NSW INDEPENDENT SCHOOLS

2013
Higher School Certificate
Preliminary Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 3 – 5

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 6 – 9

60 marks

Attempt Questions 11 – 14

Allow about 1 hour 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Student name / number ....

Marks

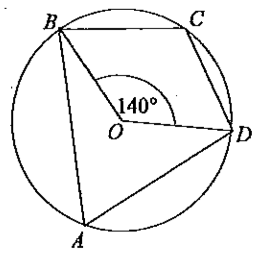
Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

---

1 1



Not to scale

$ABCD$ is a cyclic quadrilateral inscribed in a circle with centre O such that $\angle BOD = 140°$.
What is the size of $\angle BCD$?

(A) 100°

(B) 110°

(C) 120°

(D) 130°

2 $P(x)$ is an odd polynomial. When $P(x)$ is divided by $(x-2)$ the remainder is 5. 1
What is the remainder when $P(x)$ is divided by $(x+2)$?

(A) -5

(B) $-5x$

(C) $5x$

(D) 5

3 $P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0, a)$ and directrix $y = -a$, where $a > 0$. Which of the following is an expression for the distance PF? 1

(A) $at^2 - a$

(B) $2at - a$

(C) $2at + a$

(D) $at^2 + a$

Student name / number ....

Marks

4 Which of the following is an expression for $\frac{d}{dx}\left(x\sqrt{x^2+2}\right)$? 1

(A) $\frac{x}{\sqrt{x^2+2}}$

(B) $1+\frac{x}{\sqrt{x^2+2}}$

(C) $\frac{2x^2+2}{\sqrt{x^2+2}}$

(D) $\frac{2x^2+x+4}{\sqrt{x^2+2}}$

5 Which of the following is an expression for $\cos(A+B)-\cos(A-B)$? 1

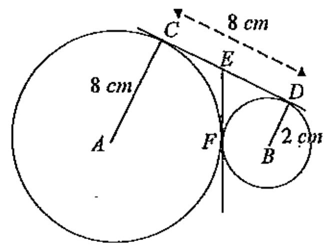
(A) $-2\sin A\sin B$

(B) $-2\cos A\cos B$

(C) $2\cos A\cos B$

(D) $2\sin A\sin B$

6 1



Not to scale

Two circles, one with centre A and radius 8cm, the other with centre B and radius 2cm, touch externally at F. C is a point on the larger circle ad D is a point on the smaller circle such that CD, of length 8 cm, is a common tangent to the two circles. The common tangent to the two circles at F meets CD at E. What is the length of FE?

(A) 6cm

(B) 5cm

(C) 4cm

(D) 3cm

Marks

7 Which of the following is an expression for $\frac{d^2}{dx^2}\left(\frac{x}{2x+1}\right)$? 1

(A) $\frac{-4}{(2x+1)^3}$

(B) $\frac{-2}{(2x+1)^3}$

(C) $\frac{2}{(2x+1)^3}$

(D) $\frac{4}{(2x+1)^3}$

8 If $x = 2at$ and $y = 3at^2$, which of the following is an expression for $\frac{dy}{dx}$? 1

(A) t

(B) $2t$

(C) $3t$

(D) $6t$

9 Which of the following is an expression for $\frac{1}{(n-1)!} + \frac{n^3+1}{(n+1)!}$? 1

(A) $\frac{n+1}{n!}$

(B) $\frac{n^2+1}{n!}$

(C) $\frac{n^2+n+1}{n!}$

(D) $\frac{n^3+n^2+1}{n!}$

10 Which of the following is an expression for $\frac{\sin 8x}{\sin x}$? 1

(A) $\cos 4x \cos 2x \cos x$

(B) $2\cos 4x \cos 2x \cos x$

(C) $4\cos 4x \cos 2x \cos x$

(D) $8\cos 4x \cos 2x \cos x$

Marks

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question on a SEPARATE page of your own paper, or writing booklet, if provided.

All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet.

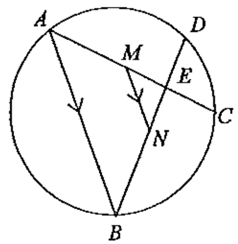
(a) Find the coordinates of the point $P(x, y)$ which divides the interval joining the points $A(-4, 5)$ and $B(5, -1)$ internally in the ratio $2:1$. 2

(b) Find the number of ways in which 2 brothers and 4 sisters can line up in a queue

(i) without restriction. 1

(ii) so that the 2 brothers are not next to each other. 2

(c)



AC and BD are two chords of a circle which intersect at E inside the circle. M is a point on AE and N is a point on BE such that $MN \parallel AB$.

(i) Give a reason why $\angle BDC = \angle BAC$. 1

(ii) Hence show that $MNCD$ is a cyclic quadrilateral. 2

(d) Let $P(x) = 2x^3 - 3x^2 - 3x + 2$.

(i) Show that $(x+1)$ is a factor of $P(x)$. 1

(ii) Hence express $P(x)$ as a product of three linear factors. 2

(e)(i) Use the substitution $t = \tan\frac{x}{2}$ to show that $\sin x + \cos x + 1 = \frac{2(t+1)}{1+t^2}$. 2

(ii) Hence solve the equation $\sin x + \cos x = -1$ for $0° \le x \le 360°$. 2

Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

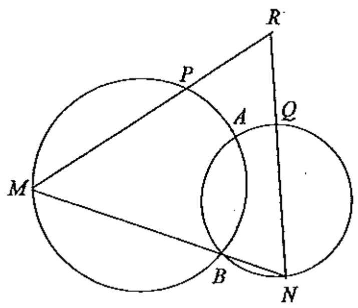
(a) Find correct to the nearest degree the acute angle between the lines $2x+y-3=0$ and $x-3y+2=0$. **2**

(b) Each multiple choice question has 1 correct answer and 3 incorrect answers. A test contains 4 multiple choice questions.

(i) Find the number of ways in which the 4 questions can be answered. **1**

(ii) Find the number of ways of getting 2 correct answers and 2 incorrect answers. **2**

(c)



Two circles intersect at A and B. M is a point on the first circle and N is a point on the second circle such that MBN is a straight line. P is a point on the first circle and Q is a point on the second circle such that MP produced and NQ produced meet at R.

(i) Give a reason why $\angle RPA = \angle MBA$. **1**

(ii) Hence show that $RPAQ$ is a cyclic quadrilateral. **2**

(d)(i) Express $\sqrt{3}\cos x+\sin x$ in the form $R\sin(x+\alpha)$ where $R>0$ and $0°<x<90°$. **2**

(ii) Hence find the range of the function $f(x)=\sqrt{3}\cos x+\sin x$. **1**

(e) $T(2at, at^2)$ is a point on the parabola $x^2=4ay$ with focus $F(0, a)$.

(i) Use differentition to show that the normal to the parabola at T has equation $x+ty-2at-at^3=0$. **2**

(ii) Hence find the coordinates of the three points on the parabola such that normals to the parabola at these points pass through the point $(0, 6a)$. **2**

Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Find the number of ways in which 2 consonants and 1 vowel can be chosen from the word NUMBER. **2**

(b) Solve the inequality $\dfrac{2x-1}{x} \geq x$. **3**

(c) A cubic equation has roots α, β and γ such that $\alpha\beta\gamma=-2$, $\dfrac{1}{\alpha\beta}+\dfrac{1}{\beta\gamma}+\dfrac{1}{\gamma\alpha}=2$ and $\alpha^2+\beta^2+\gamma^2=10$.

(i) Show that $\alpha+\beta+\gamma=-4$. **1**

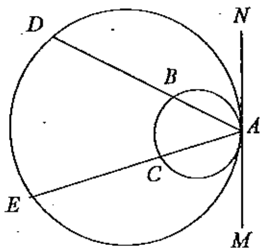
(ii) Hence find the value of $\alpha\beta+\beta\gamma+\gamma\alpha$. **2**

(d) $T(2at, at^2)$ is a point on the parabola $x^2=4ay$ with focus $F(0, a)$. The point M divides the interval FT externally in the ratio 2 : 1.

(i) Show that M is the point $M(4at, 2at^2-a)$. **1**

(ii) Hence find the Cartesian equation of the locus of M as T moves on the parabola. **2**

(e)



Two circles touch each other internally at A. MAN is the common tangent to the circles at A. ABD and ACE are two straight lines which cut the smaller circle at B and C, and the larger circle at D and E.

(i) Show that $\Delta ABC \parallel\!| \Delta ADE$. **2**

(ii) Hence show that $\dfrac{AB\times AC}{AD\times AE}=\dfrac{BC^2}{DE^2}$. **2**

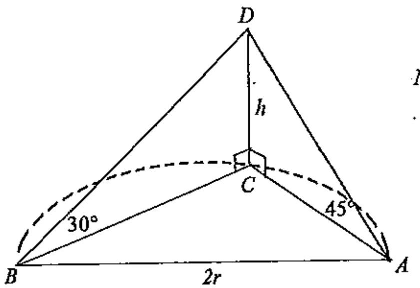
Student name / number ....

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Find the number of ways in which the letters of the word ORANGE can be arranged in a circle so that every consonant is diametrically opposite a vowel. **2**

(b)



Not to scale

AB is a diameter of a semicircular piece of horizontal ground with radius r metres. CD is a vertical flagpole of height h metres standing with its base C on the arc AB. From A and B the angles of elevation of the top D of the flagpole are 45° and 30° respectively. Show that $h = r$. **3**

(c)(i) Show that $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$. **2**

(ii) Hence find the exact value of $\tan 22\cdot 5°$. **1**

(d) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$ such that OP and OQ are perpendicular to each other.

(i) Show that $pq = -4$. **1**

(ii) Hence show that ΔOPQ has area $4a^2|p-q|$. **2**

(e) The polynomials $P(x)$ and $Q(x)$ are such that $P(x) = (x^2-1)Q(x) + Ax + B$ for some real numbers A and B.

(i) Show that $A = \frac{1}{2}\{P(1) - P(-1)\}$ and $B = \frac{1}{2}\{P(1) + P(-1)\}$. **2**

(ii) Hence show that if $P(x)$ is an even polynomial then the remainder when $P(x)$ is divided by (x^2-1) is independent of x. **2**

Section I

Questions 1-10 (1 mark each)

Question	Answer	Solution	Outcomes
1.	B	$reflex\angle BOD = 220°$ (*one revolution is 360°*) $\therefore \angle BCD = 110°$ (*∠ subtended at the centre by major arc BD is twice that subtended by the same arc at the circumference*)	PE3
2.	A	Using the remainder theorem, $P(2)=5$. Then $P(-2)=-P(2)=-5$ since $P(x)$ is odd, hence required remainder is -5	PE3
3.	D	PF is equal to the perpendicular distance from P to the horizontal line $y=-a$. $\therefore PF = at^2+a$	PE3
4.	C	$\frac{d}{dx}\left(x\sqrt{x^2+2}\right)=1.\left(x^2+2\right)^{\frac{1}{2}}+x.\frac{1}{2}\left(x^2+2\right)^{-\frac{1}{2}}.2x$ $=\left(x^2+2\right)^{-\frac{1}{2}}\left\{\left(x^2+2\right)+x^2\right\}$ $=\frac{2x^2+2}{\sqrt{x^2+2}}$	PE5
5.	A	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\therefore \cos(A+B)-\cos(A-B)=-2\sin A\sin B$	P3
6.	C	$DE=FE=CE$ (*tangents to a circle from an external point are equal*) Hence E is the centre of the circle through D, F and C. Given $DC=8$ cm, this circle has radius 4 cm. $\therefore FE=4$ cm.	PE3
7.	B	$\frac{d}{dx}\left(\frac{x}{2x+1}\right)=\frac{1.(2x+1)-x.2}{(2x+1)^2}=(2x+1)^{-2}$ $\therefore \frac{d^2}{dx^2}\left(\frac{x}{2x+1}\right)=-2(2x+1)^{-3}.2=\frac{-4}{(2x+1)^3}$	PE5
8.	C	$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=\frac{6at}{2a}=3t$	PE4
9.	B	$\frac{1}{(n-1)!}+\frac{n^3+1}{(n+1)!}=\frac{n}{n!}+\frac{(n+1)(n^2-n+1)}{(n+1)n!}=\frac{n^2+1}{n!}$	PE3
10.	D	$\sin 8x=2\sin 4x\cos 4x=4\sin 2x\cos 2x\cos 4x=8\sin x\cos x\cos 2x\cos 4x$ $\therefore \frac{\sin 8x}{\sin x}=8\cos 4x\cos 2x\cos x$	P3

Question 11

a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• finds the x coordinate of P	1
• finds the y coordinate of P	1

Answer

$A(-4,5)$ $B(5,-1)$

2 : 1

$\left(\frac{10-4}{2+1}, \frac{-2+5}{2+1}\right)$ $\therefore P(2,1)$

b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • gives the correct number of unrestricted arrangements	1
ii • counts the arrangements with the brothers next to each other	1
• finds the number of arrangements with the stated restriction by subtraction	1

Answer

i. $6!=720$ ways

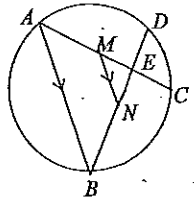
ii. Grouping the brothers as one item gives $5!$ arrangements. Hence considering the two orders of the brothers, there are $2\times 5!=240$ arrangements with the brothers next to each other.
Hence there are $6!-2\times 5!=480$ arrangements where the brothers are not next to each other.

c. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes appropriate circle property	1
ii • identifies equal corresponding angles with parallel lines	1
• applies appropriate test for a cyclic quadrilateral	1

Answer



i. $\angle BDC=\angle BAC$ (*angles in the same segment standing on the same arc BC are equal*)

ii. $\angle BAC=\angle NMC$ (*corresp. ∠'s across parallel lines are equal*)
$\therefore \angle NDC=\angle NMC$ (B, N, D collinear $\therefore \angle NDC$, $\angle BDC$ same $\angle$)
$\therefore MNCD$ is a cyclic quadrilateral (*CN subtends equal ∠'s at points M, D on the same side of CN*)

Q11 (cont)
d. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • applies factor theorem	1
ii • factorises $P(x)$ into a linear and a quadratic factor, or finds a second linear factor	1
• completes the factorisation into linear factors	1

Answer

i. $P(x)=2x^3-3x^2-3x+2$. $P(-1)=-2-3+3+2=0$ $\therefore (x+1)$ is a factor of $P(x)$.

ii. $P(x)=(x+1)(2x^2-5x+2)=(x+1)(2x-1)(x-2)$

e. Outcomes assessed : P3, P4

Marking Guidelines

Criteria	Marks
i • writes expressions for $\sin x$, $\cos x$ in terms of t	1
• takes a common denominator and simplifies	1
ii • writes equation in terms of t to find one solution for x	1
• tests $x=180°$ to find the second solution	1

Answer

i. $t=\tan\frac{x}{2} \Rightarrow \sin x+\cos x+1=\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}+1=\frac{2t+(1-t^2)+(1+t^2)}{1+t^2}=\frac{2(t+1)}{1+t^2}$

ii. For $x\neq 180°$: Substituting $t=\tan\frac{x}{2}$ transforms equation to give $\frac{2(t+1)}{1+t^2}=0$.

Then $t=-1$, $\therefore \tan\frac{x}{2}=-1$, $0°<\frac{x}{2}<180°$ $\therefore \frac{x}{2}=135°$ $\therefore x=270°$

For $x=180°$: $\sin x+\cos x=0+(-1)=-1$ $\therefore x=180°$ is also a solution.

Hence $\sin x+\cos x=-1$ for $0°\le x\le 360°$ has solutions $x=180°, 270°$

Question 12
a. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• writes an expression for $\tan\theta$ using the gradients of the lines	1
• writes the angle to the nearest degree.	1

Answer

$\tan\theta=\left|\frac{\frac{1}{3}-(-2)}{1+\frac{1}{3}\times(-2)}\right|=\left|\frac{1+6}{3-2}\right|=7$ $\therefore \theta\approx 82°$

Q12 (cont)
b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • writes the numbers of ways	1
ii • realises there are ${}^4C_2$ possibilities for selecting the two questions to be answered correctly	1
• realises the remaining questions can each be answered incorrectly in 3 ways	1

Answer

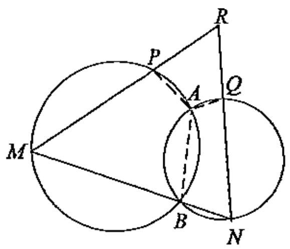
i. $4^4=256$ ii. ${}^4C_2\times 3\times 3=54$

c. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes an appropriate property of a cyclic quadrilateral	1
ii • deduces $\angle NQA=\angle RPA$	1
• applies an appropriate test for a cyclic quadrilateral	1

Answer



i. Exterior angle RPA is equal to opposite interior angle MBA in cyclic quadrilateral $MPAB$.

ii. Similarly $\angle MBA=\angle NQA$ in cyclic quad. $NQAB$

$\therefore \angle NQA=\angle RPA$

$\therefore RPAQ$ is a cyclic quadrilateral (*one exterior ∠ is equal to the interior opp. ∠*)

d. Outcomes assessed : P3, P5

Marking Guidelines

Criteria	Marks
i • writes expression with correct value of R	1
• writes expression with correct value of α	1
ii • states the range of the function	1

Answer

i. $\sqrt{3}\cos x+\sin x=2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)=2\sin\left(x+60°\right)$

ii. The range of $f(x)$ is $\{y:-2\le y\le 2\}$

Q12(cont)

e. Outcomes assessed : PE3, PE4

Marking Guidelines

Criteria	Marks
i • finds $\frac{dy}{dx}$ at T	1
• finds the equation of the normal at T	1
ii • deduces an equation for t at the required points on the parabola	1
• solves for t to find the coordinates of these points	1

Answer

i.

$$\left.\begin{aligned} x=2at &\Rightarrow \frac{dx}{dt}=2a \\ y=at^2 &\Rightarrow \frac{dy}{dt}=2at \end{aligned}\right\} \quad \therefore \frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=t \qquad \therefore \text{ normal at } T\left(2at,\,at^2\right) \text{ has gradient } -\frac{1}{t}$$

and equation $y-at^2=-\frac{1}{t}\left(x-2at\right)$, giving $x+ty-2at-at^3=0$.

ii. $(0,6a)$ lies on normal at T if $\quad 0+6at-2at-at^3=0$

$$at\,(4-t^2)=0$$

Hence the three points, with parameters $t=0,\ 2,-2,$ are $(0,0)$, $(4a,\ 4a)$, $(-4a,\ 4a)$

Question 13

a. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• counts the number of ways of selecting the consonants	1
• multiplies by the two ways of selecting the vowel	1

Answer

From NUMBER, 2 consonants and 1 vowel can be chosen in ${}^4C_2\times 2=12$ ways

b. Outcomes assessed : PE6

Marking Guidelines

Criteria	Marks
• applies an appropriate method for removing the variable denominator	1
• obtains the inequality $x<0$	1
• realises that $x=1$ is also a solution	1

Answer

$\frac{2x-1}{x}\ge x \qquad \therefore x(2x-1)\ge x^3$ and $x\ne 0$ (Multiplying both sides by x^2 which cannot be negative)

$$0\ge x\left\{x^2-(2x-1)\right\}$$

$$\therefore 0\ge x(x-1)^2 \quad \text{and} \quad x\ne 0 \qquad \therefore x<0 \text{ or } x=1 \qquad (\text{since } (x-1)^2\ge 0)$$

Q13 (cont)

c. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • expresses $\alpha+\beta+\gamma$ in terms of $\alpha\beta\gamma$ and $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$ then evaluates	1
ii • expresses $\alpha\beta+\beta\gamma+\gamma\alpha$ in terms $\alpha+\beta+\gamma$ and $\alpha^2+\beta^2+\gamma^2$	1
• evaluates $\alpha\beta+\beta\gamma+\gamma\alpha$	1

Answer

i. $\alpha+\beta+\gamma=\alpha\beta\gamma\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)=-4$

ii. $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1}{2}\left\{\left(\alpha+\beta+\gamma\right)^2-\left(\alpha^2+\beta^2+\gamma^2\right)\right\}=\frac{1}{2}\left(16-10\right)=3$

d. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • finds the coordinates of M	1
ii • eliminates t to write y in terms of x at M	1
• simplifies Cartesian equation of the locus of M into a standard form	1

Answer

i.

$F(0,a) \qquad T(2at,\,at^2)$

$2 \;:\; -1$

$$\left(\frac{2\times 2at+(-1)\times 0}{2-1},\ \frac{2\times at^2+(-1)\times a}{2-1}\right)$$

$$\therefore M\left(4at,\,2at^2-a\right)$$

ii. At M, $x=4at$ and $y=2at^2-a$.

$$\therefore y=\frac{2}{a}(at)^2-a$$

$$=\frac{2}{a}\cdot\frac{x^2}{16}-a$$

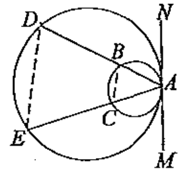
Hence locus of M has equation $x^2=8a(y+a)$

e. Outcomes assessed : PE2

Marking Guidelines

Criteria	Marks
i • quotes alternate segment theorem to deduce one pair of equal angles	1
• completes similarity proof, noting the common angle at A	1
ii • deduces sides in proportion	1
• uses ratio statements to deduce required result	1

Answer



i. $\angle NAB=\angle ACB$ and $\angle NAB=\angle AED$

(*$\angle$ between tangent and chord drawn to the point of contact is equal to $\angle$ subtended by that chord in the alternate segment*)

Hence in ΔABC, ΔADE : $\angle A$ is common and $\angle ACB=\angle AED$.

$\therefore \Delta ABC\ |||\ \Delta ADE$ (*2 angles equal and hence equiangular*)

ii. $\frac{AB}{AD}=\frac{AC}{AE}=\frac{BC}{DE}$ (*corresp. sides of similar Δ's are in proportion*)

$\therefore \frac{AB}{AD}\times\frac{AC}{AE}=\frac{BC}{DE}\times\frac{BC}{DE}$ and hence $\frac{AB\times AC}{AD\times AE}=\frac{BC^2}{DE^2}$

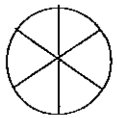
Question 14

a. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• counts the arrangements of the vowels relative to a particular vowel	1
• multiplies by the number of arrangements of the consonants in positions opposite the vowels	1

Answer



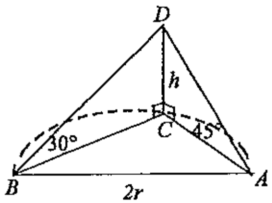
Place O on the circle, then count the arrangements relative to O.
The remaining vowels can be arranged in 4×2 ways.
Then the consonants can be arranged opposite the vowels in $3!=6$ ways.
Hence the number of arrangements is $4\times 2\times 6=48$ ways

b. Outcomes assessed : PE6

Marking Guidelines

Criteria	Marks
• finds BC and AC in terms of h	1
• deduces $\angle BCA=90°$	1
• uses Pythagoras' theorem to prove result	1

Answer



In ΔABC, $BC=h\cot 30°=h\sqrt{3}$, $AC=h$ and $\angle BCA=90°$ (*$\angle$ in a semicircle is a right angle*).

By Pythagoras' theorem, $3h^2+h^2=(2r)^2$. $\therefore 4h^2=4r^2$.

Then since $h>0$ and $r>0$, $h=r$.

c. Outcomes assessed : P3, P4

Marking Guidelines

Criteria	Marks
i • writes with common denominator $\sin 2\theta$	1
• uses appropriate double angle identities to simplify and hence establish result	1
ii • substitutes for θ to obtain required exact value	1

Answer

i. $$\begin{aligned}\operatorname{cosec}2\theta-\cot 2\theta &= \frac{1}{\sin 2\theta}-\frac{\cos 2\theta}{\sin 2\theta}\\ &= \frac{1-\cos 2\theta}{\sin 2\theta}\\ &= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}\\ &= \frac{\sin\theta}{\cos\theta}\\ &= \tan\theta\end{aligned}$$

ii. $$\begin{aligned}\tan 22\cdot 5° &= \operatorname{cosec}45°-\cot 45°\\ &= \sqrt{2}-1\end{aligned}$$

Q14 (cont)

d. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
i • uses gradients of OP and OQ to deduce result	1
ii • finds the required area or its square in terms of p and q	1
• rearranges to give required result after substitution of the value of pq	1

Answer

i. $P(2ap, ap^2)$ and $Q(2aq, aq^2)$. $OP\perp OQ \Rightarrow m_{OP}.m_{OQ}=-1$. where $m_{OP}=\dfrac{ap^2}{2ap}=\dfrac{p}{2}$.

$$\therefore \frac{p}{2}\times\frac{q}{2}=-1 \qquad \therefore pq=-4$$

ii. $Area\,\Delta OPQ=\frac{1}{2}OP.OQ$ where $OP^2=(2ap)^2+(ap^2)^2=a^2p^2(4+p^2)$.

$$\begin{aligned}\therefore (Area\,\Delta OPQ)^2 &= \tfrac{1}{4}a^2p^2(4+p^2).\,a^2q^2(4+q^2)\\ &= \tfrac{1}{4}a^4(pq)^2\{16+4(p^2+q^2)+(pq)^2\}\\ &= \tfrac{1}{4}a^4(pq)^2\{16+4((p-q)^2+2pq)+(pq)^2\}\end{aligned}$$

Substituting $pq=-4$, $(Area\,\Delta OPQ)^2=16a^4(p-q)^2$. $\therefore Area\,\Delta OPQ=4a^2|p-q|$

e. Outcomes assessed : PE2

Marking Guidelines

Criteria	Marks
i • substitutes $x=\pm 1$ to obtain simultaneous equations for A and B	1
• solves for A and B to obtain required result	1
ii • notes that $Ax+B$ is the remainder on division of $P(x)$ by (x^2-1)	1
• uses the definition of an even function to deduce $A=0$, giving the required result	1

Answer

i. $P(x)=(x^2-1)Q(x)+Ax+B \Rightarrow \begin{matrix} P(1)=A+B & (1)\\ P(-1)=-A+B & (2)\end{matrix}$

Then $\begin{matrix}(1)-(2) \Rightarrow P(1)-P(-1)=2A\\ (1)+(2) \Rightarrow P(1)+P(-1)=2B\end{matrix}$ $\quad\therefore A=\frac{1}{2}\{P(1)-P(-1)\}$ and $B=\frac{1}{2}\{P(1)+P(-1)\}$

ii. $P(x)=(x^2-1)Q(x)+Ax+B$ tells us that $(Ax+B)$ is the remainder when $P(x)$ is divided by (x^2-1).
If $P(x)$ is even, then $P(-1)=P(1)$ giving $A=0$, so that the remainder is B which is independent of x.