NSW INDEPENDENT SCHOOLS

2012
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in Question 11 – 16
- Write your student number and/or name at the top of every page

Total marks – 100

Section I - Pages 3 – 7

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 8 – 15

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

1. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

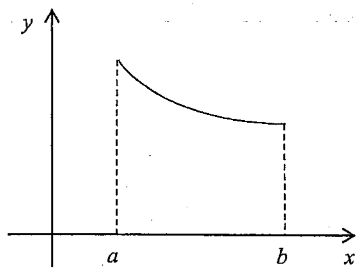
A. $k \geq -3$

B. $k \leq -3$

C. $k \geq 3$

D. $k \leq 3$

2. For the function $y = f(x)$, $a < x < b$ graphed below:



which of the following is true?

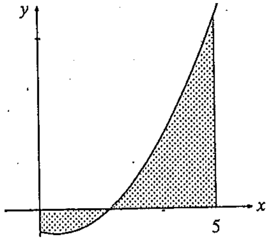
A $f'(x) > 0$ and $f''(x) > 0$

B $f'(x) > 0$ and $f''(x) < 0$

C $f'(x) < 0$ and $f''(x) > 0$

D $f'(x) < 0$ and $f''(x) < 0$

3. Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?



A $A = \left| \int_0^1 (x^2 - x - 2)\,dx \right| + \int_1^5 (x^2 - x - 2)\,dx$

B $A = \int_0^1 (x^2 - x - 2)\,dx + \left| \int_1^5 (x^2 - x - 2)\,dx \right|$

C $A = \left| \int_0^2 (x^2 - x - 2)\,dx \right| + \int_2^5 (x^2 - x - 2)\,dx$

D $A = \int_0^2 (x^2 - x - 2)\,dx + \left| \int_2^5 (x^2 - x - 2)\,dx \right|$

4. Two ordinary dice are rolled. The "score" is the sum of the numbers on the top faces. What is the probability that the score is 9?

A $\frac{3}{4}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{1}{9}$

5. $\sum_{n=3}^{7} 2n + 3 =$

A 17

B 65

C 77

D 91

6. What are the coordinates of the focus of the parabola $4y = x^2 - 8$?

A $(0, -8)$

B $(0, -7)$

C $(0, -2)$

D $(0, -1)$

7. What is the derivative of $\cos^2 3x$ with respect to x?

A $-2\sin 3x \cos 3x$

B $-6\sin 3x \cos 3x$

C $2\sin 3x \cos 3x$

D $6\sin 3x \cos 3x$

8. What is the value of $\int_0^1 \left(e^{3x}-1\right)\,dx$?

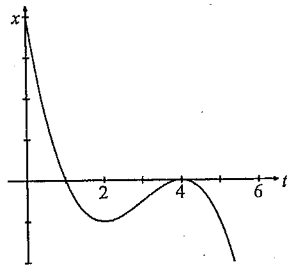
A $\frac{e^3}{3}$

B $\frac{e^3}{3}-1$

C e^3-1

D $\frac{1}{3}\left(e^3-4\right)$

9. The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph.



When was the particle at rest ?

A $t=0$

B $t=1$ and $t=4$

C $t=2$ and $t=4$

D $t=1,\ t=2$ and $t=4$

10. What are the domain and range of the function $f(x)=\sqrt{4-x^2}$?

A Domain: $-2\le x\le 2$, Range: $0\le y\le 2$

B Domain: $-2\le x\le 2$, Range: $-2\le y\le 2$

C Domain: $0\le x\le 2$, Range: $-4\le y\le 4$

D Domain: $0\le x\le 2$, Range: $0\le y\le 4$

Section II

90 marks
Attempt Question 11 – 16
Allow about 2 hours 45 minutes for this section

Answer the questions on your own paper, or writing booklets if provided.
Start each question on a new page.
All necessary working should be shown in every question.

Marks

Question 11 (15 marks)

a) Differentiate:

(i) $x\tan 2x$ **2**

(ii) $e^{4x}+\dfrac{1}{x}$ **2**

(iii) $\dfrac{3x-7}{3+2x}$ **2**

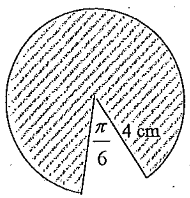
b) Find:

(i) $\int (5x-1)^9\,dx$ **2**

(ii) $\int \sin\dfrac{2x}{3}\,dx$ **2**

c) Solve $2\sin x+\sqrt{3}=0$ for $0\le x\le 2\pi$. **3**

d) Find the exact area of the shaded major sector of the circle with radius 4 cm. **2**



Marks

Question 12 (15 marks)

a) Jeff and Jonas each throw a standard die.

(i) Find the probability that they do *not* throw the same number. **1**

(ii) Find the probability that the number thrown by Jonas is less than the number thrown by Jeff. **1**

b) The sum of the first n terms of an arithmetic series is given by:

$$S_n=n(38-2n)$$

(i) Calculate S_1 and S_2. **2**

(ii) Find the first three terms of this series. **2**

(iii) Find an expression for the n-th term. **2**

c) Consider the lines $2x-y+1=0$ and $x+2y-7=0$ which intersect at point P.

(i) By solving the equations simultaneously verify that point P has coordinates $(1,3)$. **1**

The point A $(3,7)$ lies on the line $2x-y+1=0$ and point B $(-3,5)$ lies on the line $x+2y-7=0$.

(ii) Find the perpendicular distance of A from the line $x+2y-7=0$. **2**

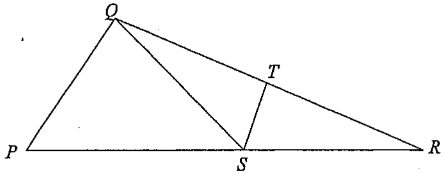
(iii) Draw a number plane showing the position of the points A, B and P and the lines $2x-y+1=0$ and $x+2y-7=0$. **1**

(iv) Show the lines $2x-y+1=0$ and $x+2y-7=0$ are perpendicular. **1**

(v) Determine, using coordinate methods, what type of right angled triangle ABP is. **2**

Question 13 (15 marks) **Marks**

a)



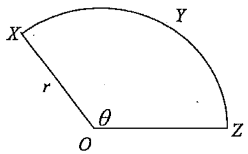
In ΔPQR, point T lies on side QR and point S lies on side PR such that $QT=TR$, $QS=QP$ and $ST\perp QT$.

Copy the diagram into your answer booklet showing all given information.

(i) Prove that $\Delta QTS\equiv\Delta RTS$. **2**

(ii) Prove that $\angle QPS=2\angle TQS$. **2**

(b) **3**



A circular sector $OXYZ$ has radius r cm and $\angle XOZ$ measures θ radians. If the arc XYZ has a length of 10π and $\theta=\frac{2\pi}{3}$ find the exact length of chord XZ.

(c) Consider the function $f(x)=x^4-4x^3$.

(i) Find the stationary points on the curve and determine their nature. **4**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve $f(x)=x^4-4x^3$ showing all important features including the intercepts. **2**

Question 14 (15 marks) **Marks**

a) Peta is saving to buy a new car. She needs $12 700 for the car. The first month she saves $25. The next month she saves $40. The following month she saves $55. **3**

If she continues to increase the amount she saves by $15 each month how many months does it take Peta to save for her car?

(b) The probability that a rare plant grows to maturity in a nursery is $\frac{1}{9}$. **3**

What is the least number of plants that would need to be in the nursery to have at least a 95% chance of one plant reaching maturity?

(c) Use Simpson's Rule with 5 function values to find an approximate value of **3**

$$\int_2^4 \frac{x}{2}\ln x\,dx.$$

(d) (i) Differentiate $\ln(\sin x)$. **2**

(ii) Sketch the curve $y=\cot x$ for $0<x<\pi$ clearly indicating where it crosses the $x-$axis. **2**

(iii) Hence, or otherwise, find the exact area bounded by the curve $y=\cot x$, the $x-$axis and the line $x=\frac{\pi}{4}$. **2**

Question 15 (15 marks)

Marks

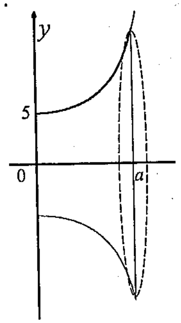
(a) Joe is trying to fill his 1 200 litre water tank. The first day he puts 100 litres into the tank, the 2^{nd} day he puts in 90 litres and the third day he puts in 81 litres. If he continues to add water in this way will he ever fill the tank? Justify your answer. **2**

(b) At the entrance to a mine there are electricity wires. The height (H) of the wires above the ground at the mine entrance varies with the outside temperature (t) and is given by the $H = 21.6 + \dfrac{120}{t-105}$, where H is in metres and t is in degrees Celsius.

(i) Find the height of the wire when the temperature is $-3°C$. **1**

(ii) The company has a piece of equipment which is 13.3 m high. All equipment entering the mine must stay at least 6.7 m below the wires. Find the maximum temperature which will allow the equipment to safely pass under the wire. **2**

(c) Charlemagne is designing a glass fruit bowl with a capacity of 2 litres. He rotates the curve $y = 5\sec\left(\dfrac{x}{6}\right)$ about the $x-$axis as shown on the diagram from $x = 0$ to $x = a$.



(i) Find the height of the bowl in *cm*. Give your answer correct to the nearest cm. Note, the bowl is on its side and the height of the bowl is the distance 0*a*. **3**

(ii) Hence find the diameter of the top of the bowl. **1**

Question 15 continued

Marks

(d) The acceleration of a particle is given by

$$\ddot{x} = 6t - 16,$$

where x is displacement in metres and t is time in seconds. Initially its velocity is 5 m s^{-1} and its displacement is 7 metres to the left of the origin.

(i) Show that the displacement of the particle is given by: **2**

$$x = t^3 - 8t^2 + 5t - 7$$

(ii) Find when and where the particle comes to rest. **3**

(iii) Does the particle ever pass through the origin? Justify your answer. **1**

Marks

Question 16 (15 marks)

(a) A tyre factory was required to install a filter to limit the pollution it emitted into the atmosphere.

The rate of emission, $\frac{dE}{dT}$, in kg per month of pollution from the factory is given by

$$\frac{dE}{dt} = 100 + \frac{300}{1+t}$$

where t is time in months measured from when the factory initially installed the filter.

(i) Find the initial rate of emission $\frac{dE}{dt}$. **1**

(ii) What will the rate of emission $\frac{dE}{dt}$ eventually tend towards? **1**

(iii) Calculate the total amount of pollution emitted by the factory in the first year of operation with the filter. **2**

(b) A radioactive plutonium isotope is decaying and releasing radiation. **2**
The amount of radiation A at time t, in years, is given by

$$A = A_0 e^{-2.89\times 10^{-5}t}$$

where A_0 is the initial amount of radiation.

Find the half-life of plutonium, to the nearest 10 years.

(c) Felicity has just commenced work and is investigating superannuation funds. She calculates that she will need \$1 000 000 in her fund when she retires in 40 years. She finds a fund guaranteeing to pay interest on each deposit at the rate of 6% per annum compounded monthly.
She intends to deposit \$$M$ into the fund at the beginning of each month for 40 years. (480 months).
After 25 years she intends to double the amount she deposits each month. That is, she will deposit a further \$$M$ into the fund for the remaining 15 years. (180 months).

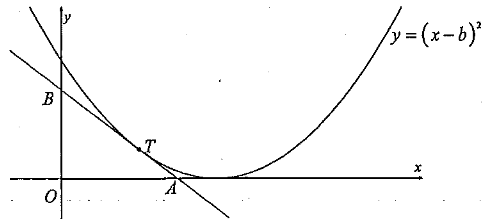
(i) Show that the value of M can be found using the formula **2**
$1000000 = 201M\left(1.005^{480} + 1.005^{180} - 2\right)$

(ii) Find the value of M. **1**

Marks

Question 16 continued

(d) The diagram shows the graph of $y = (x-b)^2$.
$T\left(t,(t-b)^2\right)$ is a point on the curve such that $0 < t < b$.
The tangent at T is drawn to intersect with the x and y axes at the points A and B respectively as shown.



(i) Show that the tangent to the curve at T is given by **2**
$y = (2x - t - b)(t - b)$.

(ii) Find the coordinates of the point T, in terms of b, which will maximise the area of ΔABO. **4**

End of paper

NSW INDEPENDENT TRIAL EXAMS – 2012
MATHEMATICS HSC TRIAL EXAMINATION
MARKING GUIDELINES

Section 1:

Question		Solution	Marks
1)		$x^2-6x-3k=0$ $\Delta\geq 0$ $\Delta=36+12k$ $36+12k\geq 0$ $12k\geq -36$ $k\geq -3$	A
2)		Decreasing function – first derivative negative Concave up – second derivative positive	C
3)		$y=x^2-x-2$ $y=(x-2)(x+1)$ x – intercept, $x=2$	C
4)		$(6,3)(5,4)(4,5)(3,6)$ $P(9)=\frac{4}{36}$ $=\frac{1}{9}$	D
5)		$\sum_{n=3}^{7}2n+3=9+11+13+15+17=65$	B
6)		$4y=x^2-8$ $x^2=4y+8$ $x^2=4(y+2)$ Vertex $(0,-2)$, Concave up. Focal length $a=1$ Focus $(0,-1)$	D
7)		$\frac{d}{dx}\cos^2 3x=2.\cos 3x\times -3\sin 3x$ $=-6\sin 3x\cos 3x$	B
8)		$\int_0^1\left(e^{3x}-1\right)dx=\left[\frac{1}{3}e^{3x}-x\right]_0^1$ $=\left(\frac{e^3}{3}-1\right)-\left(\frac{e^0}{3}-0\right)$ $=\frac{e^3}{3}-1-\frac{1}{3}$ $=\frac{e^3-4}{3}$	D
9)		At rest, stationary point.	C
10)		Domain $4-x^2\geq 0$ Range $0\leq y\leq 2$ $x^2\leq 4$ $-2\leq x\leq 2$	A

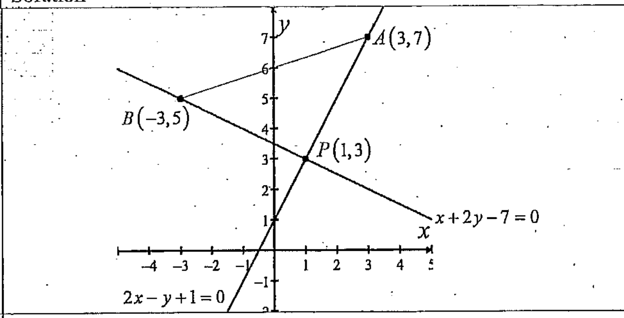
Section II

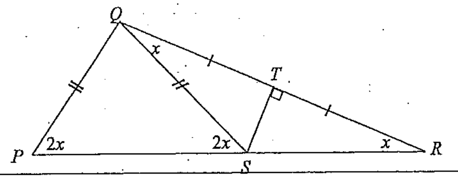
Question		Solution	Marks
11)	a)(i)	$\frac{d}{dx}(x\tan 2x)=\tan 2x+x.2\sec^2 2x$ $=\tan 2x+2x\sec^2 2x$	1 attempt at the product rule and correct differention of $\tan 2x$ 1 correct answer
	a)(ii)	$\frac{d}{dx}\left(e^{4x}+\frac{1}{x}\right)=\frac{d}{dx}\left(e^{4x}+x^{-1}\right)$ $=4e^{4x}-x^{-2}$	1 correct differentiation of exp or neg indice 1 correct answer
	a)(iii)	$\frac{d}{dx}\frac{3x-7}{3+2x}=\frac{3(3+2x)-2(3x-7)}{(3+2x)^2}$ $=\frac{9+6x-6x+14}{(3+2x)^2}$ $=\frac{23}{(3+2x)^2}$	1 for correct quotient rule 1 correct answer
11)	b)(i)	$\int(5x-1)^9\,dx=\frac{(5x-1)^{10}}{5\times 10}+c$ $=\frac{(5x-1)^{10}}{50}+c$	1 correct index 1 correct answer
	b)(ii)	$\int\sin\frac{2x}{3}dx=-\frac{3}{2}\cos\frac{2x}{3}+c$	1 correct integration of cos 1 correct "constant"
11)	c)	$2\sin x+\sqrt{3}=0$ $2\sin x=-\sqrt{3}$ $\sin x=-\frac{\sqrt{3}}{2}$ $x=\pi+\frac{\pi}{3},2\pi-\frac{\pi}{3}$ $x=\frac{4\pi}{3},\frac{5\pi}{3}$	1 correct acute angle $\frac{\pi}{3}$ 1 for $\frac{4\pi}{3}$ 1 for $\frac{5\pi}{3}$
11)	d)	$A=\frac{1}{2}\times\frac{11\pi}{6}\times 4^2$ $=\frac{44\pi}{3}cm^2$	1 for correct angle or correct formula. 1 correct answer

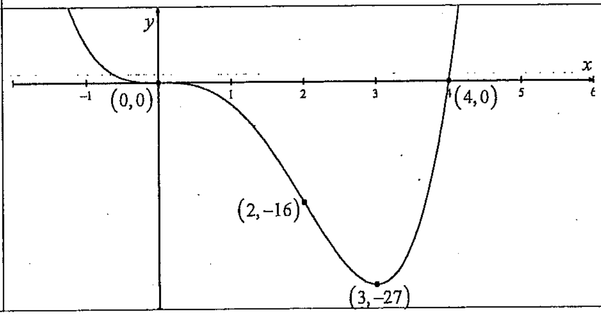
Question		Solution	Marks
12)		(see table below)	

	Jo 1	Jo 2	Jo 3	Jo 4	Jo 5	Jo 6
Je 1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
Je 2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
Je 3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
Je 4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
Je 5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
Je 6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Question		Solution	Marks
	a)(i)	$P(different)=\frac{5}{6}$	**1** correct answer
	a)(ii)	$P(E)=\frac{15}{36}$ $=\frac{5}{12}$	**1** for correct answer
12)	b)(i)	$S_n=n(38-2n)$ $S_1=1(38-2)$ $=36$ $S_2=2(38-4)$ $=2\times34$ $=68$	**1** for correct S_1 **1** for correct S_2
	b)(ii)	$T_2=S_2-S_1$ $=68-36$ $=32$ $d=-4$ $T_3=28$ 36, 32, 28,…	**1** for value of T_2 **1** for value of T_3
	b)(iii)	$T_n=36+(n-1)(-4)$ $=36-4n+4$ $=40-4n$	**1** correct formula **1** correct answer
12)	c)(i)	$2x-y+1=0$ ……… (1) $x+2y-7=0$ ……… (2) $4x-2y+2=0$ ……… (3) = (1)×2 $5x-5=0$ ……… (2)+(3) $5x=5$ $x=1$ $2-y+1=0$ $y=3$	**1** correct answer
	c)(ii)	$d=\frac{\lvert 3+2(7)-7\rvert}{\sqrt{1+4}}$ $=\frac{10}{\sqrt5}$ $=2\sqrt5$	**1** correct d **1** correct answer

Question		Solution	Marks
12)	c)(iii)		**1** correct points on correct lines.
	c)(iv)	$2x-y+1=0$ $y=2x+1$ $m_1=2$ $x+2y-7=0$ $2y=7-x$ $y=\frac{7}{2}-\frac{1}{2}x$ $m_2=-\frac{1}{2}$ $m_1m_2=2\times-\frac{1}{2}$ $=-1$ $\therefore 2x-y+1=0$ and $x+2y-7=0$ are perpendicular	**1** correct reasoning and conclusion
	c)(v)	Given lines are perpendicular $AP=2\sqrt5$ $BP=\sqrt{(1+3)^2+(3-5)^2}$ $=\sqrt{16+4}$ $=\sqrt{20}$ $=2\sqrt5$ $\therefore$ ΔAPB is a right angle isosceles triangle	**1** for right angle triangle **1** for isosceles triangle

Question	Solution	Marks
13) a)		
a)(i)	In ΔQTS and ΔRTS $TS = TS$ common (S) $\angle QTS = \angle RTS$ given $ST \perp QT$ (A) $QT = TR$ given (S) $\therefore \Delta QTS \equiv \Delta RTS$ (SAS)	**1** for 2 pairs correct sides or angles with reasons **1** for 3rd pair of sides or angles plus correct reason for congruence.
a)(ii)	Let $\angle TQS = x$ $\angle TRS = \angle TQS = x$ matching angles in congruent triangles $\angle QSP = \angle TQS + \angle TRS$ external $\angle$ of triangle = sum or 2 opp int $\angle$'s $= 2x$ $\angle QPS = \angle QSP$ base $\angle$'s of isosceles Δ, $QP = QS$ $= 2x$ $= 2\angle TQS$ since we let $\angle TQS = x$	**1** for first 2 steps **1** correct proof
13) b)	$l = \theta r$ $10\pi = \frac{2\pi}{3} r$ $r = 15$ $XZ^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos\left(\frac{2\pi}{3}\right)$ $= 675$ $XZ = \sqrt{675}$ $= 15\sqrt{3}$	**1** correct value of r **1** correct application of cosine rule **1** correct answer

Question	Solution	Marks
13) c)(i)	$f(x) = x^4 - 4x^3$ $f'(x) = 4x^3 - 12x^2$ stationary points when $f'(x) = 0$ $4x^3 - 12x^2 = 0$ $4x^2(x-3) = 0$ $x = 0, 3$ $f(0) = 0$ $f(3) = 3^4 - 4(3)^3$ $= -27$ $SP = (0,0)\ (3,-27)$ test stationary points $f''(x) = 12x^2 - 24x$ $f''(0) = 0$ test concavity x: 0^-, 0, 0^+ $f''(x)$: +ve, 0, -ve change in concavity, $\therefore (0,0)$ horizontal point of inflexion $f''(3) = 12(3)^2 - 24(3)$ $= 36 > 0$ concave up $\therefore$ minimum at $(3,-27)$	**1** for values of x **1** for y values **1** correct identification of horizontal P.O.I **1** correct identification of minimum
c)(ii)	Possible points of inflexion occur when $f''(x) = 0$ $12x^2 - 24x = 0$ $12x(x-2) = 0$ $x = 0, 2$ $f(2) = 2^4 - 4(2)^3$ $= -16$ x: 2^-, 2, 2^+ $f''(x)$: -ve, 0, +ve Change in concavity $\therefore (2,-16)$ is a P.O.I	**1** for x values. **1** for correct test & conclusion. (Origin already tested)
c)(iii)		**1** stationary points and shape for their information **1** correct intercepts and their points of inflexion

Question		Solution	Marks
14)	a)	See working below	See marks below

14) a)

$S = 25+40+55+.......=\$12700$

A.P $a=25$, $d=15$ and $S_n=12700$. Find n.

$$12700=\frac{n}{2}\left(50+(n-1)\times15\right)$$

$$25400=n(50+15n-15)$$

$$=n(35+15n)$$

$$5080=7n+3n^2$$

— 1 for identifying AP and knowing sum of an AP formula.

$$3n^2+7n-5080=0$$

$$n=\frac{-7\pm\sqrt{49-4\times3\times-5080}}{6}$$

$$=\frac{-7\pm\sqrt{61009}}{6}$$

$$=\frac{-7\pm247}{6}$$

— 1 for forming correct quadratic equation

$=40$ months (ignore negative answer)

— 1 correct answer

14) b)

$P(\text{at least }1)=1-P(\text{all dead})$

— 1 for probability of plant dying and statement

$$1-\left(\frac{8}{9}\right)^n\geq0.95$$

$$\left(\frac{8}{9}\right)^n\leq0.05$$

$$\ln\left(\frac{8}{9}\right)^n\leq\ln(0.05)$$

$$n\ln\left(\frac{8}{9}\right)\leq\ln(0.05)$$

— 1 for taking ln of both sides

$$n\geq\ln(0.05)\div\ln\left(\frac{8}{9}\right)$$

$$n\geq25.434..$$

$$n=26$$

You need to have at least 26 plants have at least a 95% chance that one grows to maturity.

— 1 for correct answer

Question		Solution	Marks
14)	c)	See working below	See marks below

14) c)

$f(x)=\frac{x}{2}\ln x$

x	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	$\ln2$	$\frac{5}{4}\ln\left(\frac{5}{2}\right)$	$\frac{3}{2}\ln(3)$	$\frac{7}{4}\ln\left(\frac{7}{2}\right)$	$2\ln(4)$
	0.693	1.1454	1.648	2.192	2.7726

— 1 for correct h

— 1 correct simpson's rule

$$\int_2^4\frac{x}{2}\ln x\,dx \approx \frac{1}{6}\left(0.693+2.7726+4(1.1454+2.192)+2(1.648)\right)$$

$$=3.352$$

— 1 correct answer

14) d)(i)

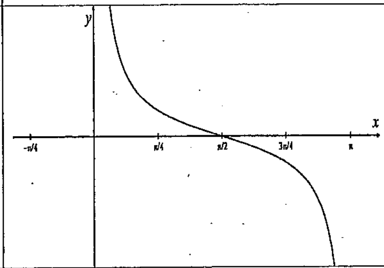
$$\frac{d}{dx}\ln(\sin x)=\frac{\cos x}{\sin x}$$

$$=\cot x$$

— 1 correct differentiation of ln function

— 1 correct differentiation of sin function

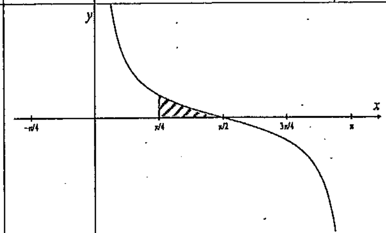
d)(ii)



— 1 for correct shape

— 1 for 1 curve and correct intercept

d)(iii)



$$A=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot x\,dx$$

$$=\left[\ln(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

— 1 for correct integration

$$=\ln\left(\sin\frac{\pi}{2}\right)-\ln\left(\sin\frac{\pi}{4}\right)$$

$$=\ln(1)-\ln\frac{1}{\sqrt2}$$

$$=0-\ln(2)^{-\frac{1}{2}}$$

— 1 for $-\ln\frac{1}{\sqrt2}$

$$=\frac{1}{2}\ln2$$

Question		Solution	Marks
15)	a)	$S=100+90+81+\ldots\ldots$ GP, where $a=100$, $r=\frac{9}{10}$ limiting sum use $S_\infty=\frac{a}{1-r}$ $S=\frac{100}{1-\frac{9}{10}}$ $=1000$ No will never fill the tank as the limiting sum is only 1000 litres	**1** for recognising limiting sum and 1000l. **1** for correct answer and reason
15)	b)(i)	$H=21.6+\frac{120}{t-105}$ $H=21.6+\frac{120}{-3-105}$ $=20.49m$	**1** correct answer
	b)(ii)	$H=21.6+\frac{120}{t-105}$ $20=21.6+\frac{120}{t-105}$ $-1.6=\frac{120}{t-105}$ $t-105=\frac{120}{-1.6}$ $t-105=-75$ $t=30°$	**1** for correct equation **1** correct answer
15)	c)(i)	2 litres equivalent to 2000 cm^3 $2000=\pi\int_0^a\left(5\sec\left(\frac{x}{6}\right)\right)^2 dx$ $=25\pi\int_0^a\sec^2\left(\frac{x}{6}\right)dx$ $\frac{80}{\pi}=\left[6\tan\left(\frac{x}{6}\right)\right]_0^a$ $\frac{80}{6\pi}=\tan\frac{a}{6}-\tan 0$ $\tan\frac{a}{6}=\frac{40}{3\pi}$ $\frac{a}{6}=\tan^{-1}\frac{40}{3\pi}$ $a=6\times1.339397..$ $a=8.03638..$ $a=8cm$ Bowl is 8 cm high.	**1** correct integral **1** for correct integration **1** correct answer
	c)(ii)	$d=2\times5\sec\left(\frac{8}{6}\right)$ $=42.5$ cm wide	**1** correct answer

Question		Solution	Marks
15)	d)(i)	$\ddot{x}=6t-16$ $\dot{x}=3t^2-16t+c$ At $t=0$, $\dot{x}=5$ $5=c$ $\dot{x}=3t^2-16t+5$ $x=t^3-8t^2+5t+k$ At $t=0$, $x=-7$ $-7=k$ $x=t^3-8t^2+5t-7$	**1** mark for velocity equation **1** mark for correctly shown
	d)(ii)	$\dot{x}=3t^2-16t+5$ $0=3t^2-16t+5$ $0=(3t-1)(t-5)$ $t=\frac{1}{3},5$ At $t=\frac{1}{3}$ $x=\left(\frac{1}{3}\right)^3-8\left(\frac{1}{3}\right)^2+5\left(\frac{1}{3}\right)-7$ $=-6\frac{5}{27}$ At $t=5$ $x=(5)^3-8(5)^2+5(5)-7$ $=-57$	**1** mark for correct times. **1** correct first location **1** correct location
	d)(iii)	Yes. Particle starts at $x=-7$, moves to the right for 1/3 of a second, stops at $x=-6\frac{5}{27}$, moves left for a further $4\frac{2}{3}$ seconds, stops at $x=-57$, then moves to the right. It doesn't stop again so it must pass through the origin.	**1**

Question		Solution	Marks
16)	a)(i)	$E=100+\frac{300}{1+t}$ At $t=0$ $E=400kg\,/\,\text{month}$	**1** correct answer
	a)(ii)	As $t\to\infty$, $E\to 100$ kg/month	**1** correct answer
	a)(iii)	$P=\int_0^{12}\left(100+\frac{300}{1+t}\right)dt$ $=\left[100t+100\ln(1+t)\right]_0^{12}$ $=(1200+100\ln 13)-(0+100\ln 1)$ $=1456kg$	**1** correct integral **1** correct answer
16)	b)		**1** for correct expression involving ½ **1** for correct answer

Question		Solution	Marks
16)	c)(i)	$A_1=M(1.005)$ $A_2=M(1.005)^2+M(1.005)$ $A_3=M(1.005)^3+M(1.005)^2+M(1.005)$ $A_{480}=M(1.005)^{480}+\ldots\ldots+M(1.005)^3+M(1.005)^2+M(1.005)$ $=M\left(1.005+1.005^2+1.005^3+\ldots\ldots 1.005^{480}\right)$ $=M\left(\frac{1.005\left(1.005^{480}-1\right)}{1.005-1}\right)$ $=201M\left(1.005^{480}-1\right)$ $B_1=M(1.005)$ $B_2=M(1.005)^2+M(1.005)$ $B_{180}=M(1.005)^{180}+\ldots\ldots+M(1.005)^3+M(1.005)^2+M(1.005)$ $=M\left(\frac{1.005\left(1.005^{180}-1\right)}{1.005-1}\right)$ $=201M\left(1.005^{180}-1\right)$ $1000000=201M\left(1.005^{480}-1\right)+201M\left(1.005^{180}-1\right)$ $=201M\left(1.005^{480}-1+1.005^{180}-1\right)$ $1000000=201M\left(1.005^{480}+1.005^{180}-2\right)$	**1** correct expression **1** correct expression
	c(ii)	$M=\frac{1000000}{201\left(1.005^{480}+1.005^{180}-2\right)}$ $=\$435.97$	**1** correct answer
16)	d)(i)	$y=(x-b)^2$ $y'=2(x-b)$ at $x=t$ $m=2(t-b)$ Equation of tangent. $y-(t-b)^2=2(t-b)(x-t)$ $y=2(t-b)(x-t)+(t-b)^2$ $=(t-b)(2(x-t)+t-b)$ $=(t-b)(2x-2t+t-b)$ $y=(t-b)(2x-t-b)$	**1** for the gradient **1** correctly shown

Question		Solution	Marks
16)	d)(ii)	Coordinates of A. $y=(t-b)(2x-t-b)$ $y=0$ $0=(t-b)(2x-t-b)$ $2x-t-b=0$ $2x=t+b$ $x=\frac{t+b}{2}$ Coordinates of B. $y=(t-b)(2x-t-b)$ $x=0$ $y=(t-b)(-t-b)$ $y=b^2-t^2$	1 for OA & OB
		Area of ΔABO $A=\frac{1}{2}\times\frac{t+b}{2}\times(b^2-t^2)$ $=\frac{1}{4}(t+b)(b^2-t^2)$	1 for area of triangle
		$A'=\frac{1}{4}((b^2-t^2)+-2t(t+b))$ $=\frac{1}{4}(b^2-t^2-2t^2-2tb)$ $=\frac{1}{4}(b^2-3t^2-2tb)$ $=-\frac{1}{4}(3t^2+2bt-b^2)$ $=-\frac{1}{4}(3t-b)(t+b)$ Max or min when $A'=0$	

16)	d)(ii) contd.	$0=-\frac{1}{4}(3t-b)(t+b)$ $3t-b=0$ or $t+b=0$ $3t=b$ $t=-b$ (disregard) $t=\frac{b}{3}$	1 for value of t
		Test $A'=-\frac{1}{4}(3t^2+2tb-b^2)$ $A''=-\frac{1}{4}(6t+2b)$ at $t=\frac{b}{3}$ $A''=-\frac{1}{4}(4b)$ $=-b$ negative $\therefore$ maximum area when $t=\frac{b}{3}$ $y=\left(\frac{b}{3}-b\right)^2$ $=\frac{4b^2}{9}$ $T\left(\frac{b}{3},\frac{4b^2}{9}\right)$	1 for value of y value.