NSW INDEPENDENT SCHOOLS

2010

Higher School Certificate

Preliminary Examination

Mathematics

Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 72

- Attempt Questions 1 – 6
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NAME / NUMBER……………………

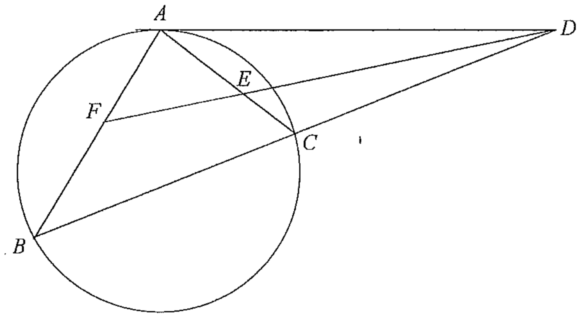
Student name / number ……………………………

Marks

Question 1 **Begin a new booklet**

(a) Simplify $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$. 2

(b)



ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D. The bisector of $\angle BDA$ cuts AC at E and AB at F.

(i) Give a reason why $\angle DAC = \angle ABC$. 1

(ii) Show that $AE = AF$. 2

(c) The polynomial $P(x) = x^3 + a^2x^2 + ax + b$ leaves a remainder of 2 when divided by x and a remainder of 13 when divided by $(x+1)$.

(i) Show that $b = 2$. 1

(ii) Find the value of a. 2

(d)(i) Express $3\cos x + \sin x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of R in simplest exact form, and the value of α in degrees correct to two decimal places. 2

(ii) Solve the equation $3\cos x + \sin x + 2 = 0$ for $0° \le x \le 360°$, giving the solutions correct to the nearest degree. 2

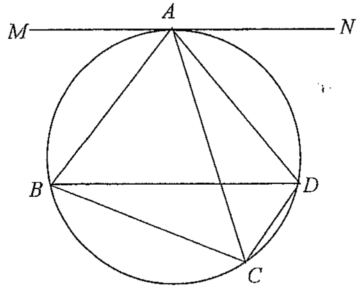
Student name / number ....

Marks

Question 2 **Begin a new booklet**

(a) Show that the equation $x^2+(k+2)x+k=0$ has two real roots for all real values of k. **2**

(b)



ABC is a triangle inscribed in a circle. MAN is the tangent to the circle at A. BD is a chord of the circle such that $BD \parallel MAN$. Show that CA bisects $\angle BCD$. **3**

(c) Solve the equation $\cos 2x+3\cos x+2=0$ for $0° \le x \le 360°$. **3**

(d)(i) Express $x^3-x^2-10x-8$ as a product of three linear factors. **2**

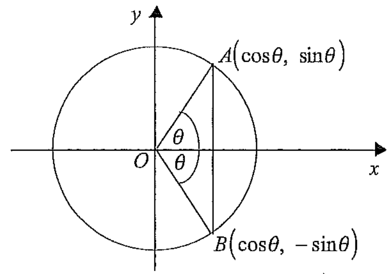
(ii) Solve the inequality $\dfrac{x^3-10x}{x^2+8} \ge 1$. **2**

Student name / number ....

Marks

Question 3 **Begin a new booklet**

(a)



$A(\cos\theta,\ \sin\theta)$ and $B(\cos\theta,\ -\sin\theta)$, $0°<\theta<90°$, are two points on the circle with centre $O(0,0)$ and radius 1. Use the cosine rule in ΔAOB to show that $\cos 2\theta = 1-2\sin^2\theta$. **2**

(b) $A(8,\sqrt{50})$ and $B(1,\sqrt{18})$ are two points. Find in simplest exact form the coordinates of the point P which divides the interval AB externally in the ratio $3:1$. **3**

(c) $P(2ap,\ ap^2)$ and $Q(2aq,\ aq^2)$ are two points on the parabola $x^2=4ay$ such that the chord PQ subtends a right angle at the focus $F(0,a)$.

(i) Show that $(p^2-1)(q^2-1)+4pq=0$. **1**

(ii) Find the value of q when $p=2$. **2**

(d) Find the number of ways in which four numerals can be chosen from the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition so that

(i) no odd numerals are chosen. **1**

(ii) at least one odd numeral is chosen. **1**

(iii) a majority of odd numerals is chosen. **2**

Student name / number ....

Marks

Question 4 **Begin a new booklet**

(a) Find the domain and range of the function $f(x)=3\sqrt{4-x^2}$. **2**

(b) α is the acute angle between the lines $x-2y=0$ and $x-y=0$.
β is the acute angle between the lines $x-y=0$ and $7x-y=0$.

(i) Find the values of $\tan\alpha$ and $\tan\beta$. **2**

(ii) Without finding the values of α and β, show exactly that $\beta=2\alpha$. **1**

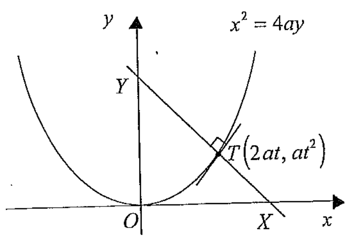
(c) Find the number of ways the letters of the word ANGLE can be arranged in a straight line so that

(i) no two consonants are next to each other. **1**

(ii) the three consonants are side-by-side. **1**

(iii) exactly two of the three consonants are side-by-side. **1**

(d)



The normal to the parabola $x^2=4ay$ at the point $T(2at, at^2)$ cuts the x-axis at X and the y-axis at Y.

(i) Use differentiation to show that the normal at T has equation $x+ty=2at+at^3$. **2**

(ii) Show that $\frac{TX}{TY}=\frac{t^2}{2}$. **2**

Student name / number ....

Marks

Question 5 **Begin a new booklet**

(a) $P(x, y)$ is a variable point which moves in the number plane so that its distance from the point $A(3,3)$ is twice its distance from the point $O(0,0)$. Find the equation of the locus of P. **2**

(b) The polynomial $P(x)=x^3+2x^2-4x-1$ has zeros α, β and γ so that $P(x)=(x-\alpha)(x-\beta)(x-\gamma)$.

(i) Find the value of $(1-\alpha)(1-\beta)(1-\gamma)$. **1**

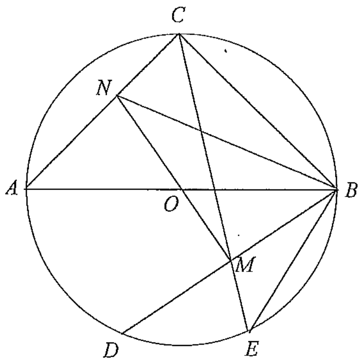
(ii) Find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$. **2**

(c) $P(2at, at^2)$ and $Q\left(\frac{-2a}{t}, \frac{a}{t^2}\right)$ are two points on the parabola $x^2=4ay$.
M is the midpoint of the chord PQ.

(i) Show that as P and Q move on the parabola $x^2=4ay$ the locus of M is a parabola with equation $x^2=2ay-2a^2$. **2**

(ii) Find the focus of the locus of M. **1**

(d)



AB is the diameter of a circle with centre O. BD is a chord with midpoint M. C is a point on the circle in the opposite segment to D. MO produced meets CA at N. CM produced meets the circle at E.

(i) Show that $BCNM$ is a cyclic quadrilateral. **2**

(ii) Show that $\angle ABN=\angle EBD$. **2**

Student name / number ....

Marks

Question 6 **Begin a new booklet**

(a) Find the coordinates of the point P on the curve $y = x\sqrt{x+3}$ where the tangent is parallel to the x-axis. **2**

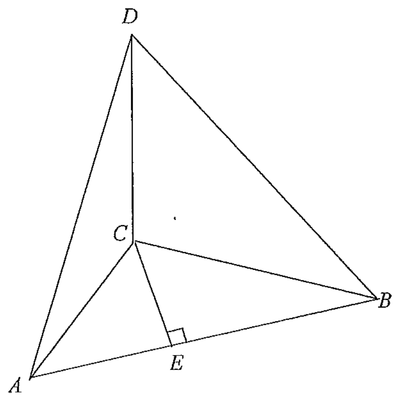
(b) Use the substitution $t = \tan\frac{x}{2}$ to show that $\dfrac{1+\sin x}{1-\cos x} = \cot\frac{x}{2} + \frac{1}{2}\text{cosec}^2\frac{x}{2}$. **3**

(c) Find the number of ways in which 3 boys and 4 girls can be arranged in a straight line so that

(i) the girls occupy both of the end positions. **1**

(ii) boys occupy exactly two of the middle three positions. **2**

(d)



CD is a vertical flagpole of height 10 metres. It stands with its base on horizontal ground. A and B are points on the ground due South and due East of C respectively. The angle of elevation of D is 45° from A and 30° from B. E is the foot of the perpendicular from C to AB.

(i) Show that $\angle ABC = 30°$. **2**

(ii) Find the angle of elevation of D from E correct to the nearest minute. **2**

Question 1

a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• factors denominators then takes common denominator	1
• simplifies by cancelling common factor	1

Answer

$$\frac{1}{p^2-pq}-\frac{1}{pq-q^2}=\frac{1}{p(p-q)}-\frac{1}{q(p-q)}=\frac{q-p}{pq(p-q)}$$

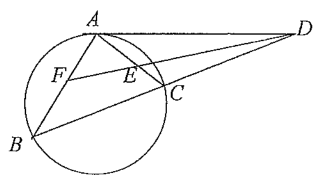
$$\therefore \frac{1}{p^2-pq}-\frac{1}{pq-q^2}=\frac{-(p-q)}{pq(p-q)}=\frac{-1}{pq}$$

b. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes alternate segment theorem	1
ii • uses exterior angle theorem (or angle sum) for appropriate triangles to deduce $\angle AEF=\angle AFE$	1
• uses appropriate triangle property to deduce $AE=AF$	1

Answer



i. $\angle DAC=\angle ABC$ since *the angle between a tangent and a chord drawn to the point of contact is equal to the angle subtended by that chord in the alternate segment.*

ii. In ΔBFD, $\angle AFD=\angle FBD+\angle FDB$ (*ext. ∠ is sum of int. opp. ∠'s*)

$\therefore \angle AFE=\angle ABC+\angle FDB$ (*∠AFD same as ∠AFE, ∠FBD same as ∠ABC*)

Similarly in ΔAED, $\angle AEF=\angle DAE+\angle ADE=\angle DAC+\angle ADF$

But $\angle ADF=\angle FDB$ (*given DF is bisector of ∠BDA*) and $\angle DAC=\angle ABC$ (*from* i.)

$\therefore \angle AEF=\angle AFE$

$\therefore AE=AF$ (*in ΔAEF, sides opp. equal ∠'s are equal*)

c. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • uses remainder theorem with division by x to deduce $b=2$	1
ii • uses remainder theorem to find quadratic equation for a	1
• finds possible values of a	1

1c. Answer

i. $P(x)=x^3+a^2x^2+ax+b$. Using remainder theorem, $P(0)=2$ $\therefore b=2$

ii. $P(-1)=13 \Rightarrow -1+a^2-a+2=13$ $\therefore a^2-a-12=0$

$(a-4)(a+3)=0$. $\therefore a=4$ or $a=-3$

d. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
i • finds value of R	1
• finds value of α	1
ii • finds one value of x	1
• finds second value of x	1

Answer

i. $3\cos x+\sin x=\sqrt{10}\left(\frac{3}{\sqrt{10}}\cos x+\frac{1}{\sqrt{10}}\sin x\right)$

$=\sqrt{10}\left(\sin\alpha\cos x+\cos\alpha\sin x\right)$ where $0°<\alpha<90°$ and $\tan\alpha=3 \Rightarrow \alpha\approx 71\cdot57°$.

$=\sqrt{10}\sin(x+\alpha)$

ii. $3\cos x+\sin x=-2$, $0°\le x\le 360°$

$\sqrt{10}\sin(x+\alpha)=-2$

$\sin(x+\alpha)=-\frac{2}{\sqrt{10}}$

$\therefore x+\alpha\approx 180°+39\cdot23°$, $360°-39\cdot23°$

$x\approx 180°+39\cdot23°-71\cdot57°$, $360°-39\cdot23°-71\cdot57°$

$x\approx 148°$, $249°$

Question 2

a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• finds expression for discriminant	1
• notes discriminant positive to deduce result	1

Answer

$x^2+(k+2)x+k=0$ $\therefore \Delta=(k+2)^2-4k=k^2+4>0$ for all real k.

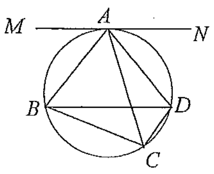
Hence equation has two real roots for all real k.

b. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
• explains why $\angle BCA=\angle BDA$	1
• explains why $\angle BDA=\angle NAD$	1
• explains why $\angle NAD=\angle ACD$	1

2b. Answer



$\angle BCA = \angle BDA$ (*$\angle$'s subtended at the circumference by the same arc AB are equal*)
$\angle BDA = \angle NAD$ (*alt. $\angle$'s with parallel lines are equal*)
$\angle NAD = \angle ACD$ (*$\angle$ between tangent and chord drawn to point of contact is equal to $\angle$ subtended by the chord in the alternate segment*)
$\therefore \angle BCA = \angle ACD$ and hence CA bisects $\angle BCD$.

c. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• uses double angle formula to rearrange as quadratic equation in $\cos x$	1
• solves this quadratic equation for $\cos x$	1
• writes three solutions for x	1

Answer

$$\cos 2x + 3\cos x + 2 = 0,\quad 0° \le x \le 360°$$
$$2\cos^2 x + 3\cos x + 1 = 0$$
$$(2\cos x + 1)(\cos x + 1) = 0$$
$$\therefore \cos x = -\tfrac{1}{2} \quad \text{or} \quad \cos x = -1$$
$$\therefore x = 120°,\ 180°,\ 240°$$

d. Outcomes assessed : PE3, PE6

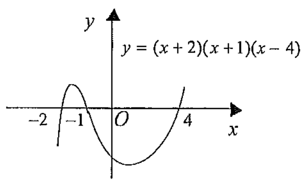
Marking Guidelines

Criteria	Marks
i • uses the factor theorem to find one linear factor	1
• finds the two remaining linear factors	1
ii • shows that the given inequality is equivalent to $x^3 - x^2 - 10x - 8 \ge 0$	1
• solves this inequality	1

Answer

i. Let $P(x) = x^3 - x^2 - 10x - 8$. Then $P(-1) = 0 \Rightarrow (x+1)$ is a factor
$$x^3 - x^2 - 10x - 8 = (x+1)(x^2 - 2x - 8)$$
$$= (x+1)(x-4)(x+2)$$

ii. $x^2 + 8 > 0$ for all x
$$\therefore \frac{x^3 - 10x}{x^2 + 8} \ge 1 \quad \Leftrightarrow \quad x^3 - 10x \ge x^2 + 8$$
$$x^3 - x^2 - 10x - 8 \ge 0$$
$$(x+2)(x+1)(x-4) \ge 0$$



By inspection of the graph, $-2 \le x \le -1$ or $x \ge 4$.

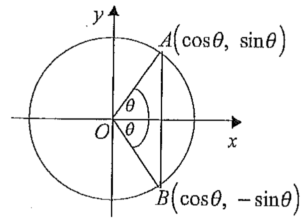
Question 3

a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• uses the cosine rule to express AB^2 in terms of $\cos 2\theta$	1
• expresses AB in terms of $\sin\theta$ and deduces the required result	1

Answer



Using the cosine rule in ΔAOB
$$AB^2 = 1^2 + 1^2 - 2.1.1.\cos 2\theta$$
$$= 2(1 - \cos 2\theta)$$
But $AB = 2\sin\theta$
$$\therefore 4\sin^2\theta = 2(1 - \cos 2\theta)$$
$$\therefore \cos 2\theta = 1 - 2\sin^2\theta$$

b. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• finds and simplifies the x coordinate of P	1
• finds an expression for the y coordinate of P	1
• gives y coordinate in simplest exact form	1

Answer

$A\left(8, \sqrt{50}\right) \qquad B\left(1, \sqrt{18}\right)$

$3 \quad : \quad -1$

$$P\left(\frac{3\times 1 + (-1)\times 8}{3 + (-1)},\ \frac{3\times\sqrt{18} + (-1)\times\sqrt{50}}{3 + (-1)}\right)$$
$$\therefore P\left(\frac{-5}{2},\ \frac{9\sqrt{2} - 5\sqrt{2}}{2}\right)$$
$$P\left(-\frac{5}{2},\ 2\sqrt{2}\right)$$

c. Outcomes assessed : P4, PE3

Marking Guidelines

Criteria	Marks
i • uses product of gradients of PF and QF is -1 to deduce result	1
ii • writes quadratic equation for q	1
• solves for q	1

Answer

i. PF has gradient $\dfrac{a(p^2-1)}{2ap} = \dfrac{(p^2-1)}{2p}$ But $\angle PFQ = 90°$. $\therefore$ *gradient PF . gradient QF* $= -1$

$$\therefore \frac{(p^2-1)}{2p}\cdot\frac{(q^2-1)}{2q} = -1. \qquad \text{Hence } (p^2-1)(q^2-1) + 4pq = 0$$

ii. $p = 2 \Rightarrow 3(q^2 - 1) + 8q = 0$
$$\therefore 3q^2 + 8q - 3 = 0$$
$$(3q-1)(q+3) = 0 \qquad \therefore q = -3 \text{ or } q = \tfrac{1}{3}$$

3d. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • states only one such combination	1
ii • writes correct answer	1
iii • counts ways of selecting three odds and one even	1
• adds to this the number of ways of selecting four odds	1

Answer

i. 2, 4, 6, 8 must be chosen. Hence only one way.

ii. ${}^9C_4 - 1 = 125$ ways

iii. Four odd *or* three odd and one even $\therefore {}^5C_4 + {}^5C_3 \times {}^4C_1 = 45$ ways

Question 4

a. Outcomes assessed : P5

Marking Guidelines

Criteria	Marks
• states domain as inequality for x	1
• states range as inequality for y	1

Answer

Domain : $x^2 \le 4 \quad \therefore \{x : -2 \le x \le 2\}$ Range : $\{y : 0 \le y \le 6\}$

b. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
i • finds exact value of $\tan\alpha$	1
• finds exact value of $\tan\beta$	1
ii • finds value of $\tan 2\alpha$ and deduces result	1

Answer

i. $\tan\alpha = \left|\dfrac{1-\frac{1}{2}}{1+1\times\frac{1}{2}}\right| = \frac{1}{3}$ $\qquad \tan\beta = \left|\dfrac{7-1}{1+7\times 1}\right| = \frac{3}{4}$

ii. $\tan 2\alpha = \dfrac{2\times\frac{1}{3}}{1-(\frac{1}{3})^2} = \frac{3}{4}$ and $0 < \tan\alpha < 1$, α acute $\Rightarrow 0 < \alpha < 45°$ $\therefore 2\alpha$ is also acute.

But there is only one acute angle with value tan ratio equal to $\frac{3}{4}$. Hence $\beta = 2\alpha$.

c. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • recognises exactly one pattern of vowels and consonants and counts arrangements	1
ii • groups the consonants then counts arrangements	1
iii • counts arrangements by subtraction from total without restriction	1

4c. Answer

i. Pattern for consonants * and vowels ° must be * ° * ° *. Hence $3! \times 2! = 12$ arrangements.

ii. Arrange [N G L], A, E in 3! ways, then arrange the three consonants in 3! ways.

Hence $3! \times 3! = 36$ arrangements.

iii. $5! - 12 - 36 = 72$ arrangements.

d. Outcomes assessed : PE3, PE4, PE5

Marking Guidelines

Criteria	Marks
i • shows that $\dfrac{dy}{dx} = t$	1
• deduces the gradient and hence the equation of the normal at T	1
ii • finds the coordinates of X and Y	1
• deduces the required ratio directly or by using intercepts on parallel lines	1

Answer

i. $x = 2at \qquad y = at^2$

$\dfrac{dx}{dt} = 2a \qquad \dfrac{dy}{dt} = 2at$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt} = t$

Hence normal at T has gradient $-\frac{1}{t}$ and equation

$y - at^2 = -\frac{1}{t}(x - 2at)$

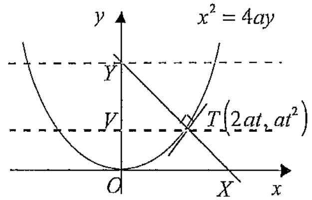
$ty - at^3 = -x + 2at$

$x + ty = 2at + at^3$

ii. $X(2at + at^3, 0)$ and $Y(0, 2a + at^2)$

Consider the set of three horizontal lines through X, T and Y. The intercepts made by this set of parallel lines on the transversals XY and OY are in proportion.

$\therefore \dfrac{TX}{TY} = \dfrac{OV}{VY} = \dfrac{at^2}{(2a + at^2) - at^2} = \dfrac{t^2}{2}$



Question 5

a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• uses the formula for the distance between points to obtain a relation between x and y	1
• simplifies the equation of the locus of P.	1

Answer

$AP = 2\,OP \quad \therefore AP^2 = 4\,OP^2$

Hence $(x-3)^2 + (y-3)^2 = 4(x^2 + y^2)$.

$3x^2 + 6x + 3y^2 + 6y = 18$

$x^2 + 2x + y^2 + 2y = 6$

$(x+1)^2 + (y+1)^2 = 8$

5b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • recognises product is $P(1)$ and evaluates	1
ii • rewrites each bracket using sum of zeros	1
• recognises product is $P(-2)$ and evaluates	1

Answer

$P(x) = x^3 + 2x^2 - 4x - 1 = (x-\alpha)(x-\beta)(x-\gamma)$

i. $(1-\alpha)(1-\beta)(1-\gamma) = P(1)$

$= -2$

ii. $\beta+\gamma = (\alpha+\beta+\gamma) - \alpha = -2-\alpha$ and similarly for $\gamma+\alpha, \quad \alpha+\beta$

$\therefore (\beta+\gamma)(\gamma+\alpha)(\alpha+\beta) = (-2-\alpha)(-2-\beta)(-2-\gamma)$

$= P(-2)$

$= 7$

c. Outcomes assessed : P4, PE3

Marking Guidelines

Criteria	Marks
i • finds coordinates of M in terms of t	1
• completes the square for the y coordinate, then eliminates t to find required equation	1
ii • rearranges equation into standard parabolic form then deduces coordinates of focus	1

Answer

i. $P(2at, at^2)$ and $Q\left(\dfrac{-2a}{t}, \dfrac{a}{t^2}\right)$. $\quad \therefore M\left(a(t-\frac{1}{t}), \ \frac{1}{2}a(t^2+\frac{1}{t^2})\right)$

Hence at M, $y = \frac{1}{2}a\left\{\left(t-\frac{1}{t}\right)^2+2\right\} = \frac{1}{2}a\left\{\dfrac{x^2}{a^2}+2\right\} = \dfrac{x^2}{2a}+a$. $\quad \therefore x^2 = 2ay - 2a^2$

ii. $x^2 = 2a(y-a)$ is the equation of a parabola with vertex $(0, a)$ and focal length $\frac{1}{2}a$.

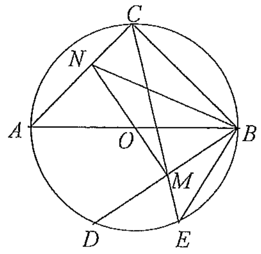
Hence focus has coordinates $\left(0, \frac{3}{2}a\right)$.

d. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • explains why one of $\angle ACB$, $\angle NMB$ is a right angle	1
• explains why the other angle is also a right angle and deduces $BCNM$ is cyclic	1
ii • deduces that $\angle ACE = \angle ABE$, $\angle NCM = \angle NBM$	1
• completes deduction that $\angle ABN = \angle EBD$	1

5d. Answer



i. $\angle ACB = 90°$ (*$\angle$ in a semi circle is a right angle*)
$\angle NMB = 90°$ (*line through centre of circle bisecting a chord is perpendicular to that chord*)
$\therefore BCNM$ is a cyclic quadrilateral. (*one pair of opp. $\angle$'s supplementary*)

ii. Considering circles ABC and $BCNM$,
$\angle ACE = \angle ABE$ (*$\angle$'s subtended at the circumference of a circle by same arc are equal*)
Similarly $\angle NCM = \angle NBM$.
Then $\angle ABN = \angle NBM - \angle DBA = \angle NCM - \angle DBA$
But $\angle NCM = \angle ACE$.
$\therefore \angle ABN = \angle ACE - \angle DBA = \angle ABE - \angle DBA = \angle EBD$

Question 6

a. Outcomes assessed : PE5

Marking Guidelines

Criteria	Marks
• uses product rule to find $\dfrac{dy}{dx}$	1
• solves $\dfrac{dy}{dx} = 0$ to find the coordinates of P.	1

Answer

$y = x\sqrt{x+3}$

$\dfrac{dy}{dx} = 1.(x+3)^{\frac{1}{2}} + x.\frac{1}{2}(x+3)^{-\frac{1}{2}}$

$= \frac{1}{2}(x+3)^{-\frac{1}{2}}\{2(x+3)+x\}$

$= \dfrac{3(x+2)}{2\sqrt{x+3}}$

Tangent is parallel to x-axis when $\dfrac{dy}{dx} = 0$

$\therefore x = -2$ and $P(-2, -2)$

b. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• writes $\sin x$ and $\cos x$ in terms of t	1
• writes LHS in terms of t, simplifies then rearranges in terms of the reciprocal of t	1
• substitutes $\cot\frac{x}{2}$ for $\frac{1}{t}$ then uses a trig. identity to complete proof.	1

Answer

$1+\sin x = 1 + \dfrac{2t}{1+t^2} = \dfrac{1+t^2+2t}{1+t^2}$

$1-\cos x = 1 - \dfrac{1-t^2}{1+t^2} = \dfrac{2t^2}{1+t^2}$

$\therefore \dfrac{1+\sin x}{1-\cos x} = \dfrac{2t+1+t^2}{2t^2}$

$= \frac{1}{t} + \frac{1}{2}\left(\frac{1}{t^2}+1\right)$

$\therefore \dfrac{1+\sin x}{1-\cos x} = \cot\frac{x}{2} + \frac{1}{2}\left(\cot^2\frac{x}{2}+1\right)$

$= \cot\frac{x}{2} + \frac{1}{2}\mathrm{cosec}^2\frac{x}{2}$

6c. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • counts arrangements	1
ii • counts ordered selections of students in middle three positions	1
• multiplies by number of arrangements of remaining children to count possible arrangements	1

Answer

i. Select and arrange the two girls on the ends, then arrange those remaining in $4 \times 3 \times 5! = 1440$ ways

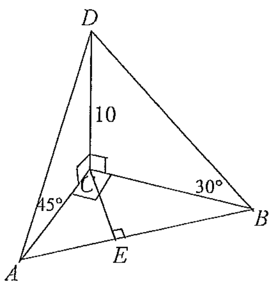
ii. Select and arrange the two boys and one girl to occupy the middle three positions in ${}^3C_2 \times {}^4C_1 \times 3!$ ways
Now arrange the remaining 4 students in the other positions in 4! ways
$\therefore {}^3C_2 \times {}^4C_1 \times 3! \times 4! = 1728$ possible arrangements.

d. Outcomes assessed : P4, PE1

Marking Guidelines

Criteria	Marks
i • finds the exact length of BC	1
• uses $AC = 10$ and trigonometry in right triangle ACB to show $\angle ABC = 30°$	1
ii • finds exact length CE	1
• finds the value of $\tan \angle DEC$ in right triangle DCE and hence $\angle DEC$	1

Answer



i. In ΔBCD, $BC = 10\cot 30° = 10\sqrt{3}$
In ΔACD, $AC = 10\cot 45° = 10$
In ΔACB, $\tan \angle ABC = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$
$\therefore \angle ABC = 30°$

ii. In ΔCBE, $CE = BC \sin 30° = 5\sqrt{3}$
In ΔDCE, $\tan \angle DEC = \frac{DC}{CE} = \frac{10}{5\sqrt{3}} = \frac{2}{\sqrt{3}}$
$\therefore \angle DEC \approx 49°6'$ (to the nearest minute)
is the size of the angle of elevation of D from E.