NSW INDEPENDENT SCHOOLS

2011
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ……………………..

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STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x, \quad x > 0$
$\int e^{ax} dx$	$= \frac{1}{a} e^{ax}, \quad a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \quad a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \quad a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \quad a \neq 0$
$\int \sec ax \, \tan ax \, dx$	$= \frac{1}{a} \sec ax, \quad a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value.

Answer the questions on your own paper or writing booklet, if provided.
Start each question on a new page.

Marks

Question 1 (12 marks)

(a) Evaluate $5e^{2.302}$ correct to 3 significant figures. **2**

(b) Factorise fully $3x^2+5x-2$. **2**

(c) Express $\frac{5}{\sqrt{3}+2}$ in the form $a\sqrt{3}+b$. **2**

(d) Solve $|2x-1|<3$. **2**

(e) If $f(x)=2\sin 3x$ find the exact value of $f'\left(\frac{\pi}{18}\right)$. **2**

(f) Express $\frac{3}{2a+3}-\frac{a-4}{a}$ as a single fraction in its simplest form. **2**

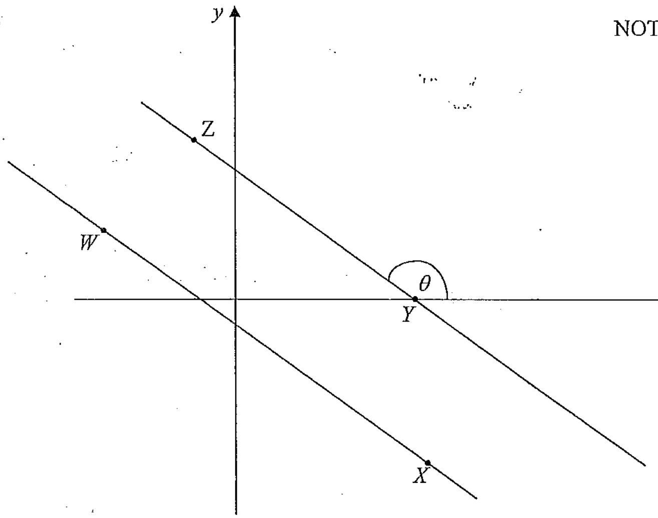
Marks

Question 2 (12 marks) Start a new writing booklet.

(a) (i) Sketch, on the same graph, the curves $y=x^2-4x+3$ and $y=x+3$. Show the x and y intercepts in the sketch. **2**

(ii) Hence shade the region defined by $\begin{cases} y<x^2-4x+3 \\ y\le x+3 \end{cases}$. **1**

(b)



NOT TO SCALE

The diagram shows the points $W(-3,\ 1)$, $X(6,\ -5)$ and $Z(-1,\ 4)$ in the Cartesian plane. The point $Y(a,\ 0)$ is the point where the line *YZ* intersects with the x–axis.

(i) Show that the gradient of *WX* is $-\frac{2}{3}$. **1**

(ii) Show that the equation of *WX* is $2x+3y+3=0$. **1**

(iii) If $WX \parallel ZY$ find the value of a, the x–coordinate of the point *Y*. **1**

(iv) Show the distance *WZ* is $\sqrt{13}$. **1**

(v) Find the perpendicular distance from *Z* to the line *WX*. **2**

(vi) Find the size of angle θ correct to the nearest minute. **2**

(vii) Find the equation of the circle centre *X* which has the line *YZ* as a tangent. **1**

Marks

Question 3 (12 marks) Start a new writing booklet.

(a) Paula has 10 playing cards in her hand, consisting of 5 different pairs of cards. She shuffles the cards and places two of them face up on the table.

(i) Find the probability that these two cards are not a pair. **1**

(ii) A third card is placed on the table. Find the probability that this card forms a pair with one of the two cards already on the table. **1**

(b) Find the domain of the function $f(x)=\frac{1}{\sqrt{4x^2-1}}$. **2**

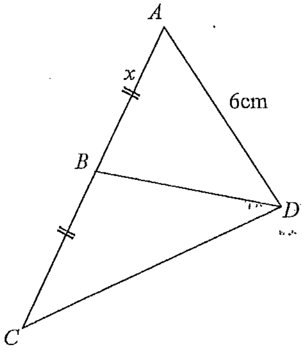
(c) Solve $2\cos^2 x-\sin x-1=0$ for $-\pi \le x \le \pi$. **4**

(d) Find the equation of the tangent to $y=2\sin 2x-3$ at the point where $x=0$. **4**

Marks

Question 4 (12 marks) Start a new writing booklet.

(a) In the diagram below, $AB=BC=x$, $AD=6\text{cm}$ and $\angle ADB=\angle ACD$.



NOT TO SCALE

(i) Show $\Delta ABD\,|||\,\Delta ADC$ **1**

(ii) Find the value of exact value of x. **2**

(b) Show that $\frac{\tan\theta}{\sec\theta-1}-\frac{\tan\theta}{\sec\theta+1}=2\cot\theta$. **2**

(c) Differentiate with respect to x

(i) $e^{x^2}\tan x$ **2**

(ii) $\frac{\ln x}{x^2}$. **2**

(d) Find the value of x if $\sum_{n=0}^{\infty}\frac{9}{x^{n+1}}=18$. **3**

Marks

Question 5 (12 marks) Start a new writing booklet.

(a) (i) Find $\int \frac{x}{2} - e^{2x} dx$. **2**

(ii) Evaluate $\int_{0}^{\pi} \left(\sin \frac{x}{2} + \sqrt{x} \right) dx$. Leave answer in exact form. **3**

(b) Maureen is raising money for charity by jumping on a Pogo Stick.
Her challenge is jump between two points, A and B, 20 times.
On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps.
On her third attempt she takes 51 jumps.
She continues this pattern for all 20 attempts.

(i) How many jumps did she make on her 20th attempt? **2**

(ii) How many jumps did she make altogether? **2**

(c) Solve for x $2\log(x-1) + \log x - \log 4x = 0$. **3**

Marks

Question 6 (12 marks) Start a new writing booklet.

(a) Consider the curve $y = x^3 - 12x^2 + 36x$.

(i) Find the x and y intercepts. **2**

(ii) Find any stationary points and determine their nature. **3**

(iii) Hence sketch for $-1 \le x \le 7$, showing all features found in parts (i) and (ii). **2**

(b) The gradient function of a curve is $\frac{dy}{dx} = 3 - \frac{2}{x^2}$. Find the equation of the curve if it passes through the point $(1, -2)$. **2**

(c) (i) A die is rolled 4 times. What is the probability that no sixes are rolled? **1**

(ii) What is the minimum number of rolls required to ensure there is a 95% chance of getting a 6? **2**

Marks

Question 7 (12 marks) Start a new writing booklet.

(a) (i) Find the exact area bounded by the curve $y=\frac{4}{x}$, the x-axis and the lines $x=1$ and $x=2$. **2**

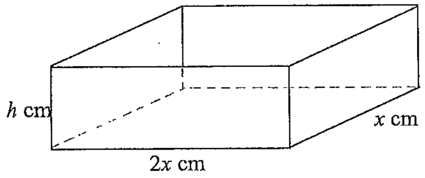
(ii) Copy and complete the function box in your writing booklet for the function $y=\frac{4}{x}$. **1**

x	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
y	4	$\frac{16}{5}$	$\frac{8}{3}$		

(iii) Use the table in part (ii) and two applications of Simpson's Rule to find an approximation for the area bounded by the curve $y=\frac{4}{x}$, the x-axis and the lines $x=1$ and $x=2$. **2**

(iv) Find the percentage error of the Simpson's Rule result. Answer correct to 4 decimal places. **1**

(b) Joe is building a small *open topped* toy box. The box is twice as long as it is wide. The box has a total external surface area of 3750 cm^2.
Note: the box does not have a lid.



(i) Show that the height h of the toy box is given by **1**

$$h=\frac{625}{x}-\frac{x}{3}.$$

(ii) Find the dimensions of the box which give a maximum volume. **3**

(iii) Joe decides that the height of the box cannot exceed $10\frac{5}{6}$ cm. **2**
Find the new dimensions of the box and hence find its volume if the surface area is to remain at 3750 cm^2.

Marks

Question 8 (12 marks) Start a new writing booklet.

(a) (i) Differentiate $\cos^2(3x)$ with respect to x. **2**

(ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\pi}\cos(3x)\sin(3x)\,dx$. **2**

(b) The area bounded by the curve $y=\sec\frac{x}{2}$, the x and y axes and the line $x=\frac{\pi}{2}$ is rotated about the x axis. Find the exact volume generated. **3**

(c) A radioactive isotope is decaying at a rate proportional to its mass according to the formula

$$M=Ae^{-kt}$$

Time t is in hours. Initially the isotope has a mass of 125 g.
After 5 hours it has decayed to a mass of 118 g.

(i) Show that $k=0.0115$ correct to 3 significant figures. **2**

(ii) Find, to the nearest gram, the mass of the isotope after 1 day. **1**

(iii) Find the time taken for the isotope to decay to a mass of 20 g. Give your answer correct to the nearest hour. **2**

Marks

Question 9 (12 marks) Start a new writing booklet.

(a) A rain water tank attached to a house holds 5600 litres of water when full.
The tank is initially empty.
During a recent thunderstorm the tank took 2 hours to fill.
The volume V litres of water flowing off the roof into the tank t minutes after the storm commences is given by

$$V=\frac{7t^2}{2700}(270-t)$$

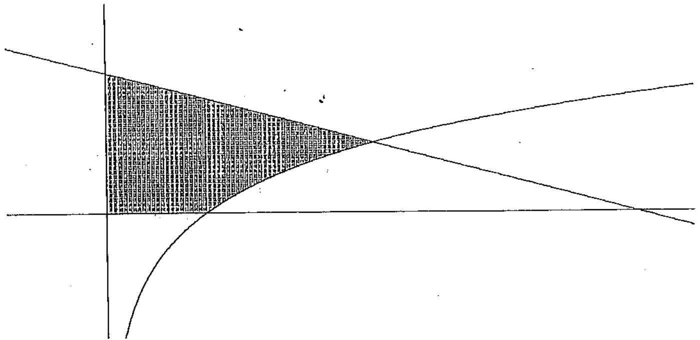
(i) Find the volume of water in the tank after 25 minutes. 1

(ii) Find the rate at which the tank is filling after one hour. 2

(iii) Find the time when the tank is filling at the fastest rate. 2

(iv) Find the amount of water which overflowed the tank if it rained for 3 hours. 2

(b) The shaded region below represents the area bounded by the x and y axes and the curves $y=2-\frac{x}{e}$ and $y=\ln x$.



(i) Show by substitution that the curves $y=2-\frac{x}{e}$ and $y=\ln x$ intersect at the point $(e,\ 1)$. 1

(ii) Hence find the exact area of the shaded region. 4

Marks

Question 10 (12 marks) Start a new writing booklet.

(a) Bob and Beryl take out a home loan of $460 000 over 30 years at 9% p.a. compounded monthly. The bank quoted them a monthly repayment of $3701.26. M represents the monthly repayment and n represents the number of payments made.

(i) Show that the amount owing after 3 months is 1

$$A_3=460000(1.0075)^3-M\left(1+1.0075+1.0075^2\right).$$

(ii) Show that the amount owing after n months can be found using the formula 1

$$A_n=460000(1.0075)^n-M\left(\frac{1.0075^n-1}{0.0075}\right).$$

(iii) Bob and Beryl decide to pay $5000 a month. 1
Find the amount owing after 10 years.

(iv) Bob and Beryl decide to start a family. They approach the bank and renegotiate their loan. After paying numerous fees and charges they agree to pay off the remaining $160 000 at 7.5% p.a. compounded monthly. 3
If their new repayment is $2500 find the time taken to repay the new loan.
You may use an adaptation of the formula in part (ii).
Give answer to the nearest month.

b) Two particles A and B start moving on the x axis at time $t=0$.
Particle A starts from a point 7 metres to the right of the origin with a velocity of $12m/s$ and has an acceleration of $-8m/s^2$.
The position of particle B is given by $x=\frac{25}{t+1}+4t$.

(i) Show that the position of particle A is given by $x=7+12t-4t^2$. 2

(ii) Describe the motion of particle A. 1

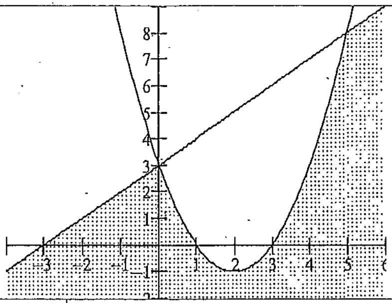
(iii) Describe the motion of particle B. 2

(iv) Use parts (ii) and (iii) to deduce that the particles meet at one point and find this point. 1

End of Paper

NSW INDEPENDENT TRIAL EXAMS – 2011
MATHEMATICS HSC TRIAL EXAMINATION
MARKING GUIDELINES

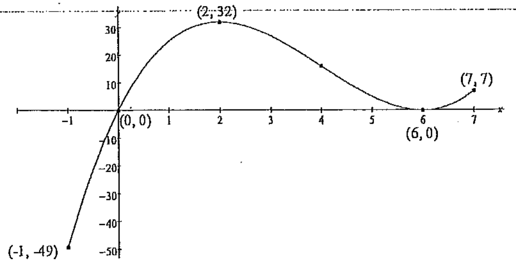
Question		Solution	Marks
1	a)	$5e^{2.302}=49.97075391$ $=50.0$	**1** for 49.97075…. **2** for sig figures
	b)	$3x^2+5x-2=3x^2+6x-x-2$ $=3x(x+2)-1(x+2)$ $=(3x-1)(x+2)$	**1** correct incomplete factorisation **2** for correct answer
	c)	$\dfrac{5}{\sqrt{3}+2}=\dfrac{5}{\sqrt{3}+2}\times\dfrac{\sqrt{3}-2}{\sqrt{3}-2}$ $=\dfrac{5\sqrt{3}-10}{3-4}$ $=-5\sqrt{3}+10$	**1** for attempted multiplication by conjugate **2** for correct solution
	d)	$\lvert 2x-1\rvert<3$ $-3<2x-1<3$ $-2<2x<4$ $-1<x<2$ OR $-(2x-1)<3 \qquad 2x-1<3$ $-2x+1<3 \qquad 2x<4$ $-2x<2 \qquad x<2$ $x>-1$ $\therefore \quad -1<x<2$	**1** for correct attempt at 2 solutions & 1 correct. **2** marks for correct solution.
	e)	$f(x)=2\sin 3x$ $f'(x)=6\cos 3x$ $f'\left(\dfrac{\pi}{18}\right)=6\cos\left(\dfrac{\pi}{6}\right)$ $=\dfrac{6\sqrt{3}}{2}$ $=3\sqrt{3}$	**1**mark for correct differentiation **2** marks for correct solution
	f)	$\dfrac{3}{2a+3}-\dfrac{a-4}{a}=\dfrac{3a-(2a+3)(a-4)}{a(2a+3)}$ $=\dfrac{3a-(2a^2-8a+3a-12)}{a(2a+3)}$ $=\dfrac{-2a^2+8a+12}{a(2a+3)}$	**1** for correct simple expression. **2** marks for correct expansion & factorisation.

Question		Solution		Marks
2	a) (i) (ii)			**1** mark for dotted parabola **2** mark for line plus all intercepts clearly marked. **3** for correct shading
	b)	(i)	$m_{wx}=\dfrac{-5-1}{6+3}$ $=\dfrac{-6}{9}$ $=-\dfrac{2}{3}$	**1** for correct substitution into the correct formula.
		(ii)	$y-1=-\dfrac{2}{3}(x+3)$ $3y-3=-2x-6$ $2x+3y+3=0$	**1** for correct substitution into the correct formula.
		(iii)	$\dfrac{4-0}{-1-a}=-\dfrac{2}{3}$ $12=2+2a$ $2a=10$ $a=5$	**1** for correct solution.
		(iv)	$d=\sqrt{(-3+1)^2+(1-4)^2}$ $=\sqrt{4+9}$ $=\sqrt{13}$	**1** for correct substitution into correct formula & evaluation.
		(v)	$d=\dfrac{\lvert 2(-1)+3(4)+3\rvert}{\sqrt{2^2+3^2}}$ $=\dfrac{\lvert -2+12+3\rvert}{\sqrt{13}}$ $=\dfrac{13}{\sqrt{13}}$ $=\sqrt{13}$	**1** for correct substitution into the correct formula **2** for correct solution
		(vi)	$\tan\theta=-\dfrac{2}{3}$ $\theta=\tan^{-1}\left(-\dfrac{2}{3}\right)$ $=146°19'$	**1** for 33°41' **2** for correct solution
		(vii)	$(x-6)^2+(y+5)^2=13$	**1** for correct solution

Question		Solution		Marks
3	a)	(i)	$P(\square pair)=\frac{8}{9}$	**1**
		(ii)	$P(pair)=\frac{2}{8}$ $=\frac{1}{4}$	**1**
	b)	$4x^2-1>0$ $4x^2>1$ $x^2>\frac{1}{4}$ $x>\frac{1}{2},\quad x<-\frac{1}{2}$		**1** for >0 and 1 correct solution **2** for both correct solutions
	c)	$2\cos^2 x-\sin x-1=0$ $2(1-\sin^2 x)-\sin x-1=0$ $2-2\sin^2 x-\sin x-1=0$ $2\sin^2 x+\sin x-1=0$ $(2\sin x-1)(\sin x+1)=0$ $2\sin x-1=0 \qquad \sin x+1=0$ $\sin x=\frac{1}{2} \qquad \sin x=-1$ $x=\frac{\pi}{6},\frac{5\pi}{6} \qquad x=-\frac{\pi}{2}$		**1** for sub $1-\sin^2 x$ **2** for factorising correct quadratic **3** for $x=\frac{\pi}{6},\frac{5\pi}{6}$ **4** for $x=-\frac{\pi}{2}$
	d)	At $x=0$ $y=2\sin(0)-3$ $y=-3$		
		Point $(0,-3)$ $y=2\sin 2x-3$ $\frac{dy}{dx}=4\cos 2x$ At $x=0$ $\frac{dy}{dx}=4$ $y+3=4(x-0)$ $y=4x-3$		**1** for y value. **2** for correct gradient function **3** for correct gradient **4** for correct tangent

Question		Solution		Marks
4	a) (i)	In ΔBAD and ΔDAC $\angle BDA=\angle DCA$ (given) $\angle BAD=\angle DAC$ (common) $\angle ABD=\angle ADC$ (3rd $\angle$'s equal) $\therefore \Delta ABD \parallel\!\mid \Delta ADC$ (equiangular)		**1** for showing 2 triangles are equilateral
	(ii)	$\frac{x}{6}=\frac{6}{2x}$ (matching sides proportional) $2x^2=36$ $x^2=18$ $x=3\sqrt{2}$		**1** for correct equation **2** for correct answer
	b)	$LHS=\frac{\tan\theta}{\sec\theta-1}-\frac{\tan\theta}{\sec\theta+1}$ $=\frac{\tan\theta(\sec\theta+1)-\tan\theta(\sec\theta-1)}{\sec^2\theta-1}$ $=\frac{\tan\theta(\sec\theta+1-\sec\theta+1)}{\tan^2\theta}$ $=\frac{2}{\tan\theta}$ $=2\cot\theta$ $=RHS$		**1** for correct expression over same denominator **2** for correct simplification
	c)	(i)	$\frac{d}{dx}\left(e^{x^2}\tan x\right)=2xe^{x^2}\tan x+e^{x^2}\sec^2 x$	**1** for diff e^{x^2} **2** for diff tan$x$ and product rule
		(ii)	$\frac{d}{dx}\left(\frac{\ln x}{x^2}\right)=\frac{x^2\frac{1}{x}-2x\ln x}{(x^2)^2}$	**1** for application of quotient rule and differentiating $\ln x$
			$=\frac{1-2\ln x}{x^3}$	**2** for correct answer
	d)	$\sum_{n=0}^{\infty}\frac{9}{x^{n+1}}=\frac{9}{x}+\frac{9}{x^2}+\frac{9}{x^3}+\ldots\ldots.$ GP. $a=\frac{9}{x},\ r=\frac{1}{x}$ $\frac{\frac{9}{x}}{1-\frac{1}{x}}=18$ $\frac{9}{x-1}=18$ $9=18x-18$ $18x=27$ $x=\frac{3}{2}$		**1** for generating series **2** for recognising infinite series and sub into correct formula **3** for correct answer

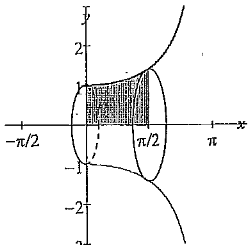
Question		Solution		Marks
5	a)	(i)	$\int \frac{x}{2} - e^{2x} dx = \frac{x^2}{4} - \frac{e^{2x}}{2} + c$	**1** for correct integration of e^{2x} **2** for correct solution
		(ii)	$\int_0^{\pi} \left(\sin\frac{x}{2} + \sqrt{x}\right) dx = \int_0^{\pi} \left(\sin\frac{x}{2} + x^{\frac{1}{2}}\right) dx$ $= \left[-2\cos\frac{x}{2} + \frac{2}{3}x^{\frac{3}{2}}\right]_0^{\pi}$ $= \left[-2\cos\frac{\pi}{2} + \frac{2}{3}(\pi)^{\frac{3}{2}}\right] - \left[-2\cos\frac{0}{2} + (0)^{\frac{3}{2}}\right]$ $= \frac{2}{3}(\pi)^{\frac{3}{2}} - (-2)$ $= 2 + \frac{2\sqrt{\pi^3}}{3}$	**1** for correct integration of function **2** for correct substitution **3** for correct evaluation and answer
	b)	(i)	45, 48, 51, 54, …… AP, $a = 45$, $d = 3$, $n = 20$ $T_{20} = 45 + 19 \times 3$ $= 102$	1 for recognising an AP **2** for correct answer
		(ii)	$S_{20} = \frac{20}{2}(45 + 102)$ $= 1470$ OR $S_{20} = \frac{20}{2}(2 \times 45 + 19 \times 3)$ $= 10(90 + 57)$ $= 1470$	1 for correct substitution into correct formula **2** for correct answer.
	c)	$2\log(x-1) + \log x - \log 4x = 0$ $\log(x-1)^2 + \log x - \log 4x = \log 1$ $\log \frac{\cancel{x}(x-1)^2}{4\cancel{x}} = \log 1$ $(x-1)^2 = 4$ $x^2 - 2x + 1 = 4$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = -1$ or $x = 3$ However, $x \neq -1$ $\therefore$ $x = 3$ is the only solution		1 for arrangement . **2** for 2 solutions. **3** for correct solution.

Question		Solution		Marks
6	a)	(i)	y intercept $= 0$. x intercepts. $y = 0$ $0 = x^3 - 12x^2 + 36x$ $0 = x(x^2 - 12x + 36)$ $0 = x(x-6)^2$ $x = 0,\ 6$	**1** for $y = 0$. **2** for $x = 0,\ 6$
		(ii)	$y = x^3 - 12x^2 + 36x$ $\frac{dy}{dx} = 3x^2 - 24x + 36$ Stationary points occur when $\frac{dy}{dx} = 0$ $3x^2 - 24x + 36 = 0$ $x^2 - 8x + 12 = 0$ $(x-2)(x-6) = 0$ $x = 2,\ 6$ Stationary Points at $(2,\ 32)$ and $(6,\ 0)$ Test stationary points. $\frac{d^2y}{dx^2} = 6x - 24$ At $(2,\ 32)$, $\frac{d^2y}{dx^2} = -12 < 0$ $\therefore$ a maximum turning point At $(6,\ 0)$, $\frac{d^2y}{dx^2} = 12 > 0$ $\therefore$ a minimum turning point	**1** for finding first derivative and equating to 0. **2** for finding the 2 stationary points. **3** for correctly testing stationary points and drawing the correct conclusion.
		(iii)		**1** for the end values **2** mark for complete & accurate sketch

Question		Solution		Marks
6	b)		$\frac{dy}{dx}=3-\frac{2}{x^2}$ $\frac{dy}{dx}=3-2x^{-2}$ $y=3x+2x^{-1}$ $y=3x+\frac{2}{x}$ $-2=3+\frac{2}{1}+c$ $-7=c$ $y=3x+\frac{2}{x}-7$	**1** for primitive function. **2** for value of c and equation.
	c)	(i)	$P\left(4\times\tilde{6}\right)=\left(\frac{5}{6}\right)^4$ $=\frac{625}{1296}$	**1** for correct expression or answer
		(ii)	$1-\left(\frac{5}{6}\right)^n>0.95$ $0.05>\left(\frac{5}{6}\right)^n$ $\ln\left(0.05\right)>n\ln\left(\frac{5}{6}\right)$ $n>\ln\left(0.05\right)\div\ln\left(\frac{5}{6}\right)$ $n>16.43$ $n=17$	**1** for inequality **2** marks for the correct answer

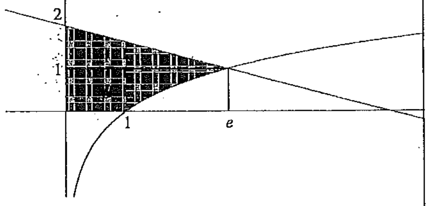
Question		Solution		Marks
7	a)	(i)	$A=\int_1^2 \frac{4}{x}dx$ $=\left[4\ln x\right]_1^2$ $=4\ln 2-4\ln 1$ $=4\ln 2$	**1** for integration **2** for correct answer
		(ii)	$\begin{array}{\|c\|c\|c\|c\|c\|c\|} \hline x & 1 & 1\frac{1}{4} & 1\frac{1}{2} & 1\frac{3}{4} & 2 \\ \hline y & 4 & \frac{16}{5} & \frac{8}{3} & \frac{16}{7} & 2 \\ \hline \end{array}$	**1** for complete table
		(iii)	$A\square\frac{\frac{1}{4}}{3}\left(4+2+4\left(\frac{16}{5}+\frac{16}{7}\right)+2\left(\frac{8}{3}\right)\right)$ $=2\frac{487}{630}$	**1** for correct substitution into correct formula **2** for correct answer
		(iv)	% Error $=\frac{2\frac{487}{630}-4\ln 2}{4\ln 2}\times 100$ $=0.0154\%$	**1** for correct answer
	b)	(i)	$SA=2x^2+2\times 2x\times h+2\times x\times h$ $3750=2x^2+6xh$ $6xh=3750-2x^2$ $h=\frac{3750}{6x}-\frac{2x^2}{6x}$ $h=\frac{625}{x}-\frac{x}{3}$	**1** for correct expression for surface area.
		(ii)	$V=2x\times x\times h$ $=2x^2\left(\frac{625}{x}-\frac{x}{3}\right)$ $=1250x-\frac{2x^3}{3}$ $V'=1250-2x^2$ $0=1250-2x^2$ $2x^2=1250$ $x^2=625$ $x=25$ $V''=-4x$ At $x=25$, $V''=-100<0$ $\therefore$ Maximum volume when $x=25$ $h=\frac{625}{25}-\frac{25}{3}$ $=16\frac{2}{3}$ Dimensions, 25cm by 50 cm by $16\frac{2}{3}$ cm.	**1** for expression for volume. **2** for $x=25$ **3** for $h=16\frac{2}{3}$

Question		Solution		Marks
7	b)	(iii)	$h=\frac{625}{x}-\frac{x}{3}$ $10\frac{5}{6}=\frac{625}{x}-\frac{x}{3}$ $65x=3750-2x^2$ $2x^2+65x-3750=0$ $x=\frac{-65\pm\sqrt{4225+30000}}{4}$ $x=\frac{-65+185}{4}$ $x=30$	1 for $x=30$
			Dimensions 30 cm by 60 cm by $10\frac{5}{6}$ cm. Volume $=30\times60\times10\frac{5}{6}$ $=19500\text{ cm}^3$	**2** for correct volume

Question		Solution		Marks
8	a)	(i)	$\frac{d}{dx}\cos^2(3x)=2\cos(3x)\times-\sin(3x)3$ $=-6\cos(3x)\sin(3x)$	**2** for totally correct answer. (Lose 1 mark for forgetting — or 3.
		(ii)	$\int_{-\frac{\pi}{6}}^{\pi}\cos(3x)\sin(3x)\,dx=-\frac{1}{6}\int_{\frac{\pi}{6}}^{\pi}-6\cos(3x)\sin(3x)\,dx$ $=-\frac{1}{6}\left[\cos^2 3x\right]_{\frac{\pi}{6}}^{\pi}$	**1** for correct integration.
			$=-\frac{1}{6}\left(\left(\cos^2 3\pi\right)-\left(\cos^2\left(\frac{\pi}{2}\right)\right)\right)$ $=-\frac{1}{6}(1)^2$ $=-\frac{1}{6}$	**2** for correct answer
	b)		$V=\pi\int_0^{\frac{\pi}{2}}\sec^2\left(\frac{x}{2}\right)dx$ 	**1** correct expression for volume
			$=2\pi\left[\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$	**2** correct integration
			$=2\pi\left(\tan\frac{\pi}{4}-\tan 0\right)$ $=2\pi\text{ u}^3$	**3** correct answer

Question		Solution		Marks
8	c)	(i)	$M=Ae^{-kt}$ when $t=0$, $M=125$ $125=Ae^0$ $A=125$	**1** for $A=125$
			$M=125e^{-kt}$ when $t=5$, $M=118$ $118=125e^{-5k}$ $\frac{118}{125}=e^{-5k}$ $-5k=\ln\left(\frac{118}{125}\right)$	**2** for showing k.
			$k=\ln\left(\frac{118}{125}\right)\div-5$ $=0.011525822$ $=0.0115$	
		(ii)	$M=125e^{-(0.0115)\times24}$ $=94.85$ (94.79... if exact k used) $=95g$	**1** for correct answer.
		(iii)	$20=125e^{-0.0115t}$ $\frac{20}{125}=e^{-0.0115t}$ $-0.0115t=\ln\left(\frac{4}{25}\right)$	**1** for correct equation
			$t=159.3549099$ (158.9978896 if exact k used) $t=159$ hours.	**2** for correct answer

Question		Solution		Marks
9	a)	(i)	$V=\frac{7(25)^2}{2700}(270-25)$ $=396.99$ $=397$ litres.	1 for correct answer
		(ii)	$V=\frac{7t^2}{10}-\frac{7t^3}{2700}$ $\frac{dV}{dt}=\frac{7t}{5}-\frac{7t^2}{900}$ at $t=60$ $\frac{dV}{dt}=\frac{7(60)}{5}-\frac{7(60)^2}{900}$ $=56$ litres/minute	1 for derivative 2 for correct answer
		(iii)	$\frac{d}{dt}\left(\frac{dV}{dt}\right)=\frac{7}{5}-\frac{7t}{450}$ $0=\frac{7}{5}-\frac{7t}{450}$ $\frac{7t}{450}=\frac{7}{5}$ $t=90$ minutes. Check maximum flow rate. $\frac{d}{dt}\left(\frac{d^2V}{dt^2}\right)=-\frac{7}{450}<0$ $\therefore$ maximum flow rate	1 for 90 minutes 1 for checking maximum
		(iv)	$t=120$ $V=\frac{7(120)^2}{2700}(270-120)$ $=5600$ litres (check only - not necessary as given) at $t=180$ $V=\frac{7(180)^2}{2700}(270-180)$ $=7560$ litres Litres overflowed $=7560-5600$ $=1960$ litres	1 for 7560 litres 2 for correct answer

9	b)	(i)	For the curve $y=2-\frac{x}{e}$ at $x=e$ $y=2-\frac{e}{e}$ $=1$ passes through the point $(e,\ 1)$ For the curve $y=\ln x$ at $x=e$ $y=\ln e$ $=1$ passes through the point $(e,\ 1)$	1 mark for showing
		(ii)	Area $A=\frac{1}{2}\times1\times e$ $=\frac{e}{2}$ For Area B, $y=\ln x$ $x=e^y$ Area $B=\int_0^1 e^y dy$ $=\left[e^y\right]_0^1$ $=e^1-e^0$ $=e-1$  $Area=\frac{e}{2}+e-1$ $=\frac{3e}{2}-1$ units^2	1 for area A 2 for changing subject to x 3 for area B 4 for total area

Question		Solution		Marks
10	a)	(i)	Interest rate $=9\div 12\div 100=0.0075$ $A_1=460000(1.0075)-M$ $A_2=A_1(1.0075)-M$ $=(460000(1.0075)-M)(1.0075)-M$ $=460000(1.0075)^2-M(1.0075)-M$ $=460000(1.0075)^2-M(1+1.0075)$ $A_3=A_2(1.0075)-M$ $=\left(460000(1.0075)^2-M(1.0075)-M\right)(1.0075)-M$ $=460000(1.0075)^3-M(1.0075)^2-M(1.0075)-M$ $=460000(1.0075)^3-M\left(1+1.0075+1.0075^2\right)$	**1** for correct answer
		(ii)	$A_n=460000(1.0075)^n-M\left(1+1.0075+1.0075^2+.....+1.007^{n-1}\right)$ $=460000(1.0075)^n-M\left(\frac{1\times\left(1.0075^n-1\right)}{1.0075-1}\right)$ $=460000(1.0075)^n-M\left(\frac{1.0075^n-1}{0.0075}\right)$	**1** for showing
		(iii)	$A_{120}=460000(1.0075)^{120}-5000\left(\frac{1.0075^{120}-1}{0.0075}\right)$ $=\$160052.87$	**1** for correct answer
		(iv)	$0=160000(1.00625)^n-2500\left(\frac{1.00625^n-1}{0.00625}\right)$ $0=160000(1.00625)^n-400000\left(1.00625^n-1\right)$ $0=160000(1.00625)^n-400000(1.00625)^n+400000$ $240000(1.00625)^n=400000$ $1.00625^n=\frac{400000}{240000}$ $1.00625^n=\frac{5}{3}$ $n\ln(1.00625)=\ln\frac{5}{3}$ $n=81.98724739$ $n=82$ months.	**1** for correct interest rate. **2** for correct substitution of information leading to $1.00625^n=\frac{5}{3}$. (2 marks for correct working and answer from an incorrect interest rate.) **3** for correct answer.

Question		Solution		Marks
10	b)	(i)	$a=-8$ $v=-8t+c$ At $t=0,\ v=12$ $12=c$ $v=-8t+12$ $x=-4t^2+12t+k$ At $t=0,\ x=7$ $7=k$ $x=-4t^2+12t+7$ $x=7+12t-4t^2$ Allow marks for taking $x=7+12t-4t^2$ and differentiating. $x=7+12t-4t^2$ $v=12-8t.$ At $t=0,\ v=12$ $a=-8$	**1** mark for velocity equation **2** mark for showing correct equation.
		(ii)	Particle stops when $v=0$ $0=-8t+12$ $8t=12$ $t=\frac{3}{2}$ $x=7+12\left(\frac{3}{2}\right)-4\left(\frac{3}{2}\right)^2$ $=16$ Particle starts 7 metres to the right of the origin and moves right for $1\frac{1}{2}$ seconds till it is 16 metres from the origin. It then turns and moves through the origin at increasing velocity. Acceleration constant at $-8m/s^2$.	**1** for description including turning point of particle.

10		(iii)	$x=\frac{25}{t+1}+4t$ $=25(t+1)^{-1}+4t$ $v=-25(t+1)^{-2}+4$ At $t=0$ $v=-21m/s$ Find where particle stops $v=0$ $0=\frac{-25}{(t+1)^2}+4$ $4=\frac{25}{(t+1)^2}$ $4(t+1)^2=25$ $(t+1)^2=\frac{25}{4}$ $t+1=\frac{5}{2}$ $t=\frac{3}{2}$ $x=\frac{25}{\frac{3}{2}+1}+4\left(\frac{3}{2}\right)$ $=16$ Particle starts at $x=25$ with a velocity of $-21m/s$ moving towards the origin for 1.5 seconds. It comes to rest at $x=16$ then moves away from the origin.	**1** for finding when and where the particle stops. **2** for description mentioning starting point & turning point.
		(iv)	Particles meet after 1.5 seconds at $x=16$.	**1** mark