NSW INDEPENDENT SCHOOLS

2009
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: .....

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

Note: $\ln x = \log_e x, \quad x > 0$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value.

Answer the questions on your own paper or writing booklet, if provided.
Start each question on a new page.

Marks

Question 1 (12 marks)

(a) Evaluate $\sqrt{\dfrac{627}{42+29}}$ correct to two significant figures. **2**

(b) Find integers a and b such that $\left(2\sqrt{3}-1\right)^2 = a\sqrt{3}+b$. **2**

(c) Solve $\dfrac{2x-1}{3}-\dfrac{1-3x}{5}=2$. **2**

(d) Find a primitive function of $3+\sin 2x$. **2**

(e) Find the values of x for which $|2-3x|>11$. **2**

(f) Factorise x^4-16. **2**

Marks

Question 2 (12 marks) Start a new writing booklet.

(a) Differentiate with respect to x:

(i) $e^{2x}\tan x$, **2**

(ii) $\dfrac{\sin x}{4-x}$. **2**

(b) Find the equation of the normal to the curve $y=\log_e x-1$ at the point $(e,\ 0)$. **2**

(c) Find:

(i) $\displaystyle\int \frac{3}{\sqrt{2x-1}}\,dx$. **2**

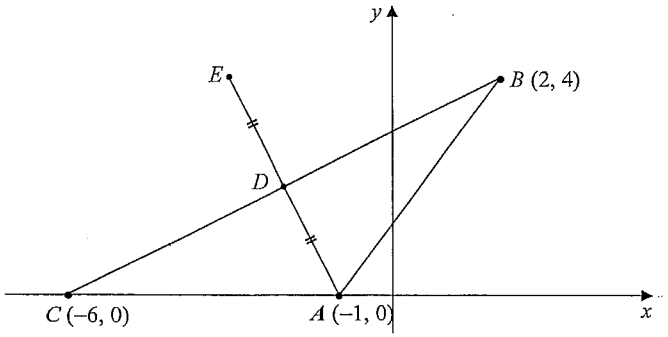
(ii) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3x\,dx$. Give your answer in exact form. **2**

(d) Solve $2\sin\theta+1=0$ for $0\le\theta\le 2\pi$. **2**

Marks

Question 3 (12 marks) Start a new writing booklet.

(a)



NOT TO SCALE

In the diagram A, B and C are $(-1,\ 0)$, $(2,\ 4)$ and $(-6,\ 0)$ respectively.
D has coordinates $(-2, 2)$ and is the midpoint or AE.

(i) Find the length of the interval AB. **1**

(ii) Find the equation of the circle with centre at B which passes through the point A. **1**

(iii) Find the size of $\angle CAB$ (to the nearest degree). **2**

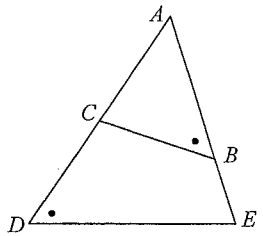
(iv) Find the midpoint of BC. **1**

(v) Show the equation of the line BC is $x-2y+6=0$. **1**

(vi) Find the perpendicular distance of A from the line BC in simplest exact form. **2**

(vii) What type of quadrilateral is $ABEC$? Give reasons for your answer. **2**

(b)



NOT TO SCALE

$\Delta ABC \,|||\, \Delta ADE$. $AC = 4$ *cm*, $AB = 6$ *cm* and $BE = 2$ *cm*.
Find the length CD. **2**

Marks

Question 4 (12 marks) Start a new writing booklet.

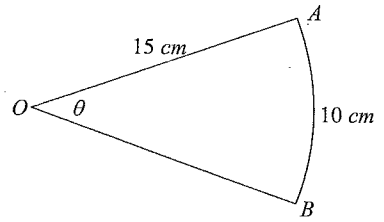
(a) Evaluate $\sum_{x=2}^{20}(3x-5)$. **3**

(b) Consider the parabola $4y = x^2 - 2x + 5$.

(i) Show the vertex is $(1,\ 1)$, **2**

(ii) Find the coordinates of the focus. **2**

(c)

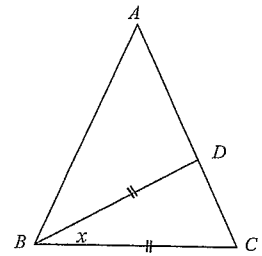


In the diagram, AB is an arc of a circle with centre O.
The arc AB is 10 *cm* and the radius OA is 15 *cm*. Find:

(i) the exact size of $\angle AOB$ in radians, **1**

(ii) the exact area of the sector AOB. **1**

(d)



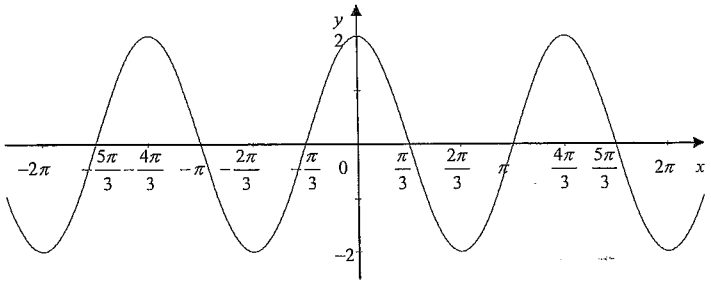
NOT TO SCALE

ΔABC is isosceles with $AB = AC$.
D lies on AC such that $\angle ABD = 3\angle DBC$ and $BD = BC$, as shown above.
Find the value of x. **3**

Marks

Question 5 (12 marks) Start a new writing booklet.

(a)



The graph above can be represented by an equation in the form $y = a\cos nx$. **2**
Find the values of a and n.

(b) A scientist measured the instantaneous velocity, in ms^{-1}, of a particle as it moved through its motion. He made 5 measurements at $t =$ 0, 1, 2, 3 & 4 seconds.
His results, for $v = f(t)$, are tabulated below.

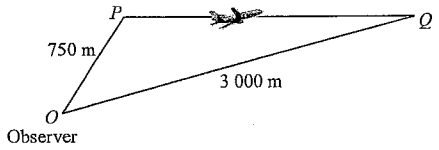
t	0	1	2	3	4
v	6	10	11	8	2

(i) Use Simpson's Rule, with five function values, to approximate the value of **2**

$$\int_0^4 f(t)\,dt$$

(ii) What does $\int_0^4 f(t)\,dt$ represent? **1**

(c)



An observer is standing at point O and sees a plane at P a distance of 750 m from O. **4**
Eight seconds later the plane is sighted at Q, a distance of 3 000 m from O.
The angles of elevation of P and Q from O are 73° and 7° respectively.
Find the speed of the plane to the nearest km/h.

(d) The gradient of a curve is given by $\frac{dy}{dx} = \frac{2x}{x^2+e}$. The curve passes through **3**
the point $(0,\ 2)$.

What is the equation of the curve?

Marks

Question 6 (12 marks) Start a new writing booklet.

(a) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$.

(i) Find the coordinates of the turning points of the graph $y = f(x)$, and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **2**

(iii) Hence sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and where the curve meets the y–axis. **2**

(iv) For what values of x is the graph of $y = f(x)$ concave up? **1**

(b) Find the values of k for which the quadratic equation $2x^2 - kx + 5 = 0$ has real roots **2**

(c) Fiona is planting exotic flowers in her garden. The probability that an individual flower survives is 0.35. Fiona plants 5 flowers. **2**

What is the probability that at least one flower survives?

Marks

Question 7 (12 marks) Start a new writing booklet.

(a) (i) Differentiate $\log_e(\cos x)$ with respect to x. **2**

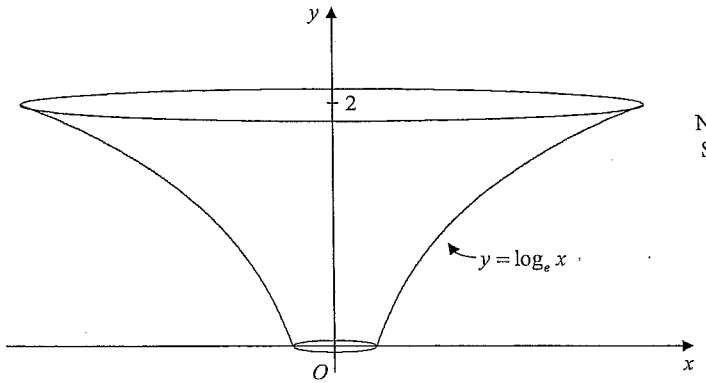
(ii) Sketch the curve $y = \tan x$ for $0 \le x < \frac{\pi}{2}$. **1**

(iii) Hence, using parts (i) and (ii) or otherwise, find the area bounded by the curve $y = \tan x$, the x axis and the line $x = \frac{\pi}{3}$. **3**

Leave answer in simplest exact form.

(b) State the domain and range of the function $y = 2\sqrt{9 - x^2}$. **3**

(c) **3**



NOT TO SCALE

A mould for vase is formed by rotating that part of the curve $y = \log_e x$ between $y = 0$ and $y = 2$ about the y axis.

Find the volume of the mould. Leave your answer in simplest exact form.

Marks

Question 8 (12 marks) Start a new writing booklet.

(a) Consider the geometric series $3 - 6y + 12y^2 - 24y^3 + ...$

(i) For what values of y does this series have a limiting sum? **2**

(ii) The limiting sum of the series is $2\frac{1}{4}$. Find the value of y. **2**

(b) The gravitational pull on the surface of the planet Zeus was found to be $20ms^{-2}$. The gravitational pull, $G\ ms^{-2}$, at a distance d metres above the surface of the planet is given by

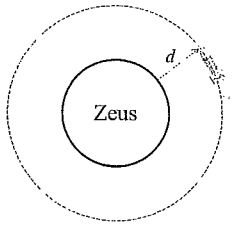
$$G = Ae^{-0.00005d}$$

where A is a constant.

(i) Write down the value of A. **1**

(ii) What is the gravitational pull of the planet Zeus 2000 metres above the planet's surface? Give your answer correct to 2 decimal places, **2**

(iii) A rocket ship, travelling at a constant speed, can only orbit the planet if the gravitational pull is less than $15ms^{-2}$. **3**



What is the minimum distance the rocket ship can orbit Zeus before the gravitational pull becomes too strong and sends it crashing into the planet's surface? (Answer to the nearest 100 metres)

(iv) At what rate is the gravitational pull decreasing 2000 metres above the surface of Zeus? **2**

Marks

Question 9 (12 marks) Start a new writing booklet.

(a) A water tank initially contains 4900 litres of water.
The tap is turned on and the water flows out at a rate given by

$$\frac{dV}{dt} = \frac{1}{8}t - 35$$

where V is the volume, in litres, of water in the tank at time t minutes.

(i) Find an expression for the volume of water in the tank at time t. **2**

(ii) Find the volume of water remaining after 30 minutes. **1**

(iii) Find the rate at which the water is flowing after 16 minutes. **1**

(iv) Find the time taken for the tank to be completely empty. **2**

(b) Jonah is saving for a cruise. He opens an "Incentive Saver Account" which pays interest at the rate of 0.4% per month compounded monthly at the end of each month.
Jonah decides to deposit $400 into the account on the first of each month.
He makes his first deposit on the 1st January 2007 and his last on the 1st July 2009.
He withdraws the entire amount, plus interest, immediately after his final interest payment on 31st July 2009.

(i) How much did Jonah deposit into his "Incentive Saver Account"? **1**

(ii) How much did Jonah withdraw from his account on the 31st July 2009? **3**

(iii) Jonah's cruise is cancelled due to swine flu. He decides to deposit $10000 into a different account which offers interest at the rate of 5% p.a. compounded quarterly for 2 years.
How much will Jonah receive at the end of the investment period? **2**

Marks

Question 10 (12 marks) Start a new writing booklet.

(a) A particle moves in a straight line so that its displacement, x metres from a fixed point O on a line, is given by

$$x = t + \frac{16}{t+1}$$

where t is measured in seconds.

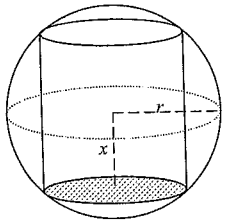
(i) Find the particle's initial position. **1**

(ii) Find expressions for the velocity and acceleration of the particle in terms of t. **2**

(iii) Find when and where the particle is at rest. **2**

(iv) Find the limiting velocity of the particle. **1**

(b) A cylinder is to be made to fit inside a sphere of radius r cm, as shown.



Let x be the distance of the base of the cylinder from the centre of the sphere as shown.

(i) Find an expression for the radius of the base of the cylinder in terms of r and x. **1**

(ii) Show the volume, V, of the cylinder is given by **2**

$$V = 2\pi x\left(r^2 - x^2\right)$$

(iii) Find, in terms of r, the maximum volume of the cylinder. **3**
Give answer in exact form.

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2009
MATHEMATICS HSC TRIAL EXAMINATION
MARKING GUIDELINES

Question 1

a) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct evaluation of expression.	1
• Correct answer correct to 2 significant figures	1

Sample Answer:-

$$\sqrt{\frac{627}{42+29}} = 2.97169$$
$$= 3.0$$

b) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct expansion of expression.	1
• Correct values of a and b.	1

Sample Answer:-

$$\left(2\sqrt{3}-1\right)^2 = 12-4\sqrt{3}+1$$
$$= 13-4\sqrt{3}$$
$$a = -4$$
$$b = 13$$

c) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct multiplication of both sides of the equation by 15.	1
• Correct solution.	1

Sample Answer:-

$$\frac{2x-1}{3} - \frac{1-3x}{5} = 2$$
$$10x-5-3+9x = 30$$
$$19x = 38$$
$$x = 2$$

d) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct integration of $\sin 2x$	1
• Correct answer including c.	1

Sample Answer:-

$$\int (3+\sin 2x)\,dx = 3x - \frac{1}{2}\cos 2x + c$$

e) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 3

Criteria	Marks
• Attempting to solve two inequalities and getting one correct	1
• Correct solution.	1

Sample Answer:-

$$|2-3x| > 11$$
$$-11 > 2-3x \qquad 2-3x > 11$$
$$3x > 13 \qquad 3x < -9$$
$$x > \frac{13}{3} \qquad x < -3$$

f) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct factorization into two factors.	1
• Correct factorization into three factors.	1

Sample Answer:-

$$x^4 - 16 = \left(x^2-4\right)\left(x^2+4\right)$$
$$= (x-2)(x+2)\left(x^2+4\right)$$

Question 2 (12 marks)

a) (i) 2 marks

Outcomes assessed: - P7, Targeted Performance Band:- 2

Criteria	Marks
• Correct application of the product rule with correct differentiation of $\tan x$.	1
• Correct solution.	1

Sample Answer:-

$$\frac{d}{dx}\left(e^{2x}\tan x\right) = 2e^{2x}\tan x + e^{2x}\sec^2 x$$
$$= e^{2x}\left(2\tan x + \sec^2 x\right)$$

a) (ii) 2 marks

Outcomes assessed: - P7, Targeted Performance Band:- 2

Criteria	Marks
• Correct application of the quotient rule.	1
• Correct solution.	1

Sample Answer:-

$$\frac{d}{dx}\left(\frac{\sin x}{4-x}\right) = \frac{(4-x)\cos x - \sin x(-1)}{(4-x)^2}$$
$$= \frac{(4-x)\cos x + \sin x}{(4-x)^2}$$

b) 2 marks

Outcomes assessed: - H3, Targeted Performance Band:- 3

Criteria	Marks
• Correct differentiation and gradient of tangent.	1
• Correct gradient of normal and equation of normal.	1

Sample Answer:-

$y = \log_e x - 1$

$\frac{dy}{dx} = \frac{1}{x}$

at $x = e$, $m_t = \frac{1}{e}$

$m_n = -e$ (Gradient of normal)

equation of normal at point $(e,\ 0)$

$y - 0 = -e(x - e)$

$y = e^2 - ex$

c) (i) 2 marks

Outcomes assessed: - P8, Targeted Performance Band:- 3

Criteria	Marks
• Write integral as a power.	1
• Correctly evaluates integral (constant c not necessary)	1

Sample Answer:-

$$\int \frac{3}{\sqrt{2x-1}}dx = 3\int (2x-1)^{-\frac{1}{2}}dx$$

$$= 3\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2}\times 2} + c$$

$$= 3\sqrt{2x-1} + c$$

c) (ii) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct integral	1
• Correct evaluation of definite integral	1

Sample Answer:-

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3x\,dx = \frac{1}{3}\left[\tan 3x\right]_{\frac{\pi}{12}}^{\frac{\pi}{9}}$$

$$= \frac{1}{3}\left(\tan\frac{\pi}{3} - \tan\frac{\pi}{4}\right)$$

$$= \frac{1}{3}\left(\sqrt{3} - 1\right)$$

d) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct rearrangement of trigonometric equation and one correct solution.	1
• Two correct solutions.	1

Sample Answer:-

$2\sin\theta + 1 = 0$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 3 (12 marks)

a) (i) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct length of interval AB.	1

Sample Answer:-

$d = \sqrt{(2--1)^2 + (4-0)^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

a) (ii) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct equation of circle.	1

Sample Answer:-

$(x-2)^2 + (y-4)^2 = 25$

a) (iii) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct acute angle.	1
• Correct answer (ignore accuracy)	1

Sample Answer:-

$$m = \frac{4-0}{2--1}$$
$$= \frac{4}{3}$$
$$\tan\theta = \frac{4}{3}$$
$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$
$$\theta = 53°$$
$$\angle CAB = 180 - 53$$
$$= 127°$$

a) (iv) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct midpoint.	1

Sample Answer:-

$$\text{Midpoint} = \left(\frac{2+-6}{2}, \frac{4+0}{2}\right)$$
$$= (-2, 2)$$

a) (v) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correctly shown	1

Sample Answer:-

$$\frac{y-0}{4-0} = \frac{x+6}{2+6}$$
$$\frac{y}{4} = \frac{x+6}{8}$$
$$2y = x+6$$
$$x - 2y + 6 = 0$$

a) (vi) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct substitution into correct formula.	1
• Correct answer	1

Sample Answer:-

$$d = \frac{|(-1) - 2(0) + 6|}{\sqrt{1+2^2}}$$
$$= \frac{5}{\sqrt{5}}$$
$$= \sqrt{5}$$

a) (vii) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 3

Criteria	Marks
• Correctly identifies rhombus.	1
• Correct reasoning	1

Sample Answer:-

Rhombus.

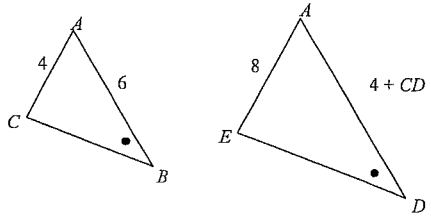
Diagonals bisect (common midpoint D) and two adjacent sides equal ($AB = AC = 5$).

b) 2 marks

Outcomes assessed: - P4, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct algebraic/numerical expression involving corresponding sides.	1
• Correct answer	1

Sample Answer:-



$$\frac{4}{8} = \frac{6}{4+CD}$$
$$\frac{1}{2} = \frac{6}{4+CD}$$
$$4 + CD = 12$$
$$CD = 8$$

Question 4 (12 marks)

a) 3 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correctly generating the series	1
• Recognising an AP and correct number of terms	1
• Correct solution	1

Sample Answer:-

$$\sum_{x=2}^{20}(3x-5)=1+4+7+.....+55$$

$$n=19$$

$$S_{19}=\frac{19}{2}(1+55)$$

$$=532$$

b) (i) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 3

Criteria	Marks
• Correctly completing the square	1
• Correctly shown	1

Sample Answer:-

$$4y=x^2-2x+5$$

$$x^2-2x=4y-5$$

$$x^2-2x+1=4y-4$$

$$(x-1)^2=4(y-1)$$

$$\therefore \text{ Vertex } =(1,\ 1)$$

b) (ii) 2 marks

Outcomes assessed: - P4

Targeted Performance Band:- 3

Criteria	Marks
• Correct value of a	1
• Correct Focus	1

Sample Answer:-

$$a=1$$

$$\text{Focus}=(1,\ 2)$$

c) (i) 1 mark

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer	1

Sample Answer:-

$$l=\theta r$$

$$10=15\theta$$

$$\theta=\frac{2}{3}$$

c) (ii) 1 mark

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer for area.	1

Sample Answer:-

$$A=\frac{1}{2}\theta r^2$$

$$=\frac{1}{2}\times\frac{2}{3}\times15^2$$

$$=75cm^2$$

d) 3 marks

Outcomes assessed: - H2, H5, Targeted Performance Bands:- 3/4

Criteria	Marks
• Correct conclusion of $\angle ACB=x$	1
• Correct conclusion of two expressions for $\angle BDC$ (must have second expression)	1
• Correct answer	1

Sample Answer:-

$\angle DBC=x$

$\angle ABD=3x$

$\angle ABC=4x$ $(\angle DBC+\angle ABD)$

$\angle ACB=4x$ (base angle of isosceles ΔABC)

$\angle BDC=4x$ (base angle of isosceles ΔBCD)

$\angle BDC=180-(x+4x)$ (angle sum of ΔBCD)

$=180-5x$

$4x=180-5x$ $(=\angle BDC)$

$9x=180$

$x=20°$

Question 5 (12 marks)

a) 2 marks

Outcomes assessed: - H5, Targeted Performance Bands:- 3

Criteria	Marks
• Correct value of a	1
• Correct value of n	1

Sample Answer:-

From graph $a = 2$

$$\text{Period} = \frac{4\pi}{3}$$

$$\text{Period} = \frac{2\pi}{n}$$

$$\frac{2\pi}{n} = \frac{4\pi}{3}$$

$$n = \frac{3}{2}$$

b) (i) 2 marks

Outcomes assessed: - H5, Targeted Performance Bands:- 2

Criteria	Marks
• Correct substitution into a correct formula	1
• Correct answer	1

Sample Answer:-

$$A = \frac{1}{3}\left(6+2+4\left(10+8\right)+2\times 11\right)$$

$$= \frac{1}{3}\times 102$$

$$= 34$$

b) (ii) 1 marks

Outcomes assessed: - H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

Distance travelled by the particle in the first 4 seconds of motion.

c) 4 marks

Outcomes assessed: - H5, Targeted Performance Band:- 2/3

Criteria	Marks
• Correct angle at O	1
• Correct use of cosine rule for distance traveled PQ	1
• Correct answer ms^{-1} (Or changing units before use of cosine rule)	1
• Correct answer km/h	1

Sample Answer:-

$$\angle O = 73 - 7$$

$$= 66°$$

$$d^2 = 750^2 + 3000^2 - 2\times 750\times 3000\cos 66$$

$$= 7732185.106$$

$$d = 2780.680691$$

$$\text{Speed} = \frac{2780.680691}{8}$$

$$= 347.5850864 m/s$$

$$= 347.5850864\times 60\times 60 \div 1000$$

$$= 1251.3063$$

$$= 1251 km/h$$

$$d^2 = 0.75^2 + 3^2 - 2\times 0.75\times 3\cos 66$$

$$= 7.732185106$$

$$= 2.780680691$$

$$\text{Speed} = \frac{2.780680691}{\left(8/(60\times 60)\right)}$$

$$= 1251.306311$$

$$= 1251 km/h$$

d) 3 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3/4

Criteria	Marks
• Correct integration of function	1
• Correct substitution of point to find c	1
• Correct evaluation of c and equation of curve.	1

Sample Answer:-

$$\frac{dy}{dx} = \frac{2x}{x^2+e}$$

$$y = \ln\left(x^2+e\right)+c$$

Sub the point $(0,\ 2)$

$$2 = \ln\left(e\right)+c$$

$$c = 1$$

$$y = \ln\left(x^2+e\right)+1$$

Question 6 (12 marks)

a) (i) 3 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct differentiation and evaluation of x values of turning points.	1
• Correct turning points.	1
• Correct determination of nature of turning points using a recognized method.	1

Sample Answer:-

$f(x)=x^3-3x^2-9x+22$

$f'(x)=3x^2-6x-9$

Stationary points occur when $f'(x)=0$

$0=3x^2-6x-9$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1,\ 3$

$f(-1)=-1-3+9+22$

$=27$

$f(3)=27-27-27+22$

$=-5$

Turning points $(-1,\ 27)$, $(3,\ -5)$

Test nature of points.

$f''(x)=6x-6$

Test $(-1,\ 27)$

$f''(-1)=-12<0$ $\therefore$ concave down. $(-1,\ 27)$ is a maximum

Test $(3,\ -5)$

$f''(3)=12>0$ $\therefore$ concave up. $(3,\ -5)$ is a minimum.

a) (ii) 2 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct identification of a possible point of inflexion	1
• Correct test and confirmation of point of inflexion	1

Sample Answer:-

$f''(x)=6x-6$

Possible points of inflexion occur when $f''(x)=0$

$6x-6=0$

$6x=6$

$x=1$

$f(1)=1-3-9+22$

$=11$

Possible point of inflexion $(1,\ 11)$

Test point of inflexion.

x	-1	1	3
$f''(x)$	-12	0	12

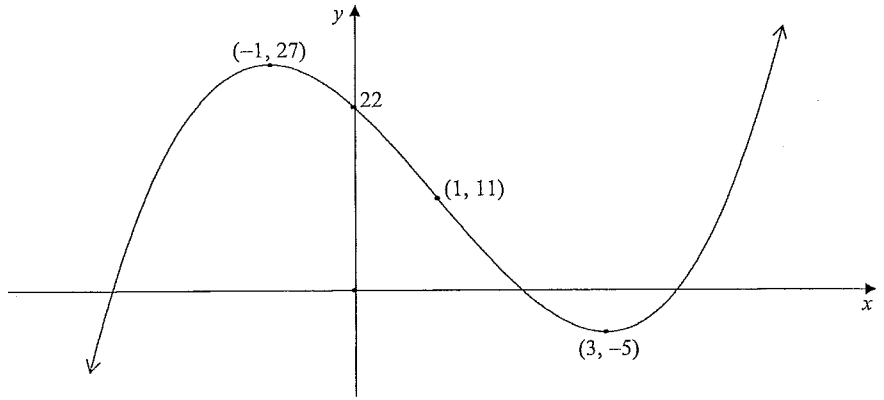
Change in concavity, $\therefore$ $(1,\ 11)$ is a point of inflexion.

a) (iii) 2 marks

Outcomes assessed: - H6, Targeted Performance Band:- 3

Criteria	Marks
• Correct shape graph	1
• Correct graph with all 4 points.	1

Sample Answer:-



a) (iv) 1 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer	1

Sample Answer:-

$x > 1$

b) 2 marks

Outcomes assessed: - P6, Targeted Performance Band:- 2

Criteria	Marks
• Correct discriminant and assumption $\Delta \geq 0$	1
• Correct solution of inequality.	1

Sample Answer:-

For real roots $\Delta \geq 0$.

$$\Delta = k^2 - 4 \times 2 \times 5$$

$$k^2 - 40 \geq 0$$

$$k^2 \geq 40$$

$$k \leq -2\sqrt{10} \text{ or } k \geq 2\sqrt{10}$$

c) 2 marks

Outcomes assessed: - H4, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct probability for not surviving.	1
• Correct solution.	1

Sample Answer:-

$$P(\text{Die}) = 1 - 0.35$$
$$= 0.65$$
$$P(\text{at least one survives}) = 1 - P(\text{all die})$$
$$= 1 - (0.65)^5$$
$$= 0.8839709375$$

Question 7 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct differentiation of $\log_e f(x)$	1
• Correct differentiation of $\cos x$ and simplification to $-\tan x$	1

Sample Answer:-

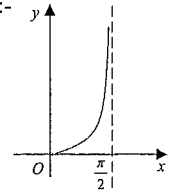
$$\frac{d}{dx}\log_e(\cos x) = \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

a) (ii) 1 marks

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct graph	1

Sample Answer:-

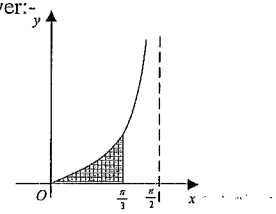


a) (iii) 3 marks

Outcomes assessed: - H3, H8, Targeted Performance Band:- 4

Criteria	Marks
• Correct statement for area and correct integration	1
• Correct solution to give $-\log_e\left(\frac{1}{2}\right)$	1
• Correct solution in correct form. $\log_e(2)$	1

Sample Answer:-



$$A = \int_0^{\frac{\pi}{3}} \tan x \, dx$$

$$= -\left[\ln(\cos x)\right]_0^{\frac{\pi}{3}}$$

$$= -\left(\ln\left(\cos\frac{\pi}{3}\right) - \ln(\cos 0)\right)$$

$$= -\left(\ln\left(\frac{1}{2}\right) - \ln 1\right)$$

$$= -\ln\left(\frac{1}{2}\right)$$

$$= \ln 2$$

b) 3 marks

Outcomes assessed: - P5, Targeted Performance Band:- 3

Criteria	Marks
• Correct domain	1
• $0 \le y \le 3$ (i.e. ignore 2)	1
• Correct range	1

Sample Answer:-

Domain: -

$9 - x^2 \ge 0$

$x^2 \le 9$

$-3 \le x \le 3$

Range:-

$0 \le y \le 6$

c) 3 marks

Outcomes assessed: - H3, H8, Targeted Performance Band:- 4/5

Criteria	Marks
• Correctly changes subject of formula to $x = e^y$	1
• Correct expression for volume	1
• Correct solution	1

Sample Answer:-

$x = e^y$

$$V = \pi \int_0^2 e^{2y} dy$$

$$= \frac{\pi}{2}\left[e^{2y}\right]_0^2$$

$$= \frac{\pi}{2}\left[e^4 - e^0\right]$$

$$= \frac{\pi}{2}\left(e^4 - 1\right) \text{ units}^3$$

Question 8 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H5

Targeted Performance Band:- 3

Criteria	Marks
• Correct value for r.	1
• Correct solution	1

Sample Answer:-

$3 - 6y + 12y^2 - 24y^3 .....$

$r = -2y$

$|-2y| < 1$

$-1 < 2y < 1$

$-\frac{1}{2} < y < \frac{1}{2}$

a) (ii) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct expression for $S_\infty = 2\frac{1}{4}$	1
• Correct solution	1

Sample Answer:-

$$S_\infty = \frac{3}{1+2y}$$

$$\frac{9}{4} = \frac{3}{1+2y}$$

$$9+18y = 12$$

$$18y = 3$$

$$y = \frac{1}{6}$$

b) (i) 1 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct solution	1

Sample Answer:-

$A = 20$

b) (ii) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct substitution into the formula	1
• Correct solution (Ignore decimal places)	1

Sample Answer:-

$G = 20e^{-0.00005d}$

$= 20e^{-0.00005\times 2000}$

$= 18.10 ms^{-2}$

b) (iii) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct substitution into the formula	1
• Correct expression $\ln\left(\frac{3}{4}\right) > -0.00005d$	1
• Correct solution	1

Sample Answer:-

$15 > 20e^{-0.00005d}$

$\frac{3}{4} > e^{-0.00005d}$

$\ln\left(\frac{3}{4}\right) > -0.00005d$

$d > \ln\left(\frac{3}{4}\right) \div (-0.00005)$

$d > 5753.64119$

$d = 5800$ metres

b) (iv) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correctly recognizes the use of the derivative	1
• Correct solution (ignore units)	1

Sample Answer:-

$G = 20e^{-0.00005d}$

$\frac{dG}{dd} = -0.001e^{-0.00005d}$

at $d = 2000$

$\frac{dG}{dd} = -0.001e^{-0.00005\times 2000}$

$= -0.001e^{-0.1}$

$= -0.0009\ ms^{-2}$ per metre.

At 2000 m, the gravitational pull is decreasing at a rate of 0.0009 ms^{-2} per metre.

Question 9 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct integration of derivative	1
• Correct equation for volume of water	1

Sample Answer:-

$\frac{dV}{dt}=\frac{1}{8}t-35$

$V=\frac{1}{16}t^2-35t+c$

At $t=0$, $V=4900$

$4900=\frac{1}{16}(0)^2-35(0)+c$

$c=4900$

$V=\frac{1}{16}t^2-35t+4900$

a) (ii) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct solution	1

Sample Answer:-

At $t=30$

$V=\frac{1}{16}(30)^2-35(30)+4900$

$=3906.25$ litres.

a) (iii) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct solution	1

Sample Answer:-

At $t=16$

$\frac{dV}{dt}=\frac{1}{8}(16)-35$

$=-33$

Water is flowing out of the tank at the rate of 33 litres per minute.

a) (iv) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct interpretation using either $\frac{dV}{dt}=0$ or $V=0$	1
• Correct solution for either method.	1

Sample Answer:-

$\frac{dV}{dt}=0$

$\frac{1}{8}t-35=0$

$\frac{1}{8}t=35$

$t=280$ minutes

$V=0$

$\frac{1}{16}t^2-35t+4900=0$

$t=\frac{35\pm\sqrt{1225-1225}}{2\left(\frac{1}{16}\right)}$

$t=280$ minutes

b) (i) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 5

Criteria	Marks
• Correct solution.	1

Sample Answer:-

Jonah deposited $=31\times400$

$=\$12400$

b) (ii) 3 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correctly identifying G.P and 31 terms	1
• Correct substitution into correct formula	1
• Correct solution.	1

Sample Answer:-

$S=400(1.004)+400(1.004)^2+400(1.004)^3+.....+400(1.004)^{31}$

G.P, $a=400(1.004)$, $r=1.004$, $n=31$

$S=\frac{400(1.004)(1.004^{31}-1)}{1.004-1}$

$=13226.28557$

$=\$13226$

b) (iii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 2/3

Criteria	Marks
• Correct formula and interest rate.	1
• Correct solution.	1

Sample Answer:-

$A = P\left(1+\frac{r}{100}\right)^n$, $P = 10000$, $r = 5/4 = 1\frac{1}{4}\%$, $n = 8$

$A = 10000(1.0125)^8$

$= 11044.86101$

$= \$11045$

Question 10 (12 marks)

a) (i) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct integration of derivative	1

Sample Answer:-

At $t = 0$

$x = 0 + \frac{16}{1}$

$= 16$ metres to the right of O.

a) (ii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct velocity equation	1
• Correct acceleration equation	1

Sample Answer:-

$x = t + \frac{16}{t+1}$

$= t + 16(t+1)^{-1}$

$\dot{x} = 1 - 16(t+1)^{-2}$

$= 1 - \frac{16}{(t+1)^2}$

$\ddot{x} = 32(t+1)^{-3}$

$= \frac{32}{(t+1)^3}$

a) (iii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correctly equating velocity to 0 and finding time t	1
• Correctly find where particle is at rest.	1

Sample Answer:-

$\dot{x} = 1 - \frac{16}{(t+1)^2}$

$0 = 1 - \frac{16}{(t+1)^2}$

$1 = \frac{16}{(t+1)^2}$

$(t+1)^2 = 16$

$t+1 = \pm 4$

$t = 3$ seconds

$x = t + \frac{16}{t+1}$

$x = 3 + \frac{16}{4}$

$x = 3 + 4$

$x = 7$ metres to the right of the origin.

a) (iv) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

As $t \to \infty$

$\dot{x} = 1 - \frac{16}{(\infty+1)^2}$

$\dot{x} = 1\ ms^{-1}$

b) (i) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

Base radius $= \sqrt{r^2 - x^2}$

b) (ii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct height of $2x$	1
• Correct formula	1

Sample Answer:-

$V = \pi(\text{Base Radius})^2 \times \text{height}$

$= \pi\left(r^2 - x^2\right) \times 2x$

$= 2\pi x\left(r^2 - x^2\right)$

b) (iii) 3 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 5/6

Criteria	Marks
• Equating derivative to 0 and finding x.	1
• Testing maximum	1
• Correct maximum volume.	1

Sample Answer:-

$V = 2\pi\left(xr^2 - x^3\right)$

$\frac{dV}{dx} = 2\pi\left(r^2 - 3x^2\right)$

Maximum occurs when $\frac{dV}{dx} = 0$

$0 = 2\pi\left(r^2 - 3x^2\right)$

$3x^2 = r^2$

$x^2 = \frac{r^2}{3}$

$x = \frac{r}{\sqrt{3}}$

Test for Maximum

$\frac{d^2V}{dx^2} = -12\pi x$

at $x = \frac{r}{\sqrt{3}}$

$\frac{d^2V}{dx^2} = -\frac{12\pi r}{\sqrt{3}} < 0$ $\therefore$ a maximum volume when $x = \frac{r}{\sqrt{3}}$

Maximum Volume

$V = 2\pi.\frac{r}{\sqrt{3}}\left(r^2 - \frac{r^2}{3}\right)$

$= \frac{2\pi r}{\sqrt{3}}.\frac{2r^2}{3}$

$= \frac{4\pi r^3}{3\sqrt{3}}$

$= \frac{4\sqrt{3}\pi r^3}{9}$ cubic units