NSW INDEPENDENT SCHOOLS

2013
Higher School Certificate
Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 3 – 7

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 8 – 12

60 marks

Attempt Questions 11 – 14

Allow about 1 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

1. What is the value of $\frac{18.81-3.47}{2.79+7.75}$ correct to 2 significant figures?

A. 1.4

B. 1.45

C. 1.46

D. 1.5

2. What are the values of a and b if $\frac{5-2\sqrt{2}}{1+\sqrt{2}}=a+b\sqrt{2}$?

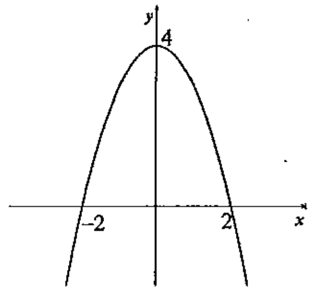
A. $a=-9,\quad b=7$

B. $a=9,\quad b=-7$

C. $a=-7,\quad b=9$

D. $a=7,\quad b=-9$

3. Which equation corresponds to the graph below?



A. $y=2-x^2$

B. $y=4-x^2$

C. $y=x+4$

D. $y=x^2+4$

4. If $\tan\theta=-\frac{4}{5}$ and $\cos\theta>0$, what is the value of $\sin\theta$?

A. $\frac{4}{\sqrt{41}}$

B. $\frac{5}{\sqrt{41}}$

C. $-\frac{4}{\sqrt{41}}$

D. $-\frac{5}{\sqrt{41}}$

5. Which of the following equations has solutions $x=2$ and $x=-3$?

A. $x^2-5x-6=0$

B. $x^2+5x-6=0$

C. $x^2-x-6=0$

D. $x^2+x-6=0$

6. The function $f(x)=\dfrac{x^2-1}{x}$ is:

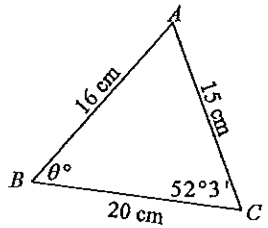
A. an even function

B. an odd function

C. Neither odd nor even function

D. a zero function.

7. Which of the following is a correct expression involving θ in triangle ABC?



Not to scale

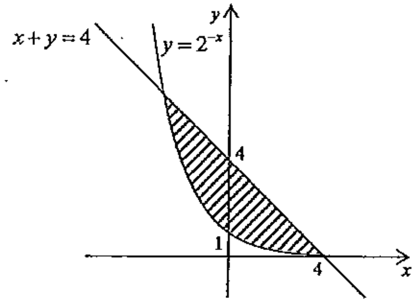
A. $15^2=16^2+20^2+2\times16\times20\cos\theta$

B. $\cos\theta=\dfrac{16^2+20^2-15^2}{2\times20\times15}$

C. $\dfrac{15}{\sin\theta}=\dfrac{16}{\sin 52°3'}$

D. $\dfrac{\sin\theta}{16}=\dfrac{\sin 52°3'}{15}$

8. Which pair of inequalities defines the shaded region?



A. $\begin{cases}x+y\geq4\\ y\geq2^{-x}\end{cases}$

B. $\begin{cases}x+y\leq4\\ y\geq2^{-x}\end{cases}$

C. $\begin{cases}x+y\geq4\\ y\leq2^{-x}\end{cases}$

D. $\begin{cases}x+y\leq4\\ y\leq2^{-x}\end{cases}$

9. Which parabola has a vertex at $(-1,2)$ and directrix $y=1$?

A. $(x-1)^2=4(y+2)$

B. $(x+1)^2=4(y-2)$

C. $(x-1)^2=8(y+2)$

D. $(x+1)^2=8(y-2)$

10. The quadratic equation $x^2-3x+1=0$ has roots α and β.
The value of $\alpha^2+\beta^2$ is ?

A. 1

B. 3

C. 7

D. 9

Section II

60 marks
Attempt Question 11 – 14
Allow about 1 hours 45 minutes for this section

Answer each question on a SEPARATE page of your own paper or writing booklet, if provided.

All necessary working should be shown in every question.

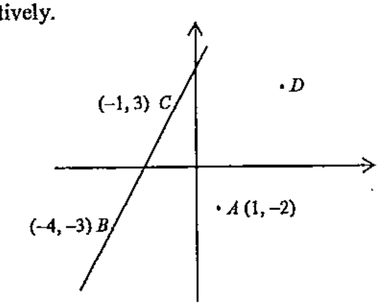
Marks

Question 11 (15 marks). Use a SEPARATE writing booklet.

(a) Factorise $2x^2+7x-4$ **2**

(b) Solve $|4-3x|=3-4x$ **3**

(c) In the diagram below the points A, B and C have coordinates $(1,-2)$, $(-4,-3)$ and $(-1,3)$ respectively.



(i) Calculate the exact length of interval BC. **2**

(ii) Find the gradient of BC. **1**

(iii) Hence, show that the equation of BC is $y=2x+5$. **1**

(iv) Find, to the nearest degree, the acute angle between the x–axis and the line BC. **1**

(v) Find the perpendicular distance between A and the line BC. **2**

(vi) Find the coordinates of D, in the first quadrant, so that $ABCD$ is a parallelogram. **2**

(vii) Find the exact area of the parallelogram $ABCD$. **1**

Marks

Question 12 (15 marks). Use a SEPARATE writing booklet.

(a) Find the equation of the line through the point of intersection of the lines $6x-5y=3$ and $4x+y=-11$ and also through the point $(2,1)$. **3**

(b) Solve $2\sin x+1=0$, for $0°\le x\le 360°$. **2**

(c) Differentiate with respect to x.

(i) $\dfrac{x^3-7x+1}{x}$ **2**

(ii) $2\sqrt{x}\left(3x^2-7\right)$ **2**

(iii) $\dfrac{5x}{x^2-1}$ **2**

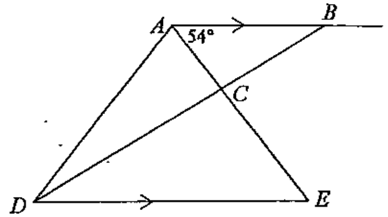
(d) Find the equation of the tangent to the curve $y=\dfrac{2}{x-1}$ at the point $P(3,1)$. **2**

(e) Show that $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$ if $x=4\sin A$ and $y=3\cos A$. **2**

Marks

Question 13 (15 marks). Use a SEPARATE writing booklet.

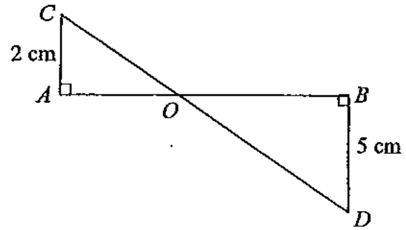
(a) In the diagram $AB \parallel DE$, $AE=DE$, $AE\perp BD$ and $\angle BAC=54°$.



Copy the diagram into your answer booklet showing all given information.

Calculate $\angle ADB$ giving reasons. **2**

(b) In the diagram, AOB and COD are straight lines. $AC\perp AB$ and $AB\perp BD$.



(i) Prove $\Delta ACO \,|||\, \Delta BDO$. **2**

(ii) If CD is 35 cm, find the length of OD. **2**

(c) The lengths of the sides of triangle ABC are 5.2 cm, 7.3 cm and 6.7 cm.

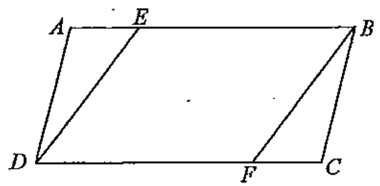
(i) Calculate the size of the smallest angle in ΔABC. Give the answer correct to the nearest minute. **2**

(ii) Hence find the area of the triangle. Give the answer correct to the nearest square cm. **1**

Marks

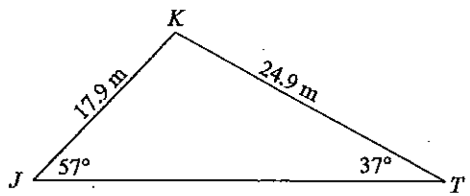
Question 13 continued. (15 marks).

(d) In the diagram $ABCD$ is a parallelogram and $AE = CF$, prove $\angle ADE = \angle CBF$. **3**



(e) Two boys, Joe and Tom, are flying kites in the local park when their kites collide midair.
The angles of elevation of Joe and Tom's kites are 57° and 37° respectively.
Joe's kite string is 17.9 metres and Tom's kite string was 24.9 m at the time of impact.

Using the information and the diagram given find the distance between Joe (*J*) and Tom (*T*) when the kites collide at point *K*. **3**
Give your answer correct to the nearest metre.



Marks

Question 14 (15 marks). Use a SEPARATE writing booklet.

(a) Find $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$ **1**

(b) Find the values of k for which $kx^2 = 6x + 9$ has no real roots. **2**

(c) Differentiate $f(x) = x^2 - x$ from first principles by using the definition **2**
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

(d) Solve $2^{2x} - 9(2^x) + 8 = 0$. **3**

(e) For the function $f(x) = \sqrt{4 - x^2} + 3$, find:

(i) the domain **2**

(ii) the range. **2**

(f) Find the values of a, b and c if $2x^2 + 3x + 1 = ax(x+2) + b(x+2) + c$. **3**

NSW INDEPENDENT TRIAL EXAMS – 2013

MATHEMATICS – YR 11 PRELIMINARY EXAMINATION
MARKING GUIDELINES

Section I

1	2	3	4	5	6	7	8	9	10
D	A	B	C	D	B	C	B	B	C

Section II.

Question11		Solution	Notes
a)		$2x^2+7x-4=(2x-1)(x+4)$	2 marks
b)		$\lvert 4-3x\rvert=3-4x$ $4-3x=3-4x$, $1=-x$, $x=-1$ OR $-(4-3x)=3-4x$, $-4+3x=3-4x$, $7x=7$, $x=1$ Test solutions: $\lvert 4-3(-1)\rvert=3-4(-1)$ $=7$ is a solution $\lvert 4-3(1)\rvert=3-4(1)$ $=-1$ is NOT a solution	1 for attempt at 2 solutions & $x=-1$ 1 for $x=1$ 1 for test & conclusion
c)	(i)	$BC=\sqrt{(-1+4)^2+(3+3)^2}$ $=\sqrt{9+36}$ $=\sqrt{45}$ $=3\sqrt{5}$	1 for correct substitution into correct formula 1 for solution
	(ii)	$m=\frac{3+3}{-1+4}$ $=\frac{6}{3}$ $=2$	1 mark
	(iii)	$y-3=2(x+1)$ $y-3=2x+2$ $y=2x+5$ OR $y+3=2(x+4)$ $y+3=2x+8$ $y=2x+5$	1 mark for correctly shown
	(iv)	$\tan\theta=2$ $\theta=63°26'$	1 mark
	(v)	$d=\frac{\lvert 2(1)-(-2)+5\rvert}{\sqrt{4+1}}$ $=\frac{9}{\sqrt{5}}$	1 for substitution in correct formula. 1 for the correct answer
	(vi)	(4, 4)	1 for each correct coordinate
	(vii)	$A=3\sqrt{5}\times\frac{9}{\sqrt{5}}$ $=27\text{ u}^2$	1 correct answer

Question12		Solution	Notes
a)		$6x-5y-3+k(4x+y+11)=0$ $6(2)-5(1)-3+k(4(2)+1+11)=0$ $12-5-3+20k=0$ $20k=-4$ $k=-\frac{1}{5}$ $6x-5y-3-\frac{1}{5}(4x+y+11)=0$ $5(6x-5y-3)-1(4x+y+11)=0$ $30x-25y-15-4x-y-11=0$ $26x-26y-26=0$ $x-y-1=0$	1 correct substitution into a correct formula. 1 correct k 1 correct equation
b)		$2\sin x+1=0$ $2\sin x=-1$ $\sin x=-\frac{1}{2}$ $x=(180+30)°,(360-30)°$ $x=210°,\ 330°$	1 for each correct answer
c)	(i)	$\frac{d}{dx}\left(\frac{x^3-7x+1}{x}\right)=\frac{d}{dx}(x^2-7+x^{-1})$ $=2x-x^{-2}$ $=2x-\frac{1}{x^2}$ OR $\frac{d}{dx}\left(\frac{x^3-7x+1}{x}\right)=\frac{x(3x^2-7)-(x^3-7x+1)}{x^2}$ $=\frac{3x^3-7x-x^3+7x-1}{x^2}$ $=\frac{2x^3-1}{x^2}$	1 for simplifying expression 1 for solution
	(ii)	$\frac{d}{dx}2\sqrt{x}(3x^2-7)=\frac{d}{dx}\left(6x^{\frac{5}{2}}-14x^{\frac{1}{2}}\right)$ $=15x^{\frac{3}{2}}-7x^{-\frac{1}{2}}$	1 for expansion (or use of product rule) 1 for solution
	(iii)	$\frac{d}{dx}\frac{5x}{x^2-1}=\frac{5(x^2-1)-5x\times 2x}{(x^2-1)^2}$ $=\frac{5x^2-5-10x^2}{(x^2-1)^2}$ $=\frac{-5-5x^2}{(x^2-1)^2}$	1 for application of quotient rule. 1 correct solution
d)		$y=2(x-1)^{-1}$ $y'=-2(x-1)^{-2}$ $=\frac{-2}{(x-1)^2}$ at $x=3$ $y'=-\frac{1}{2}$ $y-1=-\frac{1}{2}(x-3)$ $2y-2=-x+3$ $x+2y=5$	1 for gradient 1 for equation
e)		$\frac{16\sin^2 A}{16}+\frac{9\cos^2 A}{9}=\sin^2 A+\cos^2 A$ $=1$	1 for substitution 1 for show

Question13		Solution	Notes
a)		$\angle AED = 54°$ alternate $\angle$'s, $AB \parallel DE$ $\angle EAD = \frac{1}{2}(180 - 54)$ base angles of isosceles ΔAED $= 63°$ $\angle ABC = 180 - (90 + 54)$ $\angle$ sum of ΔABC $= 36°$ $\angle ADB = 180 - (36 + 54 + 63)$ $\angle$ sum of ΔABD $= 27°$	1 for 2 angles, 1 for solution
b)	(i)	In ΔACO and ΔBDO $\angle AOC = \angle BOD$ vertically opposite $\angle$'s equal $\angle OAC = \angle OBD$ given they equal 90° $\Delta ACO \parallel\!\mid \Delta BDO$ equiangular	1 1
	(ii)	Let $OD = x$ and $\therefore CO = 35 - x$ $\frac{BD}{AC} = \frac{OD}{OC}$ $\frac{5}{2} = \frac{x}{35 - x}$ $5(35 - x) = 2x$ $175 - 5x = 2x$ $175 = 7x$ $x = 25$ cm	1 1
c)	(i)	$\cos\theta = \frac{6.7^2 + 7.3^2 - 5.2^2}{2 \times 6.7 \times 7.3}$ $= \frac{71.14}{97.82}$ $\theta = 43°21'$	1 1
	(ii)	$A = \frac{1}{2} \times 6.7 \times 7.3 \times \sin 43°21'$ $= 16.785$ $= 17$ cm^2	1
d)		In ΔAED and ΔCFB, $AD = CB$ opposite sides of $\parallel$ogram equal $\angle DAE = \angle BCF$ opposite angles of $\parallel$ogram equal $AE = FC$ given $\therefore$ $\Delta AED \equiv \Delta CFB$ S.A.S $\therefore$ $\angle ADE = \angle CBF$ corresponding angles in congruent triangles equal.	1 for 2 equal sides or angles, 1 for $\equiv \Delta's$ and 1 for correct reason for angles
e)		$\angle JKT = 180 - (57 + 37) = 86°$ $JT^2 = 17.9^2 + 24.9^2 - 2 \times 17.9 \times 24.9 \times \cos 86°$ $= 878.2376842$ $JT = 29.635$ $JT = 30$ m OR $\frac{JT}{\sin 86°} = \frac{24.9}{\sin 57°}$ $JT = \frac{24.9 \sin 86°}{\sin 57°}$ $JT = 29.62$ $JT = 30m$	1 for $\angle K$ 1 for correct formula (cosine rule or sine rule) 1 correct answer (nearest m)

Question14		Solution	Notes
a)		$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$ $= 4$	1
b)		$kx^2 = 6x + 9$ $kx^2 - 6x - 9 = 0$ No real roots when $\Delta < 0$. $36 - 4 \times k \times (-9) < 0$ $36 + 36k < 0$ $36k < -36$ $k < -1$	1 1
c)		$f(x) = x^2 - x$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$ $= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$ $= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 1)}{\cancel{h}}$ $= \lim_{h \to 0} 2x + h - 1$ $= 2x - 1$	1 1
d)		$2^{2x} - 9(2^x) + 8 = 0$ Let $m = 2^x$ $m^2 - 9m + 8 = 0$ $(m - 8)(m - 1) = 0$ $m = 1,\ 8$ $2^x = 1,\ 2^x = 8$ $x = 0,\ 3$	2 marks for 1 and 8 1 for solution
e)	(i)	$4 - x^2 \geq 0$ $x^2 \leq 4$ $-2 \leq x \leq 2$	1 1
	(ii)	Range - $0 \leq \sqrt{4 - x^2} \leq 2$ Range - $3 \leq f(x) \leq 5$	1 1

f)		$2x^2+3x+1=ax(x+2)+b(x+2)+c$ Let $x=-2$ $2(-2)^2+3(-2)+1=0+0+c$ $8-6+1=c$ $3=c$ Let $x=0$ $1=2b+3$ $b=-1$ Let $x=-1$ $2(-1)^2+3(-1)+1=-a-1+3$ $2-3+1=-a+2$ $a=2$	1 1 1