NSW INDEPENDENT SCHOOLS

2010

Higher School Certificate

Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NAME / NUMBER……………………

STUDENT NAME/NUMBER________________

Total marks – 84
Attempt Questions 1 - 7
All questions are of equal value.

Start each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1. (12 marks)

(a) Evaluate $\sqrt[3]{6.91\times10^{-5}}$ correct to three significant figures. **2**

(b) Factorize $2x^2+x-28$. **2**

(c) Simplify $\frac{2x+3}{3}-\frac{x+2}{4}$. **2**

(d) Express $\left(2\sqrt{3}+1\right)\left(2-\sqrt{3}\right)$ in the form $a\sqrt{3}+b$. **2**

(e) A pair of jeans were discounted by 15% to a selling price of \$63.75. Find the original marked price of the jeans before the discount was applied. **2**

(f) Solve the following inequality. $|3x-5|\geq 2$. **2**

Question 2. (12 marks) Start a new writing booklet. **Marks**

(a) Solve the equation $\frac{2x-1}{5}=\frac{3x+2}{4}$. **2**

(b) Simplify $\frac{x^2-9}{x^2+x-12}$. **2**

(c) Consider $f(x)=\begin{cases}-3 & if\ x\leq -1\\ 2x-3 & if\ x>-1\end{cases}$. Evaluate $f(-1)+f(1)$. **1**

(d) Determine whether the function $f(x)=\frac{1}{x^2-4}$ is odd, even or neither odd nor even. WORKING MUST BE SHOWN. **1**

(e) **Sketch** graphs of the following functions and state the **domain** of each.

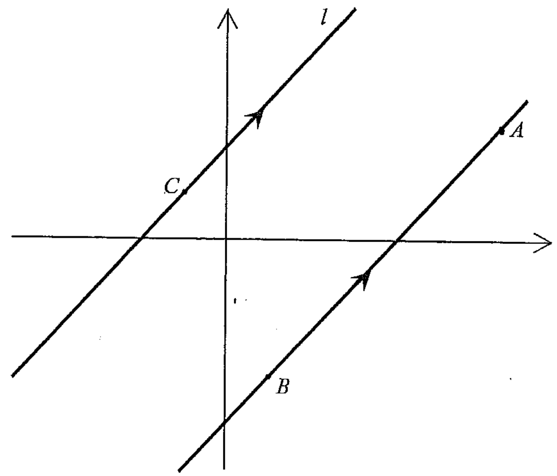
(i) $y=\frac{3}{2x-1}$. **2**

(ii) $y=|2-3x|$. **2**

(f) Solve $2\sin^2 x=1$ where $-180°\leq x\leq 180°$. **2**

Question 3. (12 marks) Start a new writing booklet. **Marks**

(a)



NOT TO SCALE

The line l passes through $C(-1,\ 2)$ and has equation $y=2x+4$.
The point B has coordinates $(1,\ -6)$ and the line AB is parallel to line l.
Copy the diagram into your examination booklet writing the coordinates of B and C onto this diagram.

(i) Find the length of the interval BC. **1**

(ii) Find the midpoint of BC. **1**

(iii) Write down the slope of the line l and find the angle l makes with the positive x–axis. **2**

(iv) Show that AB has equation $y=2x-8$. **1**

(v) If P is a point which lies on AB and on the line $y=2$, find the coordinates of P. **1**

(vi) Find the perpendicular distance of P from the line l. **2**

(vii) Find the size of $\angle ABC$ to the nearest minute. **1**

Question 3 continued on next page.

Question 3. continued. **Marks**

(b) A regular polygon has interior angles measuring $156°$. **1**
How many sides does the polygon have?

(c) The curve $y = ax^2 - 2x - 14$ has a gradient of 10 when $x = 2$. Find the value of a. **2**

Question 4. (12 marks) Start a new writing booklet. **Marks**

(a) Consider $f(x) = x^2 - 5x$

(i) Using $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ or otherwise differentiate $f(x)$ from **first principles** to show that $f'(x) = 2x - 5$. **2**

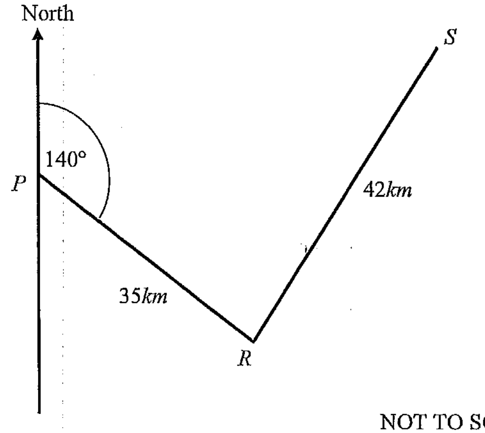
(ii) Find the gradient of the tangent when $x = 1$. **1**

(iii) Find the equation of the normal through the point $(1, -4)$. **2**

(b) Find the exact value of $\cot 330°$. **2**

Question 4. continued. **Marks**

(c)



NOT TO SCALE

A tourist drives $35km$ from the town of Pine Vale (P) on a bearing of $140°$ T to the town of Radiatagrove (R).
He then drives $42km$ on a bearing of $38°$ T to the town of Spruceville (S).

Copy this **diagram** into your writing booklet.

(i) Show that $\angle PRS = 78°$. **1**

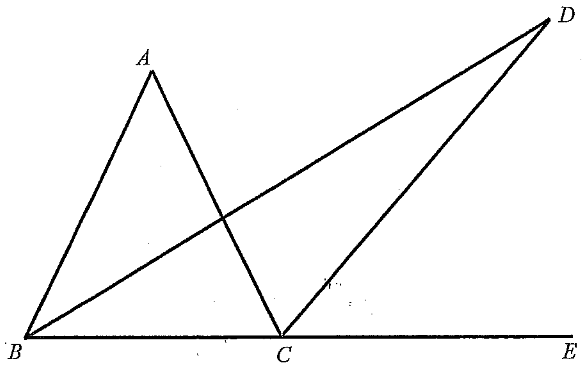
(ii) Show that the distance from Spruceville to Pine Vale (SP) is $49km$, correct to the nearest kilometre. **1**

(iii) Show the size of $\angle SPR = 57°$ to the nearest degree. **1**

(iv) Hence, or otherwise, find the bearing of Pine Vale from Spruceville. Show all necessary working.. **2**

Question 5. (12 marks) Start a new writing booklet. **Marks**

(a)



NOT TO SCALE

ABC is an isosceles triangle in which $AB = AC$ and $\angle BAC = 64°$.
BC is produced to E. BD bisects $\angle ABC$ and CD bisects $\angle ACE$.

Copy or trace the diagram into your writing booklet and mark on it all the given information.

(i) Find the size of $\angle ABC$ giving reasons. **1**

(ii) Find the size of $\angle BDC$ giving reasons. **2**

(b) By expressing $\sec\theta$ and $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$, show that **1**

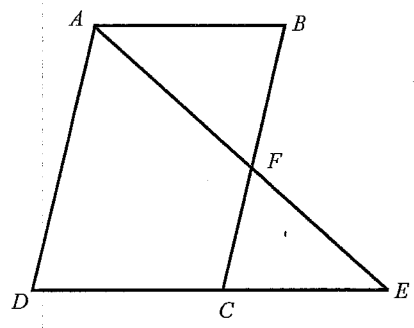
$$\sec^2\theta - \tan^2\theta = 1.$$

(c) If $\sin\theta = -\frac{4}{11}$ and $\tan\theta > 0$ find the exact value of $\cos\theta$. **2**

(d) Given the equation $3x^2 + 7x - 4 = 0$ has roots α and β, without finding α or β evaluate $\alpha^2 + \beta^2$. **3**

Question 5. continued. **Marks**

(e)



$ABCD$ is a parallelogram. DC is produced to E. AE cuts BC at F.
$AD = 16cm$, $CE = 9cm$ and $BF = 10cm$.

(i) Prove that $\triangle ABF$ is similar to $\triangle ECF$. **2**

(ii) Find AB. **1**

Question 6. (12 marks) Start a new writing booklet. **Marks**

(a) For the parabola $(x-2)^2 = 8(y+3)$

(i) Find the coordinates of the vertex. **1**

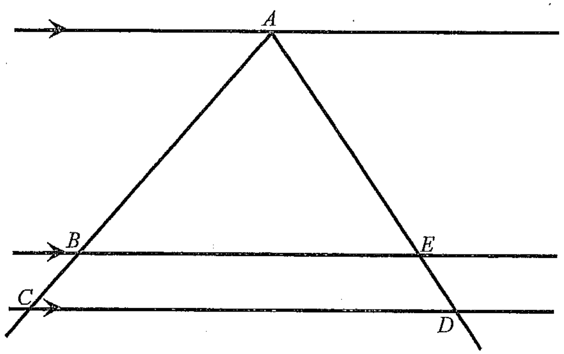
(ii) Find the value of the focal length. **1**

(iii) Find the coordinates of the focus. **1**

(iv) Find the equation of the directrix. **1**

(v) Sketch the parabola labelling the vertex, focus and directrix. **1**

(b)



NOT TO SCALE

$AB = 7cm$, $BC = 4cm$, $ED = 6cm$. Find AD giving reasons. **2**

(c) Express $9x^2 + 2x - 5$ in the form $ax(x+1) + b(x+1) + c$. **3**

(d) For what values of k will the expression $kx^2 - 4x + k$ always be positive? **2**

Question 7. (12 marks) Start a new writing booklet. **Marks**

(a) Evaluate $\lim_{x \to \infty} \frac{3x^2 - 2x - 6}{4 - 2x^2}$. **1**

(b) Find the value of k for which the equation $x^2 - (k+4)x + (k-3) = 0$ has

(i) one root equal to -2. **1**

(ii) roots which are reciprocals of each other. **1**

(iii) roots which are equal in absolute value but opposite in sign. **1**

(c) Find all real numbers x which satisfy the equation $x^4 = 8(x^2 + 6)$. **2**

(d) Differentiate

(i) $\frac{3x^2 - 5}{2x + 1}$. **2**

(ii) $\sqrt[5]{(2x+7)^2}$. **2**

(e) Find, as a relationship between a, b and c, the condition for the quadratic equation in x **2**

$$(a^2 - b^2)x^2 + 2b(a-c)x + (b^2 - c^2) = 0$$

to have equal roots. Simplify your answer as far as possible.

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2010
MATHEMATICS – YR 11 PRELIMINARY
EXAMINATION - MARKING GUIDELINES

Question 1

a) $\sqrt[3]{6.91\times10^{-5}} = 0.4103564$ (1)
$= 0.0410$ (1)

b) $2x^2+x-28 = 2x^2+8x-7x-28$
$= 2(x+4)-7(x+4)$ (1)
$= (2x-7)(x+4)$ (1)

c) $\frac{2x+3}{3}-\frac{x+2}{4} = \frac{4(2x+3)-3(x+2)}{12}$ (1)
$= \frac{8x+12-3x-6}{12}$
$= \frac{5x+6}{12}$ (1)

d) $(2\sqrt3+1)(2-\sqrt3) = 4\sqrt3-6+2-\sqrt3$ (1)
$= 3\sqrt3-4$ (1)

e) $85\%\times Cost = \$63.75$ (1)
$Cost = 63.75\div0.85$
$Cost = \$75.00$ (1)

f) $|3x-5|\ge2$
$3x-5\ge2$
$3x\ge7$
$x\ge\frac{7}{3}$ (1)
OR
$-(3x-5)\ge2$
$-3x+5\ge2$
$-3x\ge-3$
$x\le1$ (1)
$\therefore x\le1$ or $x\ge2\frac{1}{3}$

Question 2

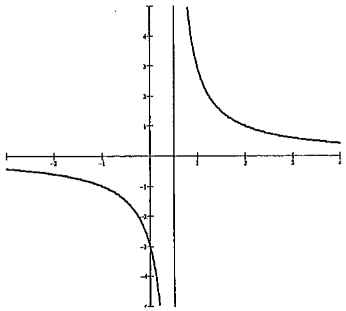
a) $\frac{2x-1}{5}=\frac{3x+2}{4}$
$4(2x-1)=5(3x+2)$
$8x-4=15x+10$ (1)
$-14=7x$
$x=-2$ (1)

b) $\frac{x^2-9}{x^2+x-12}=\frac{(x-3)(x+3)}{(x+4)(x-3)}$ (1)
$=\frac{x+3}{x+4}$ (1)

c) $f(-1)=-3$
$f(1)=2(1)-3$
$=-1$
$f(-1)+f(1)=-3+-1$
$=-4$ (1)

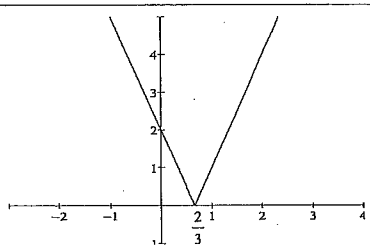
d) $f(x)=\frac{1}{x^2-4}$
$f(a)=\frac{1}{a^2-4}$
$f(-a)=\frac{1}{(-a)^2-4}$
$=\frac{1}{a^2-4}$
$=f(a)$
$\therefore$ *even function* (1)

e) (i)



Domain:- All real x, but $x\ne\frac{1}{2}$ (2)

e) (ii)

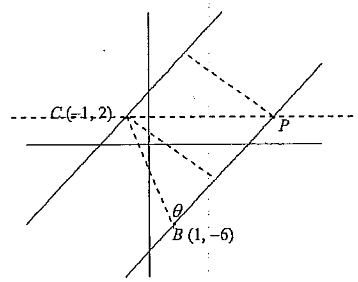


Domain:- All Real x. (2)

f) $\sin^2x=\frac{1}{2}$
$\sin x=\pm\frac{1}{\sqrt2}$ (2)
$x=45°,135°,-45°,-135°$

Question 3

(a)



(i) $BC=\sqrt{(1--1)^2+(-6-2)^2}$
$=\sqrt{2^2+(-8)^2}$
$=\sqrt{4+64}$
$=\sqrt{68}$ (1)
$=2\sqrt{17}$

(ii) Midpoint $=\left(\frac{1+-1}{2},\frac{-6+2}{2}\right)$
$=\left(\frac{0}{2},\frac{-4}{2}\right)$
$=(0,-2)$ (1)

(iii) $m=2$ (1)
$\tan\theta=2$
$\theta=63°26'$ (1)

(iv) $y+6=2(x-1)$ (1)
$y+6=2x-2$
$y=2x-8$ MUST SHOW

(v) Solve $y=2$ and $y=2x-8$
$2x-8=2$
$2x=10$
$x=5$ (1)
$P\ (5,2)$

(vi) $d=\frac{|2(5)-(2)+4|}{\sqrt{1^2+2^2}}$ (1)
$=\frac{|10-2+4|}{\sqrt{1+4}}$
$=\frac{12}{\sqrt5}$ (1)
$=\frac{12\sqrt5}{5}$

(vii) $\sin\theta=\left(\frac{12}{\sqrt5}\div2\sqrt{17}\right)$
$\theta=\sin^{-1}\left(\frac{6}{\sqrt{85}}\right)$
$\theta=40°36'$ (1)

b) $\frac{(n-2)180}{n}=156$
$(n-2)180=156n$
$180n-360=156n$
$24n=360$
$n=15$ (1)

c) $y=ax^2-2x-14$
$y'=2ax-2$ (1)
At $x=2$ $y'=10$
$10=2a(2)-2$
$12=4a$
$a=3$ (1)

Question 4

a) i)

$f(x)=x^2-5x$

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to0}\frac{(x+h)^2-5(x+h)-(x^2-5x)}{h}$ (1)

$=\lim_{h\to0}\frac{x^2+2xh+h^2-5x-5h-x^2+5x}{h}$

$=\lim_{h\to0}\frac{h(2x+h-5)}{h}$ (1)

$=\lim_{h\to0}(2x+h-5)$

$=2x-5$

ii)

$f'(x)=2x-5$

$f'(1)=2(1)-5$

$=-3$ (1)

iii)

Gradient of normal $=\frac{1}{3}$

$y+4=\frac{1}{3}(x-1)$ (1)

$3y+12=x-1$

$x-3y-13=0$ (1)

b)

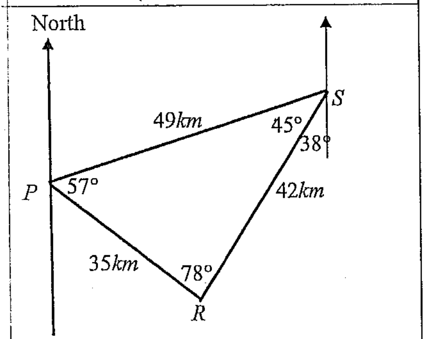
$\cot330^\circ=\frac{1}{\tan330^\circ}$

$=-\frac{1}{\tan30^\circ}$ (1)

$=-\frac{1}{\frac{1}{\sqrt3}}$

$=-\sqrt3$ (1)

c)



i)

$\angle PRS=(180-140)+38$

$=40+38$ SHOW (1)

$=78^\circ$

ii)

$SP^2=35^2+42^2-2\times35\times42\times\cos78$

$SP=\sqrt{35^2+42^2-2\times35\times42\times\cos78}$ (1)

$=49$ km

iii)

$\frac{\sin\theta}{42}=\frac{\sin78}{49}$

$\sin\theta=\frac{\sin78}{49}\times42$ SHOW (1)

$\theta=\sin^{-1}\left(\frac{\sin78}{49}\times42\right)$

$\theta=57^\circ$

OR

$\cos\theta=\frac{49^2+35^2-42^2}{2\times49\times35}$

$=\frac{1862}{3430}$

$\theta=\cos^{-1}\left(\frac{19}{35}\right)$

$=57^\circ$

iv)

$\angle PSR=180-(57+78)$ (1)

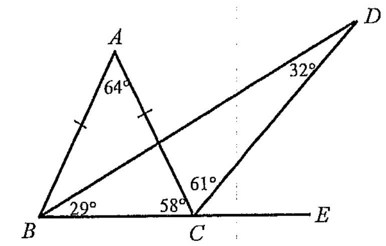
$=45^\circ$

Bearing $=180+38+45$

$=263^\circ$ (1)

Question 5

a)



i)

$\angle ABC=(180-64)\div2$

$=58^\circ$

(base $\angle$'s of isosceles $\triangle$ are equal) (1)

ii)

$\angle DBC=58\div2$ (base $\angle$'s of isosceles $\triangle$

$=29^\circ$ are equal)

$\angle ACB=58^\circ$ (base $\angle$'s of isosceles $\triangle$=)

$\angle ACE=180-58$

$=122^\circ$($\angle$sum of straight line BCE)

$\angle ACD=\frac{1}{2}\angle ACE$

$=\frac{1}{2}\times122$ (1)

$=61^\circ$ (given CD bisects $\angle ACE$)

$\angle BDC=180-(29+(61+58))$

$=180-148$

$=32^\circ$ ($\angle$ sum of $\triangle BCD$) (1)

b)

$\sec^2\theta-\tan^2\theta=1$

$LHS=\sec^2\theta-\tan^2\theta$

$=\frac{1}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$

$=\frac{1-\sin^2\theta}{\cos^2\theta}$ (1)

$=\frac{\cos^2\theta}{\cos^2\theta}$

$=1$

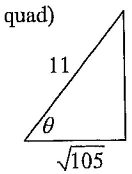
$=RHS$

c)

$\sin\theta=-\frac{4}{11}$ (3rd or 4th quad)

$\tan\theta>0$ (1st or 3rd quad)

$\therefore\theta$ in 3rd quadrant



$\cos\theta=-\frac{\sqrt{105}}{11}$ (1) (1)

d)

$\alpha+\beta=-\frac{7}{3}$, $\alpha\beta=-\frac{4}{3}$ (1 each)

$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

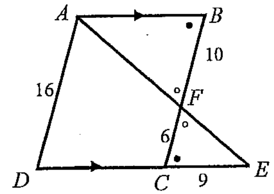
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(-\frac{7}{3}\right)^2-2\left(-\frac{4}{3}\right)$

$=\frac{73}{9}$ (1)

$=8\frac{1}{9}$

e)



i)

$AB\parallel DE$ (opp sides of parallelogram ||)

In $\triangle ABF$ and $\triangle ECB$

$\angle AFB=\angle EFC$ (vertically opposite $\angle$'s =)

$\angle ABF=\angle ECF$ (alternate $\angle$'s $AB\parallel DE$)(1)

$\therefore\triangle ABF\,|||\,\triangle ECF$ (Equiangular) (1)

ii)

$FC=16-10$

$=6$ (opp sides of ||ogram =)

$\frac{AB}{CE}=\frac{BF}{CF}$ (corresponding sides in

$\frac{AB}{9}=\frac{10}{6}$ similar $\triangle$'s are proprtional)

$AB=\frac{10}{6}\times9$

$=15$ (1)

Question 6

e) $(x-2)^2 = 8(y+3)$

(i) Vertex $= (2,-3)$ (1)

(ii) $(x-2)^2 = 4(2)(y+3)$

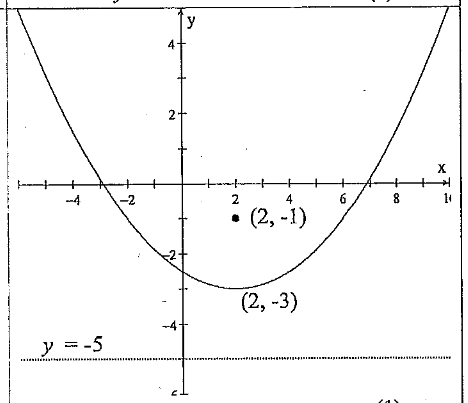
$a = 2$ (1)

(iii) Draw sketch as you go to help answer the question. Get parabola with the correct concavity.

Focus $= (2,-1)$ (1)

(iv) Directrix $y=-5$ (1)

(v)



(1)

b) $\frac{AE}{ED} = \frac{AB}{BC}$ (|| lines cut off intercepts in the same ratio)

$\frac{AE}{6} = \frac{7}{4}$

$AE = \frac{7}{4} \times 6$

$AE = 10\frac{1}{2}$ (1)

$AD = AE + ED$

$= 10\frac{1}{2} + 6$

$= 16\frac{1}{2}$ (1)

c) $9x^2 + 2x - 5 = ax(x+1) + b(x+1) + c$

Let $x=-1$

$9-2-5 = c$

$c = 2$ (1)

$9x^2+2x-5 = ax(x+1)+b(x+1)+2$

Let $x = 0$

$-5 = b+2$

$b = -7$ (1)

$9x^2+2x-5 = ax(x+1) - 7(x+1) + 2$

Let $x = 1$

$9+2-5 = 2a-14+2$

$6 = 2a-12$

$2a = 18$

$a = 9$ (1)

$9x^2+2x-5 = 9x(x+1) - 7(x+1) + 2$

d) $kx^2 - 4x + k$

Positive definite, $k > 0$ (1)

V< 0

$16 - 4k^2 < 0$

$4k^2 > 16$

$k^2 > 4$

$k > 2,\quad k < -2$

$k > 2$ (1)

Question 7

a) $\lim_{x\to\infty} \frac{3x^2-2x-6}{4-2x^2} = \lim_{x\to\infty} \frac{\frac{3\cancel{x^2}}{\cancel{x^2}} - \frac{2\cancel{x}}{x^{\cancel{2}}} - \frac{6}{x^2}}{\frac{4}{x^2} - \frac{2\cancel{x^2}}{\cancel{x^2}}}$

$= \lim_{x\to\infty} \frac{3 - \frac{2}{x} - \frac{6}{x^2}}{\frac{4}{x^2} - 2}$

$= -\frac{3}{2}$ (1)

b) $x^2 - (k+4)x + (k-3) = 0$

Let roots be α and β

$\alpha + \beta = k+4,\quad \alpha\beta = k-3$

i) One root $= -2$

$(-2)^2 - (k+4)(-2) + k - 3 = 0$

$4 + 2k + 8 + k - 3 = 0$

$3k = -9$

$k = -3$ (1)

OR

Let α and -2 be the roots.

$\alpha - 2 = k + 4$

$\alpha = k+6$

$-2\alpha = k-3$

$-2(k+6) = k-3$

$-2k - 12 = k - 3$

$3k = -9$

$k = -3$

ii) Let the roots be α and $\frac{1}{\alpha}$.

Use product of roots.

$1 = k - 3$

$k = 4$ (1)

iii) Let the roots be α and $-\alpha$.

Use the sum of roots.

$0 = k+4$

$k = -4$ (1)

c) $x^4 = 8(x^2+6)$

$x^4 = 8x^2 + 48$

$x^4 - 8x^2 - 48 = 0$

Let $m = x^2$

$m^2 - 8m - 48 = 0$

$(m-12)(m+4) = 0$

$m = 12$ or $m = -4$ (1)

$x^2 = 12$

$x = \pm\sqrt{12}$

$x^2 = -4$ (no real solution)

$\therefore\ x = \pm 2\sqrt{3}$ (1)

d) i) $\frac{d}{dx}\left(\frac{3x^2-5}{2x+1}\right) = \frac{6x(2x+1) - 2(3x^2-5)}{(2x+1)^2}$ (1)

$= \frac{12x^2 + 6x - 6x^2 + 10}{(2x+1)^2}$

$= \frac{6x^2 + 6x + 10}{(2x+1)^2}$ (1)

ii) $f(x) = \sqrt[5]{(2x+7)^2}$

$= (2x+7)^{\frac{2}{5}}$ (1)

$f'(x) = \frac{2}{5}(2x+7)^{-\frac{3}{5}}.2$

$= \frac{4}{5}(2x+7)^{-\frac{3}{5}}$ (1)

$= \frac{4}{5\,.\sqrt[5]{(2x+7)^3}}$

e) Consider the discriminant

V$= 4b^2(a-c)^2 - 4(a^2-b^2)(b^2-c^2)$

$= 4b^2(a^2 - 2ac + c^2) - 4(a^2b^2 - b^4 - a^2c^2 + b^2c^2)$

$= \cancel{4a^2b^2} - 8b^2ac + \cancel{4b^2c^2} \cancel{-4a^2b^2} + 4b^4 + 4a^2c^2 \cancel{-4b^2c^2}$

$= 4(b^4 - 2b^2ac + a^2c^2)$ (1)

$= 4(b^2 - ac)^2$

For equal roots V$= 0$

$4(b^2-ac)^2 = 0$

$b^2 = ac$ (1)