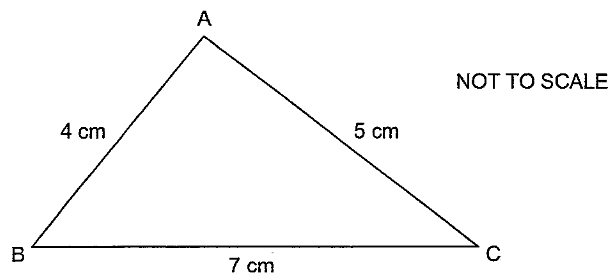
(a) Evaluate: $\sqrt[3]{e^{2.4}-1}$ correct to 5 significant figures. **2**

(b) Given: $\frac{5}{\sqrt{3}-1}=a\sqrt{3}+b$, find the values of a and b. **2**

(c) Solve, giving your answer(s) in exact form: $2x^2-5x-4=0$. **2**

(d) Find a primitive of: $\frac{1}{x^2}+\frac{1}{x}$. **2**

(e)



In the diagram above, find the size of the largest angle.
Give your answer correct to the nearest degree. **2**

(f) Simplify: $\frac{5}{m-2}-\frac{2}{m-3}$. **2**

STUDENT NUMBER/NAME: ……………………..

Question 2 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) Differentiate with respect to x:

(i) $\frac{\cos x}{x-1}$. **2**

(ii) $(3x^2-7)^5$. **2**

(b) Solve: $|2x-3|<1$. **2**

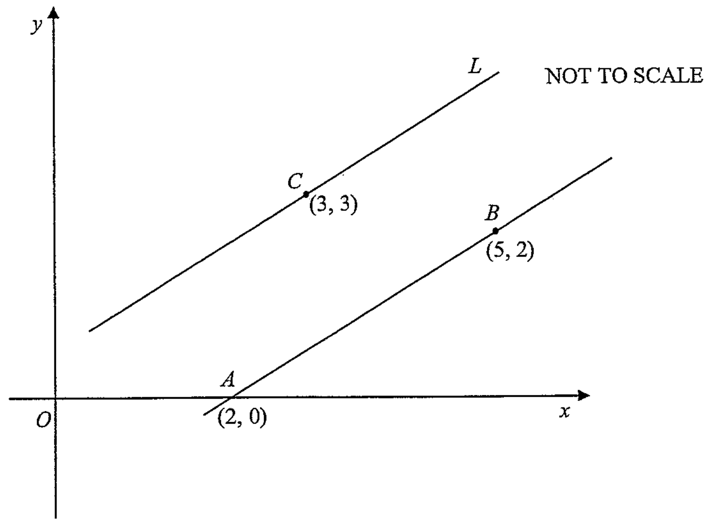
(c) (i) Find: $\int \frac{x}{x^2+2}dx$. **2**

(ii) Evaluate: $\int_0^{\frac{2\pi}{3}} \sin 2x\,dx$. **3**

(d) Use the change of base rule to evaluate: $\log_8 4$. **1**

Question 3 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a)



In the diagram above the points A(2,0), B(5,2) and C(3,3) are shown.
Copy or trace the diagram onto your worksheet.

(i) Find the exact length of AB. **1**

(ii) SHOW that the equation of AB is $2x-3y-4=0$. **1**

(iii) Find the exact perpendicular distance from C to AB. **1**

(iv) The line L passing through C has equation $2x-3y+3=0$. Show that L is parallel to AB. **2**

(v) D is a point on L such that the length of DC is $\frac{\sqrt{13}}{2}$ units. What type of quadrilateral is ABCD? Give reasons. **1**

(vi) Calculate the area of ABCD. **1**

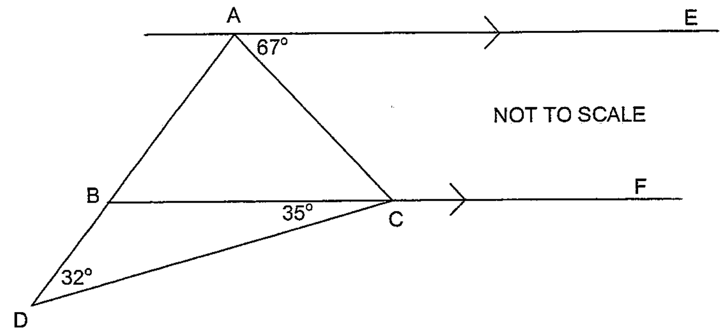
(b) Solve: $2\cos A=-\sqrt{3},\ for\ 0\le A\le 2\pi$. **2**

(c) Find the equation of the tangent to the curve $y=x^2\ln x$ at the point P on it where $x=e$. **3**

STUDENT NUMBER/NAME: ....

Question 4 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a)



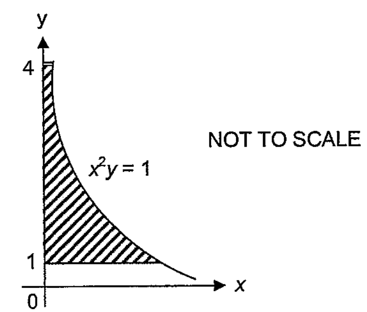
In the diagram above AE is parallel to BF.

$\angle ADC=32°$, $\angle BCD=35°$ and $\angle CAE=67°$.

(i) Show that ΔABC is isosceles. **3**

(ii) Find the size of $\angle BAC$. **1**

(b)



The shaded region above shows the area bounded by the graph $x^2y=1$, $(x>0)$, the y-axis and the lines $y=1$ and $y=4$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis. Give your answer in exact form. **3**

Question 4 continues on the next page

Question 4 (continued) **Marks**

(c) During the drought of the last few years, the water level in the local dam in the township of Wallaville was reduced to 2.5% of its capacity.

In the first week of drought breaking rain, the inflow added 3% of capacity to the amount of water in the dam (i.e. the dam was 5.5% full).

In the next week the inflow added 3.5% of capacity to the amount of water in the dam.

In the third week 4% of capacity was added.

This pattern continued so that each week an extra 0.5% of capacity was added to the dam until it was full.

(i) What percentage of capacity was added to the dam in the 10^{th} week? **1**

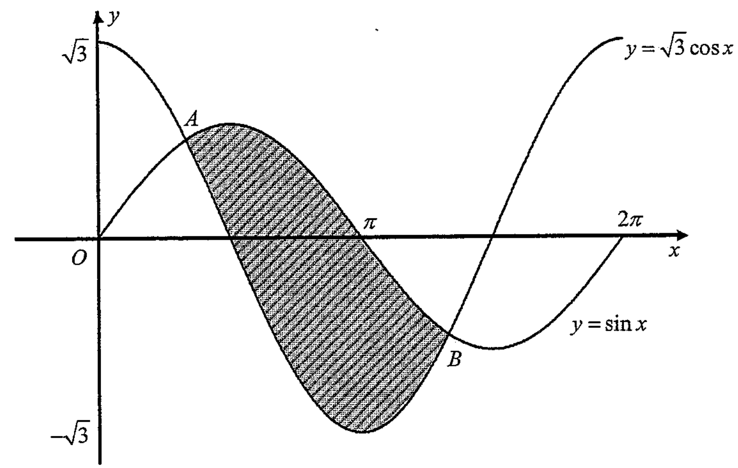
(ii) What percentage of capacity was in the dam after 10 weeks? **2**

(iii) How many weeks would it have taken to fill the dam? **2**

STUDENT NUMBER/NAME: ……………………..

Question 5 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a)



The diagram shows the graphs $y = \sin x$ and $y = \sqrt{3}\cos x$, $0 \le x \le 2\pi$.
The graphs intersect at points A and B.

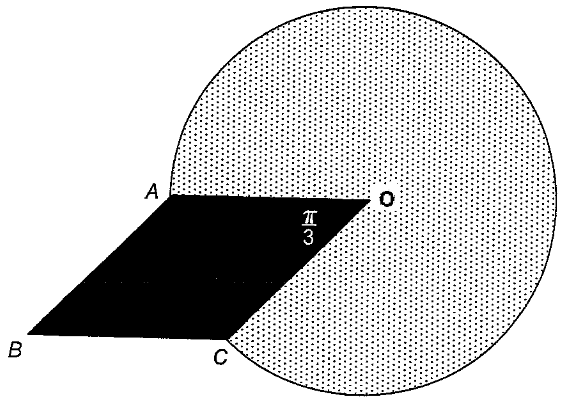
(i) Show that point A has coordinates $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ and find the coordinates of B. **1**

(ii) Find the area enclosed by the two graphs. **3**

Question 5 continues on the next page

Question 5 (continued) **Marks**

(b) A concrete viewing platform is to be built at a mountain lookout. The platform is formed from a rhombus $AOCB$ with side $AO = 5\text{m}$ and $\angle AOC = \frac{\pi}{3}$, and the major sector of a circle centre O, radius AO. The concrete is 200mm thick. The platform is illustrated in the diagram below.



NOT TO SCALE

(i) Show that reflex $\angle AOC = \frac{5\pi}{3}$. **1**

(ii) Calculate the area of the platform. **3**

(iii) Find the volume of concrete used to make the platform. **1**

(c) Given $\tan A = \frac{\sqrt{15}}{7}$ and $\pi \leq A \leq 2\pi$, find the exact value of $\operatorname{cosec} A$. **3**

STUDENT NUMBER/NAME: ……………………..

Question 6 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) A nurseryman has developed a new frost resistant strain of a grape vine. Testing has shown that the probability that the plant survives a heavy frost is 9 in 10.

Three of these plants are selected at random and exposed to a heavy frost.

What is the probability that:

(i) All three plants survive the frost? **1**

(ii) None of the plants survive the frost? **2**

(iii) Exactly two plants survive the frost? **2**

(iv) At most two plants survive the frost? **2**

(b) Evaluate: $\sum_{x=0}^{4}\left(\sin\frac{\pi x}{4}\right)$ **2**

(c) Given the infinite series: $\frac{x}{3}+\frac{2x^2}{9}+\frac{4x^3}{27}+\ldots$:

(i) Show that it is a geometric series. **1**

(ii) Find the values of x such that the series has a limiting sum and find the sum (in terms of x). **2**

Question 7 (12 marks) Use a SEPARATE page or writing booklet — **Marks**

(a) A function is defined as $f(x)=x^3-3x^2$.

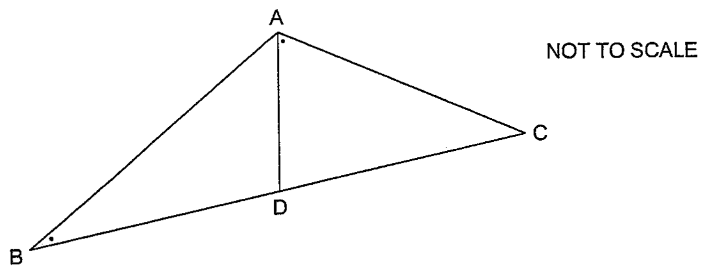
(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **1**

(iii) Sketch the graph of $y=f(x)$ indicating clearly the stationary points, the point of inflexion and the x-intercepts. **3**

(iv) Find the minimum value of the function in the interval $-2 \le x \le 3$. **1**

(b) In the diagram $\angle CAD = \angle ABC$.



Copy or trace the diagram onto your worksheet.

(i) Prove that ΔCAD is similar to ΔCBA. **3**

(ii) Hence or otherwise show that $AC^2 = CD.CB$. **1**

Question 8 (12 marks) Use a SEPARATE page or writing booklet — **Marks**

(a) The number of worms, N, in a worm farm at time t weeks, is given by the formula:

$N = N_0e^{kt}$ where N_0 and k are constants.

Initially there were 200 worms placed in the worm farm. After 2 weeks the number of worms had doubled.

(i) Find the value of N_0 and show that the value of k is 0.3466. **2**

(ii) How many worms were in the farm after 10 weeks? **1**

(iii) Find the rate of increase in the number of worms at 10 weeks. **2**

(iv) How many weeks would it take for the number of worms to increase by 900%? **2**

(b) Show that the quadratic equation $x^2+(p-3)x-(2p+1)=0$, where p is real, has real distinct roots? **3**

(c) If $A=\sin\beta$ express $1+\cot^2\beta$ in terms of A. **2**

Question 9 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) (i) Copy and complete the table below for the function $f(x)=(x-1)^{-2}$, giving the values correct to 3 significant figures. 1

x	2	2.5	3	3.5	4
$f(x)$					

(ii) Using Simpson's Rule with 5 function values, find an approximate value for: 2

$$\int_2^4 (x-1)^{-2}\,dx.$$

(b) The local swimming pool has been closed and is being drained for repairs. The rate, at which the amount of water in the pool, (P kilolitres) at time t hours after draining has commenced, is decreasing is given by:

$$\frac{dP}{dt}=-30(20-t)$$

Initially the pool held 6000 kilolitres.

(i) Express P as a function of t. 2

(ii) Find how much water was in the pool after 5 hours. 1

(iii) How long does it take to empty the pool? 2

(c) The equation of a parabola is: $2y=x^2-4x+6$.

(i) Find the coordinates of the vertex, V, and the focus, S. 2

(ii) Find the equation of the directrix. 1

(iii) Draw a neat sketch of the graph of this parabola showing the information obtained in (i) and (ii) above. 1

STUDENT NUMBER/NAME: ……………………..

Question 10 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) Alana has borrowed \$17 000 to buy a new car. The interest on the loan is 18% per annum paid monthly. The loan is to be repaid in equal monthly instalments of \$$P$ over a term of 5 years.

Let the amount owing on the loan after n months be \$$A_n$.

(i) Show that the amount \$$A_1$ owing after one month is given by: $A_1=\$\{(17000\times1.015)-P\}$. 1

(ii) Show that the amount \$$A_3$ owing after 3 months is given by: $A_3=\$\{(17000\times1.015^3)-P(1+1.015+1.015^2)\}$. 2

(iii) Write down a similar expression for the amount owing after 5 years (60 months). 1

(iv) Calculate the monthly instalment \$$P$ paid on the loan. 2

(v) How much would Alana have saved by paying cash for the car? 1

(b) A particle is moving in a straight line so that at time t seconds its displacement from the origin is x metres. Initially the particle is 1 metre to the left of the origin.

The velocity of the particle is given by $v=2\cos t-1$.

(i) Express the displacement x as a function of t. 2

(ii) At what time is the particle first at rest? 2

(iii) Find the position of the particle at this instant. 1

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2008
MATHEMATICS YR 12 EXAMINATION
MARKING GUIDELINES

Question 1

Part	Answer	Mark
(a)	$2.156097806 = 2.1561$ (1 mark each)	2
(b)	$\frac{5(\sqrt{3}+1)}{3-1} = \frac{5\sqrt{3}+5}{2}$ (1 mark) $a = \frac{5}{2} = b$ (1 mark)	2
(c)	$x = \frac{+5 \pm \sqrt{5^2 - 4.2.(-4)}}{2(2)}$ (1 mark) $= \frac{5 \pm \sqrt{57}}{4}$ (1 mark)	2
(d)	$\int\left(x^{-2} + \frac{1}{x}\right)dx = \frac{x^{-1}}{-1} + \ln x + C$ $\ln x - \frac{1}{x} + C$ (1 mark + 1 mark) – don't deduct mark if *C* is missing	2
(e)	$\cos A = \frac{4^2 + 5^2 - 7^2}{2.4.5}$ $= -\frac{1}{5}$ (1 mark) $A = 101.53\ldots$ $= 102°$ (1 mark)	2
(f)	$\frac{5(m-3) - 2(m-2)}{(m-2)(m-3)}$ (1 mark) $= \frac{5m - 15 - 2m + 4}{(m-2)(m-3)}$ $= \frac{3m - 11}{(m-2)(m-3)}$ (1 mark)	2

Question 2

Part	Answer	Mark
(a)(i)	$\frac{d}{dx}\left(\frac{\cos x}{x-1}\right) =$ $= \frac{(x-1)\cos x - \cos x(1)}{(x-1)^2}$ (1 mark) $= \frac{\sin x - x\sin x - \cos x}{(x-1)^2}$ (1 mark)	2
(a)(ii)	$\frac{d}{dx}(3x^2 - 7)^5$ $= 5(3x^2 - 7)^4 6x$ (1 mark) $= 30x(3x^2 - 7)^4$ (1 mark)	2

Question 2 continues on the next page

Part	Answer	Mark
(b)	$\lvert 2x - 3 \rvert < 1$ $2x - 3 < 1$ OR $2x\text{-}3 > -1$ (1 mark) $2x < 4$ $\quad 2x > 2$ $x < 2$ $\quad x > 1$ $1 < x < 2$ (1 mark)	2
(c)(i)	$\frac{1}{2}\int \frac{2x\,dx}{x^2 + 2}$ $= \frac{1}{2}\ln(x^2 + 2) + C$ (1 mark + 1 mark)	2
(c)(ii)	$\int_0^{\frac{\pi}{3}} \sin 2x\,dx$ $= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{2\pi}{3}}$ (1 mark) $= \frac{1}{2}\left[-\cos\frac{4\pi}{3} - (-\cos 0)\right]$ $= \frac{1}{2}\left[-\left(-\frac{1}{2}\right) - (-1)\right]$ (1 mark + 1 mark) $= \frac{3}{4}$	3
(d)	$\log_8 4 = \frac{\log_2 4}{\log_2 8}$ $= \frac{2}{3}$ (1 mark)	1

Question 3

Part	Answer	Mark
(a)(i)	$AB = \sqrt{(5-2)^2 + (2-0)^2} = \sqrt{13}$ (1 mark)	1
(a)(ii)	$AB: \frac{y-0}{x-2} = \frac{2-0}{5-2}$ $3y = 2x - 4$ (1 mark) $2x - 3y - 4 = 0$ SHOW	1
(a)(iii)	$p = \left\lvert\frac{2(3) - 3(3) - 4}{\sqrt{2^2 + 3^2}}\right\rvert$ $= \frac{7}{\sqrt{13}}$ (1 mark)	1
(a)(iv)	AB: $3y = 2x - 4$ $y = \frac{2}{3}x - 4$ $m_1 = \frac{2}{3}$ (1 mark) Similarly for L : $m_2 = \frac{2}{3}$ i.e. $AB \parallel L$ as $m_1 = m_2$ (1 mark)	2

Question 3 continues on the next page

Question 3 continued

(a)(v)	ABCD is a trapezium. Pair of opposite sides parallel, but not equal. (1 mark)	1
(a)(vi)	$A=\frac{1}{2}\left[\sqrt{13}+\frac{1}{2}\sqrt{13}\right].\frac{7}{\sqrt{13}}$ $=\frac{21}{4}units^2$	1
(b)	$\cos A=-\frac{\sqrt{3}}{2}$, quads 2,3 (1 mark) $A=\frac{5\pi}{6},\frac{7\pi}{6}$ (1 mark)	2
(c)	$\frac{dy}{dx}=2x\ln x+x^2\cdot\frac{1}{x}$ (1 mark) $=2x\ln x+x$ At $x=e,\ y=e^2\ln e=e^2$ and $y'=3e=m$ (1 mark) Equation of tangent is: $y-e^2=3e(x-e)$, $y=3ex-2e^2$ (1 mark)	3

Question 4

Part	Answer	Mark
(a)(i)	$\angle EAC=67°$ (Data) $\angle BCA=\angle EAC$ (Alt $\angle$'s, AE$\parallel$BF) (1 mark) $\therefore\angle BCA=67°$ $\angle ABC=\angle BDC+\angle BCD$ (Ext $\angle$ = sum of 2 interior opp.) (1 mark) $=32°+35°=67°$ $\therefore\Delta ABC$ is isosceles (pr. of equal angles) (1 mark) Deduct one mark only for lack of reasons	3
(a)(ii)	$\angle BAC+\angle BCA+\angle ABC=180°$ (Angle sum of ΔABC) $\angle BAC+67°+67°=180°$ $\angle BAC=46°$ (1 mark)	1
(b)	$x^2y=1$ $\quad$ ie. $x^2=\frac{1}{y}$ $V=\pi\int_1^4 x^2dy=\pi\int_1^4\frac{1}{y}dy$ (1 mark) $=\pi[\ln y]_1^4=\pi[\ln 4-\ln 1]$ (1 mark) $=\pi\ln 4$ (1 mark)	3
(c)(i)	Amounts of inflows each week: 3 + 3.5 + 4 + …. This is an Arithmetic series ($a=3.5$, $d=0.5$). The initial 2.5% is a separate term. $T_{10}=3+(10-1)0.5$ $=7.5\%$ added in 10th week (1 mark)	1
(c)(ii)	$S_{10}=\frac{10}{2}[2\times3+(10-1)\times0.5]$ (1 mark) $=52.5\%$ inflow in 10 weeks Dam holds 52.5% + 2.5% = 55% (1 mark)	2

Question 4(c) continues on the next page

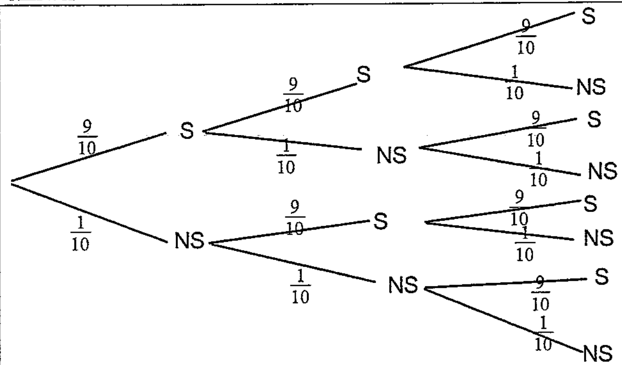
Question 4(c) continued

(c)(iii)	When 100% full, 97.5% has been added (in n weeks). $S_n=\frac{n}{2}[2\times3+(n-1)\times0.5]$ $97.5=\frac{n}{2}(0.5n+5.5)$ (1 mark) $390=n^2+11n$ $(n-15)(n+26)=0$ $n=15$ $(n\neq-26)$ It took 15 weeks to fill the dam (1 mark)	2

Question 5

Part	Answer	Mark
(a)(i)	$y=\sin x\quad y=\sqrt{3}\cos x$ $\sin x=\sqrt{3}\cos x$ $\frac{\sin x}{\cos x}=\tan x=\sqrt{3}\quad\therefore x=\frac{\pi}{3}(A),\ \frac{4\pi}{3}(B)$ $\therefore A=\left(\frac{\pi}{3},\sin\frac{\pi}{3}\right)=\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$ Similarly $B=\left(\frac{4\pi}{3},-\frac{\sqrt{3}}{2}\right)$ (1 mark)	1
(a)(ii)	$Area=\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}}(\sin x-\sqrt{3}\cos x)dx=\left[-\cos x-\sqrt{3}\sin x\right]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$ (1 mark) $=\left(-\cos\frac{4\pi}{3}-\sqrt{3}\sin\frac{4\pi}{3}\right)-\left(-\cos\frac{\kappa}{3}-\sqrt{3}\sin\frac{\pi}{3}\right)$ $=\left[-\left(-\frac{1}{2}\right)-\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)\right]-\left[-\left(\frac{1}{2}\right)-\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)\right]$ (1 mark) $=4units^2$ (1 mark)	3
(b)(i)	Reflex $\angle AOB$ = Circle - Acute $\angle AOB=2\pi-\frac{\pi}{3}=\frac{5\pi}{3}$ (1 mark)	1
(b)(ii)	Area = Rhombus + Major Sector AOB = 2 x ΔAOB $+\frac{1}{2}r^2\theta$ (1 mark) $=2\times\frac{1}{2}\times5\times5\times\sin\frac{\pi}{3}+\frac{1}{2}\times5^2\frac{5\pi}{3}$ (1 mark) $=\frac{25\sqrt{3}}{2}+\frac{125\pi}{6}$ $\approx87.1\ metres^2$ (1 mark)	3
(b)(iii)	Vol = Area x Thickness $=\left(\frac{25\sqrt{3}}{2}+\frac{125\pi}{6}\right)\times0.2$ $=5\left(\frac{\sqrt{3}}{2}+\frac{5\pi}{6}\right)metres^3$ (1 mark or approx. value 1 mark) $\approx17.42\ metres^3$	1
(c)	A is in 3rd Quadrant as $\tan A$ is + (Q 1,3) and $\pi\le A\le2\pi$ (Q 3,4) (1 mark) If X is the third side in the Δ, $X^2=(\sqrt{15})^2+7^2$, $X=8$ (1 mark) $\therefore\operatorname{cosec}A=-\frac{8}{\sqrt{15}}$ (1 mark)	3

Question 6

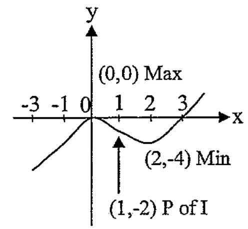
Part	Answer	Mark
(a)(i)	 $P(3S)=\left(\frac{9}{10}\right)^3=\frac{729}{1000}$ (1 mark)	1
(a)(ii)	$P(None\ S)=\left(\frac{1}{10}\right)^3=\frac{1}{1000}$ ($\frac{1}{10}$ 1 mark + $\frac{1}{1000}$ 1 mark)	2
(a)(iii)	$P(Exactly\ 2S)=3\times\left(\frac{1}{10}\right)\times\left(\frac{9}{10}\right)^2=\frac{243}{1000}$ ($\frac{1}{10}\times\left(\frac{9}{10}\right)^2$ 1 mark + $\frac{243}{1000}$ 1 mark)	2
(a)(iv)	$P(At\ most\ 2)=1-P(3S)=1-\frac{729}{1000}=\frac{271}{1000}$ (1 mark + 1 mark)	2
(b)	$\sum_{x=0}^{4}\left(\frac{\sin x\pi}{4}\right)=\sin 0+\sin\frac{\pi}{4}+\sin\frac{2\pi}{4}+\sin\frac{3\pi}{4}+\frac{\sin 4\pi}{4}$ (1 mark) $=0+\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}+0=1+\frac{2}{\sqrt{2}}=1+\sqrt{2}$ (1 mark)	2
(c)(i)	$a=\frac{x}{3},\ r=\frac{2x}{3}=\frac{T_2}{T_1}=\frac{T_3}{T_2}$ (1 mark)	1
(c)(ii)	Limiting Sum exists if $\lvert r\rvert<1$ $-1<\frac{2x}{3}<1$ $-\frac{3}{2}<x<\frac{3}{2}$ (1 mark) $S=\frac{a}{1-r}=\frac{\frac{x}{3}}{1-\frac{2x}{3}}=\frac{x}{3-2x}$ (1 mark)	2

Question 7

Part	Answer	Mark
(a)(i)	$f(x)=x^3-3x^2$ $f'(x)=3x^2-6x=0$ for stationary points (1 mark) $x=0,2$ $f''(x)=6x-6=0$ for points of inflexion $x=1$ At $x=0, f(x)=0, f''(x)=-6$ (concave down) $\therefore (0.0)$ is a Maximum T.P. Similarly $(2,-4)$ is a Minimum T.P. (1 mark + 1 mark)	3
(a)(ii)	At $x=1$, $f''(1^+)>0$, $f''(1^-)<0$, $f''(x)=0$, and changes sign. $f(1)=-2$ (1,-2) is a point of inflexion (1 mark)	1

Question 7(a) continues on the next page

Question 7(a) continued

(a)(iii)	 NOT TO SCALE (1 mark each for stat.pts, POI and x-intercepts done correctly)	3
(a)(iv)	From the graph, minimum occurs at either x = -2 or x = 2 At $x=2$, $f(2)=-4$, $f(-2)=-20$ Minimum in the given interval is -20 (1 mark)	1
(b)(i)	In Δ's CAD, CBA. $\angle CAD=\angle CBA$ (data) $\angle ACD$ is common (1 mark) $\angle CDA=\angle CAB$ (Angle sum of each triangle is 180°) (1 mark) $\therefore \Delta CAD$ is similar to ΔCBA (Angles equal) (1 mark)	3
(b)(ii)	$\frac{CD}{AC}=\frac{AC}{CB}$ (Corr. sides in similar Δ's) $\therefore AC^2=CD.CB$ (1 mark)	1

Question 8

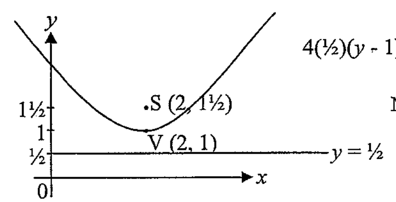
Part	Answer	Mark
(a)(i)	$N=N_0e^{kt}$ At $t=0,\ N=200.\ \therefore N_0=200$ At $t=2,\ N=400$ (1 mark) $\therefore 400=200e^{2k}$ $2k=\ln 2$ $k=0.3466$ (to 4 decimal places) (1 mark)	2
(a)(ii)	$t=10,\ N=200e^{0.3466\times 10}$ $=6402$ worms (1 marks)	1
(a)(iii)	Rate of increase $=\frac{dN}{dt}$ $=kN_0e^{kt}$ $=kN$ (1 mark) $=0.3466\times 6402$ $=2219$ worms/week (1 mark)	2
(a)(iv)	900% increase = increase of 1800. i.e. $N=2000$ (1 mark) $2000=200e^{0.3466t}$ $t=6.6$ weeks (1 mark)	2
(b)	Roots are real and different if $\Delta>0$ (1 mark) $\Delta=(p-3)^2-4\times1\times(-[2p+1])$ (1 mark) $=p^2+2p+13$ $=(p+1)^2+12$ $\therefore \Delta\geq 12$ (1 mark) $\therefore$ Roots are real and different.	3

Question 8(c) continues on the next page

Question 8(c) continued

(c)	$1+\cot^2\beta = 1+\dfrac{\cos^2\beta}{\sin^2\beta} = \dfrac{\sin^2\beta+\cos^2\beta}{\sin^2\beta}$ (1 mark) $= \dfrac{1}{\sin^2\beta} = \dfrac{1}{A^2}$ (1 mark)	2

Question 9

Part	Answer	Mark
(a)(i)	x: 2, 2.5, 3, 3.5, 4 $f(x)$: 1, 0.444, 0.25, 0.16, 0.111	1
(a)(ii)	$\int_2^4 (x-1)^{-2}\,dx \approx \left(\frac{0.5}{3}\right)\left[(1+0.111)+4\times(0.444+0.16)+2\times 0.25\right]$ (1 mark) $=\frac{0.5}{3}\times 4.027$ $= 0.671$	2
(b)(i)	$\dfrac{dP}{dt} = -600+30t, \quad P = -600t+15t^2+C$ When $t=0$, $P=6000 \rightarrow C=6000$ (1 mark) $\therefore P = 15t^2-600t+6000$ (1 mark)	2
(b)(ii)	When $t=5$, $P = 15\times 5^2 - 600\times 5+6000$ $= 3375$ kilolitres	1
(b)(iii)	If the pool is empty, $P=0$. $15t^2-600t+6000=0$ (1 mark) $(t-20)^2=0$ $t=20$ hours i.e. the pool is empty after 20 hours (1 mark)	2
(c)(i)	$2y-2 = x^2-4x+4$ $4\times\frac{1}{2}\times(y-1) = (x-2)^2$ Vertex V = (2, 1), focal length $a=\frac{1}{2}$ and focus S = $(2, 1\frac{1}{2})$ (1 mark + 1 mark)	2
(c)(ii)	Directrix is $y=\frac{1}{2}$ (1 mark)	1
(c)(iii)	 $4(½)(y - 1) = (x - 2)^2$ NOT TO SCALE	1

Question 10

Part	Answer	Mark
(a)(i)	$A_1 = 17000 + 1.5\%$ of $17000 - P = 17000\times 1.015 - P$ (1 mark)	1
(a)(ii)	$A_2 = A_1 + 1.5\%$ of $A_1 - P = 1.015\times A_1 - P = 1.015\times(17000\times 1.015 - P) - P$ $= 17000\times 1.015^2 - P(1+1.015)$ (1 mark) Similarly: $A_3 = 17000\times 1.015^3 + P(1+1.015+1.015^2)$ (1 mark)	2
(a)(iii)	$A_{60} = 17000^{60} + P(1+1.015+1.015^2+\ldots+1.015^{59})$ (1 mark)	1

Question 10(a) continues on the next page

Question 10(a) continued

(a)(iv)	If the loan is paid off in 5 years, $A_{60} = 0$. $P = \dfrac{17000\times 1.015^{60}}{G}$ where $G = \dfrac{1(1.015^{60}-1)}{1.015-1}$ (1 mark) $P = \$431.69$ (1 mark)	2
(a)(v)	Savings = \$(431.69 x 60 – 17000) = \$8901.40 (1 mark)	1
(b)(i)	$v = \dfrac{dx}{dt} = 2\cos t - 1$ $\therefore x = 2\sin t - t + C$ (1 mark) $-1 = 2\times\sin 0 - 0 + C$ $C = -1$ $x = 2\sin t - t - 1$ (1 mark)	2
(b)(ii)	$v=0$ $2\cos t - 1 = 0$ i.e. $\cos t = \frac{1}{2}$ (1 mark) First time particle is at rest when $t = \dfrac{\pi}{3}$ (1 mark)	2
(b)(iii)	Position of particle: $x = 2\sin\dfrac{\pi}{3} - \dfrac{\pi}{3} - 1$ $= \sqrt{3} - \dfrac{\pi}{3} - 1$ (1 mark)	1

The Yr 12 Trial examination and marking guidelines/suggested answers have been produced to help prepare students for the HSC to the best of our ability.

Individual teachers/schools may alter parts of this product to suit their own requirement.