SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
YEARLY EXAMINATIONS 2006

FORM I

MATHEMATICS

Examination date

Wednesday 8th November 2006

Time allowed

1 hour 30 minutes

Instructions

All ten questions may be attempted.
All ten questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Calculators are not to be used.

Collection

Write your name, class and master clearly on the front and on the tear-off sheet.
Hand in all the writing paper in a single well-stapled bundle.
Bundle the tear-off sheet with the question it belongs to.
Keep the printed examination paper and bring it to your next Mathematics lesson.

1A: TCW	1B: JMR	1C: JCM	104: BDD
105: KWM	106: FMW	107: SJE	108: LYL

Checklist

Writing paper required.
Candidature: 183 boys.

Examiner

FMW&LYL

QUESTION ONE   Start a new page.

(a) Evaluate:

   (i) $\dfrac{2}{3} \times \dfrac{1}{3}$

   (ii) $\dfrac{1}{8} + \dfrac{3}{4}$

   (iii) $5 \cdot 19 + 0 \cdot 043$

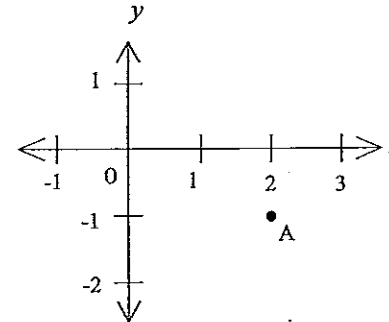(b) Find:

   (i) $-6 + 3$

   (ii) $1 - 6$

(c) Simplify:

   (i) $10x - 7x$

   (ii) $y^{10} \div y^4$

(d) Solve $2x = -18$.

(e)

Write down the coordinates of the point A in the diagram above.

(f) The 26 letters of the alphabet are written on cards and placed in a bag. If one card is drawn at random, what is the probability that it is the letter Z?

(g) Find 10% of \$60.

QUESTION TWO Start a new page.

(a) (i) Write down the 20th even number.

    (ii) Write down the 20th odd number.

(b) Expand $x(2x + 5)$.

(c) Express in decimal form:

    (i) 27%

    (ii) $\dfrac{3}{8}$

(d) Find:

    (i) $35 \div 0.7$

    (ii) $1\frac{1}{6} \div \frac{2}{3}$

    (iii) $\frac{8}{13} - \frac{2}{3}$

(e) Use a divisibility test to find out if 3 is a factor of 331 120. Explain your reasoning.

QUESTION THREE Start a new page.

(a) Express $1\frac{1}{5}$ as a percentage.

(b) Evaluate:

    (i) $-13 \div -2$

    (ii) $6 - (-10) \times 2$

(c) Find the cost of 5 apples if 9 apples cost \$4·50.

(d) Solve the following equations:

    (i) $3y - 8 = 10$

    (ii) $20 - x = 3x + 4$

(e) Keith rolls a 12-sided die with faces numbered 1 to 12. What is the probability of obtaining:

    (i) a multiple of 3,

    (ii) not a multiple of 3.

QUESTION FOUR Start a new page.

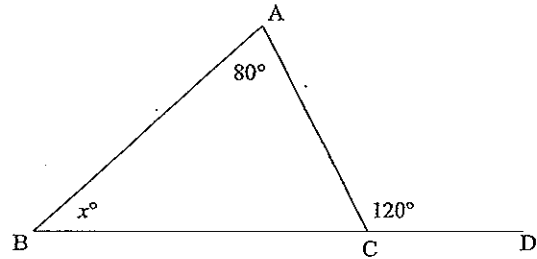(a) (i) Copy and complete the following table using the rule $y = 2x - 3$.

x	-1	0	1	2
y				

    (ii) Plot the points resulting from the table above on a number plane.
    Note: Use a ruler to draw your axes. Use 1 cm for 1 unit on both axes.

(b) Simplify:

    (i) $\left(m^3\right)^4$

    (ii) $3y^3 - 2y - 6y^3 - 8y$

    (iii) $\dfrac{p}{5} + \dfrac{2p}{3}$

    (iv) $6n^3 \times 3n^2$

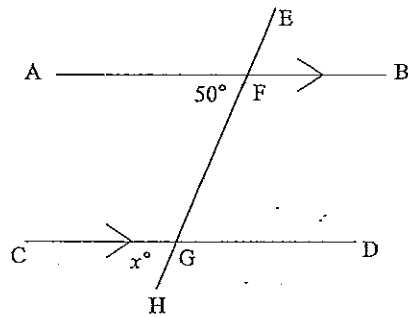(c) If $p = 3$ and $q = -4$, evaluate $p^3 - q$.

QUESTION FIVE   Start a new page.

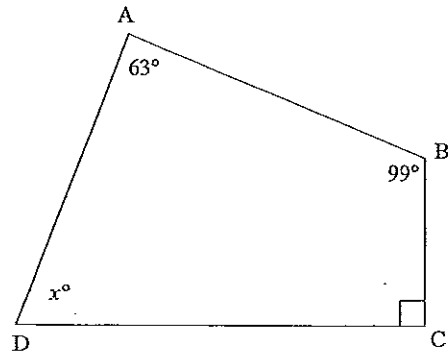(a) Find the value of x in each diagram below, giving reasons:

(i)

(ii)

(iii)

(b) Tear off the last sheet of this examination and do the constructions described there. Bundle the sheet with the rest of Question 5.

QUESTION SIX   Start a new page.

(a) Given that $18.5 \times 4.85 = 89.725$, evaluate:

   (i) 185×485

   (ii) $89.725 \div 1.85$

(b) Five doses of 12 mL are consumed from a full 250 mL bottle of medicine. What percentage of the medicine is left? Show your working.

(c) Solve the following problem by forming and then solving an equation.

   If a number is doubled and then increased by 6 it will be 13 more than its original value. Find the number.

(d) Expand and simplify:

$$3(x-2) + 2(x-4)$$

(e) (i) Express 432 as a product of its prime factors.

   (ii) Given that $60 = 2^2 \times 3 \times 5$, find the highest common factor of 60 and 432.

QUESTION SEVEN   Start a new page.

(a) Suppose $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{4, 6, 8\}$ and $D = \{0, 1, 2, 3, ..., 9, 10\}$.

  (i) State TRUE or FALSE for each of the following:

  (α) $7 \in D$

  (β) $D \subset B$

  (γ) $|D| = 10$

 (δ) $C \cap A = C$

  (ii) List all the subsets of C.

(b) 70 people were surveyed about whether they liked rock or classical music. 42 liked rock, 34 liked classical and 15 liked both types of music. Using a Venn diagram, determine how many people liked:

  (i) rock but not classical,

  (ii) neither rock nor classical.

(c) Draw two copies of the Venn diagram below and shade in the region represented by:
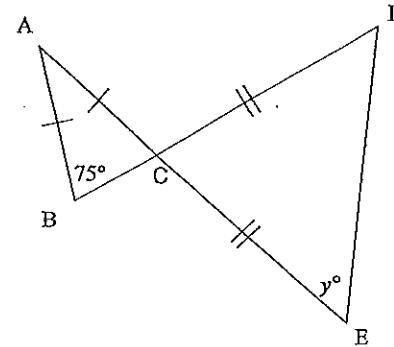
  (i) $X \cup Y$
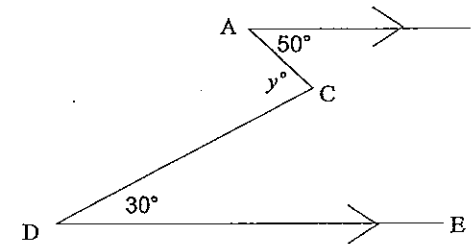
  (ii) $\overline{X \cap Y \cap Z}$
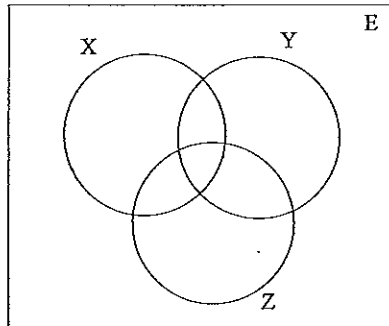
QUESTION EIGHT   Start a new page.

(a) Find the value of y in the each diagram below, giving reasons:

  (i)

  (ii)

(b) Simplify:

  (i) $\dfrac{3x^2}{10y} \times \dfrac{5y^2}{x^3}$

  (ii) $30m^6 n^8 \div -5m^2 n^7$

  (iii) $\dfrac{3m + 6m + m}{(5m^3)^2}$

QUESTION NINE   Start a new page.

(a) (i) Evaluate:

  (α) $(0{\cdot}16)^2$

  (β) $\sqrt{0{\cdot}16}$

  (ii) Hence, arrange the following decimals in ascending order:
  $0{\cdot}16$, $(0{\cdot}16)^2$, $\sqrt{0{\cdot}16}$

(b) For all numbers m and n, the operation $m@n$ is defined by $m@n = 3m^2n$.

  (i) Find y if $2@y = 54$.

  (ii) Find $\sqrt[3]{3@8}$.

  (iii) Find $(a@b)@c$.

(c) What digit is in the 52nd decimal place of $\dfrac{1}{35}$? Show your working.

QUESTION TEN   Start a new page.

(a) A palindromic number remains unchanged when its digits are reversed. For example, 636 and 175 571 are palindromic numbers.

  (i) How many 2 digit palindromic numbers are there?

  (ii) Find the total number of 3 digit numbers which are not palindromic.

(b) Samantha makes fruit drinks at the markets.

  (i) On Saturday, she mixes a 3 litre jug of apple drink containing 25% real juice with a 2 litre jug of orange drink containing 35% real juice. What percentage of the new mixture is real juice?

  (ii) On Sunday, she mixes the same 3 litre jug of apple drink with a different 2 litre jug of orange drink. What percentage of the 2 litre jug must be real juice if the final mixture is to contain 50% real juice?

(c) Given $x = 1$, $y = -2$ and $z = 3$, find the value of:

$$\cfrac{1}{x + \cfrac{x}{x + y + \cfrac{x + y}{x + y + z}}}$$

(d) The product of a particular six digit number and 5, is obtained by removing the last digit and placing it at the front. Find the number, clearly showing your reasoning.

END OF EXAMINATION

(1)(a)(i) $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ ✓

note - boys, the marking scheme is just a guide.

(ii) $\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8}$

$= \frac{7}{8}$ ✓

(iii) $5.190 +$
$\underline{0.043}$
5.233 ✓

(b)(i) $-6 + 3 = -3$ ✓
(ii) $1 - 6 = -5$ ✓

(c)(i) $10x - 7x = 3x$ ✓
(ii) $y^{10} \div y^4 = y^6$ ✓

(d) $2x = -18$
$x = -9$ ✓

(e) $(2, -1)$ ✓

(f) $\frac{1}{26}$ ✓

(g) $\frac{10}{100} \times 60 = \$6$ ✓ (ignore omission of units)

/11

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(2)

(a)(i) 40 ✓
(ii) 39 ✓

(b) $x(2x + 5) = 2x^2 + 5x$ ✓

(c)(i) $27\% = 0.27$ ✓

(ii) $8 \overline{) 3.0^60^40}$ 0.375 ✓

(d)(ii) $1\frac{1}{6} \div \frac{2}{3} = \frac{7}{6_2} \times \frac{3^1}{2}$ ✓

$= \frac{7}{4}$

$= 1\frac{3}{4}$

(i) $35 \div 0.7$
$= 350 \div 7$
$= 50$ ✓

(iii) $\frac{8}{13} - \frac{2}{3} = \frac{24 - 26}{39}$ ✓

$= -\frac{2}{39}$ ✓

(e) $3 + 3 + 1 + 1 + 2 + 0 = 10$
10 is not divisible by 3, so 3 is not a factor of 331120

(some reasoning required)

/11

③ (a) $\dfrac{6}{5} \times \dfrac{100}{1} = 120\%$ ✓

(b) (i) $-13 \div -2 = 6\frac{1}{2}$ ✓

(ii) $6 - (-10) \times 2 = 6 - (-20)$ ✓
$$= 26 \checkmark$$

(c) 9 apples cost $4.50
1 apple costs $\dfrac{4.50}{9}$ ✓✓

5 apples cost $\dfrac{4.50}{9} \times 5 = \$2.50$

(d) (i) $3y - 8 = 10$
$3y = 18$ ✓
$y = 6$ ✓

(ii) $20 - x = 3x + 4$ ✓
$16 = 4x$ ✓✓
$x = 4$ ✓

(e) (i) multiples of 3 are $3, 6, 9, 12$
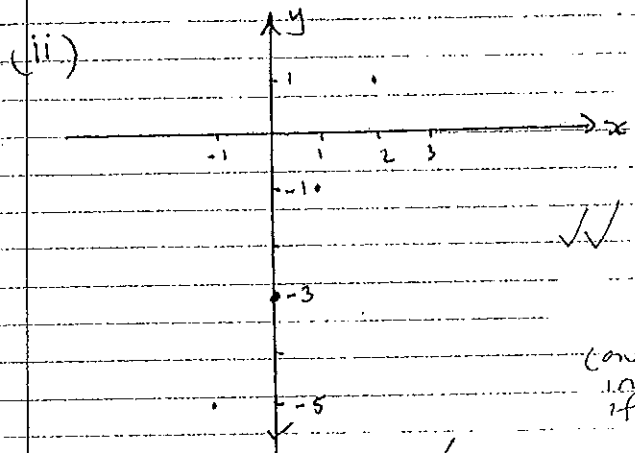
$P(E) = \dfrac{4}{12}$
$= \dfrac{1}{3}$ ✓

(ii) $P(E) = 1 - \dfrac{1}{3}$ ✓
$= \dfrac{2}{3}$ ✓ /11

④ (a) (i)

x	-1	0	1	2
y	-5	-3	-1	1	✓✓

(√x for at least one correct

(ii)

✓✓ (for points, must agree with (i))

(only penalise incorrect scale etc. if very poor)

(b) (i) $(m^3)^4 = m^{12}$ ✓

(ii) $3y^3 - 2y - 6y^3 - 8y = -3y^3 - 10y$ ✓

(iii) $\dfrac{p}{5} + \dfrac{2p}{3} = \dfrac{3p + 10p}{15}$ ✓
$= \dfrac{13p}{15}$ ✓

(iii) $6n^3 \times 3n^2 = 18n^5$ ✓✓
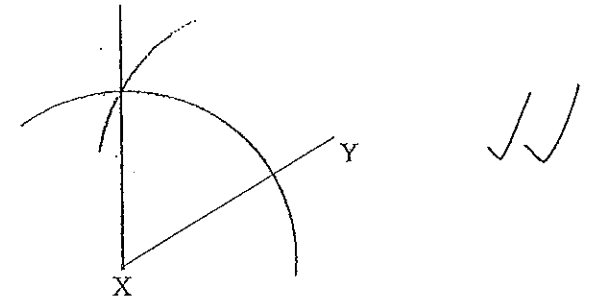
(c) $p^3 - q = 3^3 - (-4)$
$= 27 + 4$ ✓
$= 31$ /11

⑤ (a) (i) $x + 80 = 120$ (exterior angle of $\triangle ABC$)
$x = 40$

(ii) $x = 50$ (corresponding angles, $AB \parallel CD$)

(iii) $x + 63 + 99 + 90 = 360$ (angle sum of
$x = 108$ quadrilateral)

(b) see following sheet

QUESTION FIVE

(b) Using a pencil, ruler and compasses only, complete the following constructions. Do not erase any construction markings.

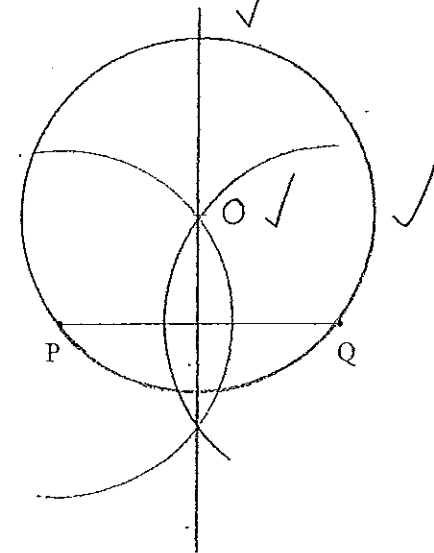(i) Construct a 60° angle at the endpoint X of the interval XY below.

(ii) The interval PQ is shown below.

(α) Construct the perpendicular bisector of the interval PQ.

(β) Mark the point O, 2 cm above PQ on the bisector you have just drawn.

(γ) Draw the circle, with centre O, which passes through the points P and Q.

6 (a) (i) $185 \times 485 = 89725$ ✓ ✓

(ii) $89.725 \div 1.85 = 48.5$ ✓

(b) $250 - 5 \times 12 = 250 - 60$ ✓

$= 190 \, mL$ ✓

$\dfrac{190}{250} \times 100\% = 76\%$ ✓

(c) let the number be m ✓
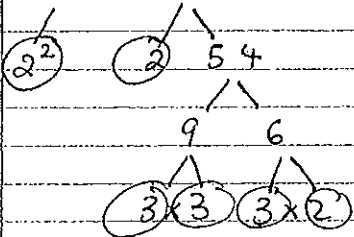
$2m + 6 = m + 13$

$m = 7$ ✓

the number is 7.

(d) $3(x-2) + 2(x-4) = 3x - 6 + 2x - 8$ ✓

$= 5x - 14$ ✓

(e) (i) 432 $432 = 2^4 \times 3^3$ ✓✓

4×108

2^2 2 54

9 6

3×3 3×2

(ii) $HCF = 2^2 \times 3$ ✓

$= 12$

H

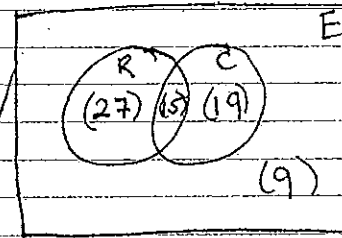**7 (a)** (i) (4) T ✓

(3) F ✓

(18) F ✓

(6) T ✓

(ii) $\emptyset, \{4\}, \{6\}, \{8\}, \{4,6\}, \{6,8\}, \{4,8\},$

$\{4,6,8\}$ ✓

($\sqrt{}$ x for 4 or more correct)
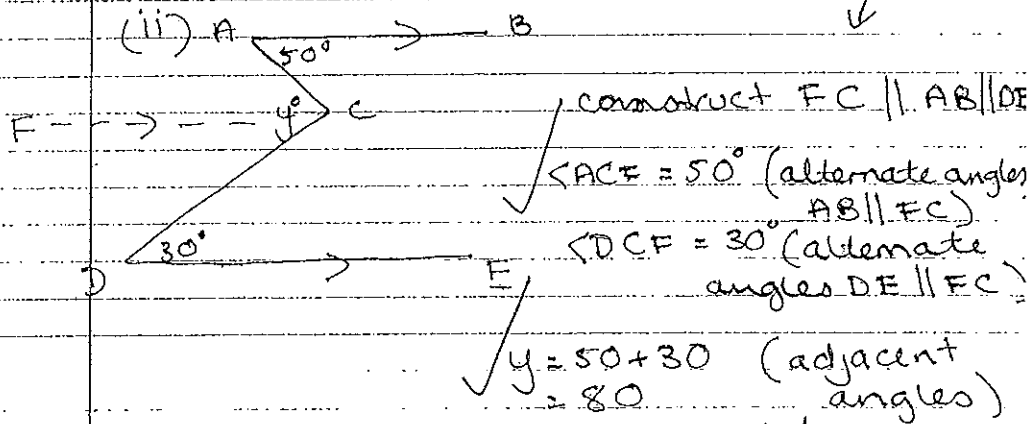
(b)

(i) 27 ✓

(ii) 9 ✓

(c)

✓

✓

11
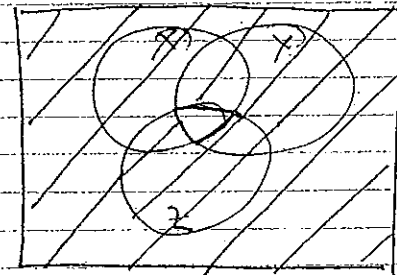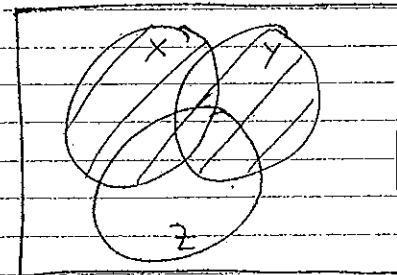
⑧ (a)(i) ∠ACB = 75° (angles opposite equal sides)
      ∠DCE = 75° (vertically opposite angles)
      ∠CDE = y° (angles opposite equal sides)

   2y + 75 = 180 (angle sum of △DCE)
      2y = 105
      y = 52½°   ↑ (must have reasons)

(ii) A ⟶ B
       50°
   F ⟶ — — y° C   construct FC ∥ AB ∥ DE
              30°   ∠ACF = 50° (alternate angles
   D ⟶ E   AB ∥ FC)
                             ∠DCF = 30° (alternate
                                 angles DE ∥ FC)

              y = 50 + 30 (adjacent
                = 80   angles)

(b)(i) $\dfrac{3x^2}{10y} \times \dfrac{5y^2}{x^3} = \dfrac{3y}{2x}$

(ii) $30m^6n^8 \div -5m^2n^7 = -6m^4n$

(iii) $\dfrac{3m + 6m + m}{(5m^3)^2} = \dfrac{10m}{25m^6}$
                = $\dfrac{2}{5m^5}$

⑨ (a)(i) (α) $(0.16)^2 = 0.0256$
          (β) $\sqrt{0.16} = 0.4$

(ii) in ascending order we have:
        $(0.16)^2, 0.16, \sqrt{0.16}$

(b)(i) $2 @ y = 3 \times 2^2 \times y$
              = 12y
        12y = 54
          y = 4½

(ii) $\sqrt[3]{3 @ 8} = \sqrt[3]{3 \times 3^2 \times 8}$
              = $\sqrt[3]{3^3 \times 2^3}$
              = 3 × 2
              = 6

(iii) $(a @ b) @ c = (3a^2 b) @ c$
              = $3 \times (3a^2 b)^2 \times c$
              = $27 a^4 b^2 c$

(c)   0.0̇285714̇
     35) 1.00000000
            70 × × × × × ×
           300
           280   52 = 1 + 6 × 8 + 3
           200
           175   the 52nd decimal
           250   place is a 5
           245
            50
            35
           150
           140
           100 etc.

⑩ (a) (i) 11, 22, 33, ... , 99

there are 9

(ii) total number of 3 digit numbers is
$$9 \times 10 \times 10 = 900$$

palindromic 3 digit numbers are
101, 111, 121, 131, ... , 191
202, 212, 222, ... , 292
etc. $\Bigg\} \begin{array}{l} 10 \times 9 \\ = 90 \end{array}$

909, 919, 929, ... , 999 $\Bigg\}$

So the total number of 3 digit numbers
which are not palindromic is
$$900 - 90 = 810$$

(b)
(i) $\dfrac{25}{4 \,+100} \times 3 + \dfrac{35}{100} \times 2 = \dfrac{3}{4} + \dfrac{7}{10}$

$= \dfrac{15 + 14}{20}$

$= \dfrac{29}{20}$

$\dfrac{\frac{29}{20}}{5} \times 100 = \dfrac{29}{20} \times \dfrac{1}{5} \times 100$

$= 29\%$

(ii) 5% of 5L $= 2\frac{1}{2}$ L

$2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$

$\dfrac{1\frac{3}{4}}{2} \times 100 = \dfrac{7}{4} \times \dfrac{1}{2} \times 100$

$= 7 \times 12\frac{1}{2}$

$= 87\frac{1}{2}\%$

(c)
$$\cfrac{1}{x + \cfrac{x}{x+y+\cfrac{x+y}{x+y+z}}} = \cfrac{1}{1 + \cfrac{1}{1-2+\cfrac{1-2}{1-2+3}}}$$

$= \cfrac{1}{1 + \cfrac{1}{-1 + \cfrac{-1}{2}}}$

$= \cfrac{1}{1 + \cfrac{1}{-\frac{3}{2}}}$

$= \cfrac{1}{1 - \cfrac{2}{3}}$

$= \cfrac{1}{\frac{1}{3}}$

$= 3$

(d) let the five digit number be
x and the last digit be y.
Then $\quad x + 100000\, y = 5(10x + y)$

$x + 100000\, y = 50x + 5y$

$49x = 99995\, y$

$7x = 14285\, y$

So $14285\, y$ is divisible by 7

But 14285 is not divisible
by 7 so y is divisible
by 7. As y is a 1 digit
number y must be 7
and x is 14285.

Therefore the number is
14 2 8 5 7