SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2009

Time Allowed: 70 minutes

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new side of a page.
- A table of standard integrals is supplied.

Name:

Q1	Q2	Q3	Q4	Q5	Total
/12	/12	/12	/12	/12	/60

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x > 0$

Question 1 (12 marks)

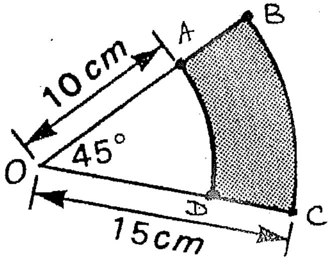
a) Express 1.45 radians in degrees and minutes (correct to nearest minute) 1

b) Find the exact value of $\tan\frac{2\pi}{3}$ 1

c) Solve $\tan x = \sqrt{2} - 1$, for $0^\circ \le x \le 360^\circ$ (leaving your answer correct to the nearest minute) 2

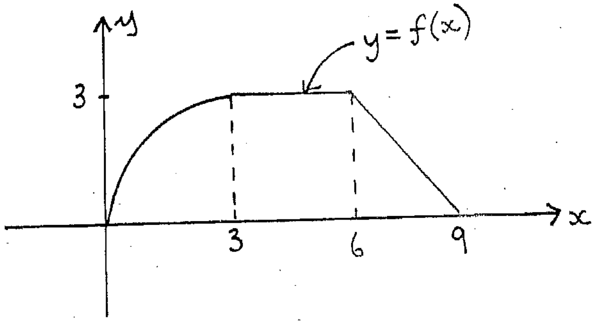
d) i) Express $45°$ in radians, in terms of π. 1

ii) Find the **area** of the shaded section ABCD, below (in terms of π) 2

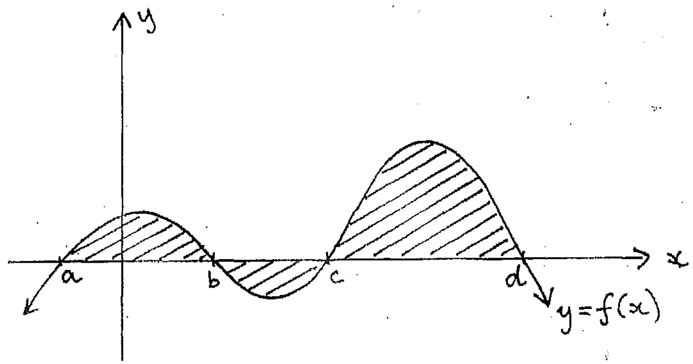


iii) Find the **perimeter** of the shaded section ABCD, above (in terms of π) 2

e) Find $\int_0^9 f(x)\,dx$, given the sketch below (in exact form). 2



f)



To calculate the shaded area above, would the evaluation of $\int_a^d f(x)dx$ give the correct solution? Explain your answer. 1

Question 2 (Start a new page) (12 marks)

a) Find k, if $\int_0^3 kx^2dx = 4$ 2

b) Evaluate $log_3 2$, correct to 2 decimal places 1

c) Solve $log_x 27 = \frac{3}{2}$ 2

d) Simplify $log_5 125 - log_5 \frac{1}{25} - log_5 \sqrt{5}$ 2

e) Differentiate the following:

i) $y = 3\ ln\,5x$ 1

ii) $y = ln(2 - 3x)$ 1

iii) $y = e^{2x}$ 1

iv) $y = 2cos3x$ 2

Question 3 (Start a new page) (12 marks)

a) Find $\frac{d}{dx}(\sqrt{x}.\,lnx)$ 2

b) Evaluate $\int_1^e \frac{2}{x}\,dx$ 2

c) Find $\int \frac{3-x}{12x-3-2x^2}\,dx$ 2

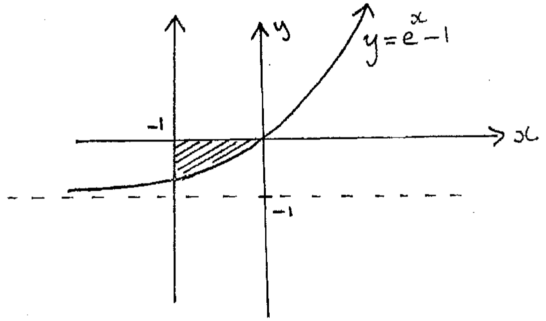
d) Find the equation of the tangent to the curve $y = tan2x$, at the point where $x = \frac{\pi}{6}$ 3

e) The curve $y = \frac{1}{x^2}$ is called a truncus. It is rotated around the y axis from $y = 1$ to $y = 6$. Find the volume of the solid formed (in exact form). 3

Question 4 **(Start a new page)** **(12 marks)**

a) Find $\int e^{7-2x}dx$ 1

b) Find the area of the shaded section below. that is bounded by the x axis, the curve $y = e^x$-1, and the line $x = -1$ (in exact form) 3



c) Differentiate $y = \frac{cos2x}{e^x}$ 2

d) i) Sketch $y = 2sin3x$, for $0 \leq x \leq \frac{\pi}{3}$ 2

ii) Find the area of the region bounded by $y = 2sin3x$, and the x axis. in your sketch above. 3

e) Find $\lim_{x \to 0} \frac{sinx}{2x}$ 1

Question 5 **(Start a new page)** **(12 marks)**

a) i) Find $\frac{d}{dx}(sinx - xcosx)$ 3

ii) Hence, find $\int_0^{\frac{\pi}{2}} x.\sin x\, dx$. 2

b) Consider the curve $y = xlnx$

i) Find its domain 1

ii) Find any stationary points on the curve, and determine their nature. 3

iii) Explain why the curve has no points of inflexion. 1

iv) Sketch the curve, showing any stationary points, and where curve cuts the x and y axes, if it does so. 2

S.T.H.S 2UNIT HSC TASK 3 JUNE 2009

QUESTION 1

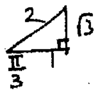
a) $1.45^c = \frac{1.45 \times 180}{\pi}$

$= 83° 5'$

b) $\tan \frac{2\pi}{3} = \tan\left(\pi - \frac{\pi}{3}\right)$ (S | A ✓ / T | C)

$= -\tan \frac{\pi}{3}$

$= -\sqrt{3}$



c) $\tan x = \sqrt{2} - 1$ (S | A ✓ / ✓ T | C)

$= .414$

$\therefore x = 22°30', \ 202°30'$

d) i) $45° = \frac{\pi}{4}$

ii) $A = \frac{1}{2} \cdot 15^2 \cdot \frac{\pi}{4} - \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{4}$

$= \frac{225\pi}{8} - \frac{100\pi}{8}$

$= \frac{125\pi}{8}$ cm^2

iii) $P = 10 + 10 \cdot \frac{\pi}{4} + 15 \cdot \frac{\pi}{4}$

$= \left(\frac{25\pi}{4} + 10\right)$ cm

e) $\int_0^9 f(x)\,dx = \frac{\pi \cdot 3^2}{4} + 9 + \frac{9}{2}$

$= \frac{9\pi}{4} + \frac{27}{2}$

f) Incorrect solution – need to take abs. value of area below axis

QUESTION 2

a) $\int_0^3 kx^2\,dx = 4$

$\left[\frac{kx^3}{3}\right]_0^3 = 4$

$9k - 0 = 4$

$9k = 4$

$k = 4/9$

b) $\log_3 2 = \frac{\log_e 2}{\log_e 3}$

$= 0.63$

c) $\log_x 27 = \frac{3}{2}$

$\therefore x^{3/2} = 27$

$x = 27^{2/3}$

$x = 9$

d) $\log_5 125 - \log_5 \frac{1}{25} - \log_5 \sqrt{5}$

$= \log_5 5^3 - \log_5 5^{-2} - \log_5 5^{1/2}$

$= 3\log_5 5 + 2\log_5 5 - \frac{1}{2}\log_5 5$

$= 3 + 2 - \frac{1}{2}$

$= 4\frac{1}{2}$

e) i) $y = 3\ln 5x \ \therefore \frac{dy}{dx} = \frac{3}{x}$

ii) $y = \ln(2-3x) \therefore \frac{dy}{dx} = \frac{-3}{2-3x}$

iii) $y = e^{2x} \ \therefore \frac{dy}{dx} = 2e^{2x}$

iv) $y = 2\cos 3x \ \therefore \frac{dy}{dx} = -6\sin 3x$

QUESTION 3

a) $u = \sqrt{x} = x^{1/2}$ $\quad v = \ln x$

$u' = \frac{1}{2}x^{-1/2}$ $\quad v' = \frac{1}{x}$

$= \frac{1}{2\sqrt{x}}$

$\therefore \frac{d}{dx}(\sqrt{x}\ln x) = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}$

b) $\int_1^e \frac{2}{x}\,dx = \left[2\ln x\right]_1^e$

$= 2\ln e - 2\ln 1$ (ln 1 = 0)

$= 2$

c) $\int \frac{3-x}{12x-3-2x^2}\,dx$

$= \frac{1}{4}\int \frac{12-4x}{12x-3-2x^2}\,dx$

$= \frac{1}{4}\ln\left(12x-3-2x^2\right)$

d) $y = \tan 2x$ $\quad y = \tan\frac{\pi}{3}$

$\frac{dy}{dx} = 2\sec^2 2x$ $\quad \therefore y = \sqrt{3}$

at $\left(\frac{\pi}{6}, \sqrt{3}\right)$ $\quad m = 2\sec^2 2 \times \frac{\pi}{6}$

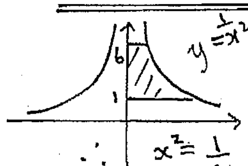
$m = 2\sec^2 \frac{\pi}{3}$

$m = 8$

$\therefore$ tangent

$y - \sqrt{3} = 8\left(x - \frac{\pi}{6}\right)$

e)



$\therefore x^2 = \frac{1}{y}$

$V = \pi \int_1^6 \frac{1}{y}\,dy$

$= \pi\left[\ln y\right]_1^6$

$V = \pi(\ln 6 - \ln 1)$

$= \pi \ln 6$ units^3

QUESTION 4

a) $\int e^{7-2x}\,dx = \frac{-1}{2}e^{7-2x} + C$

b) $A = \left|\int_{-1}^{0} e^x - 1\,dx\right|$

$= \left|\left[e^x - x\right]_{-1}^{0}\right|$

$= \left|(1+0) - (e^{-1}+1)\right|$

$= \left|-\frac{1}{e}\right|$

$= \frac{1}{e}$ unit^2

c) $u = \cos 2x$ $\quad v = e^x$

$u' = -2\sin 2x$ $\quad v' = e^x$

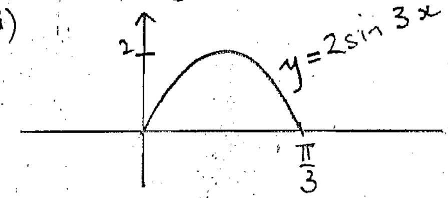
$\frac{dy}{dx} = \frac{-2e^x \sin 2x - e^x \cdot \cos 2x}{e^{2x}}$

$= \frac{e^x(-2\sin 2x - \cos 2x)}{e^{2x}}$

$= \frac{-2\sin 2x - \cos 2x}{e^x}$

d) period $\frac{2\pi}{3}$ $\quad$ amp $= 2$

i)



ii) $A = \int_0^{\pi/3} 2\sin 3x \, dx$

$= \left[-\frac{2}{3}\cos 3x\right]_0^{\pi/3}$

$= \left[-\frac{2}{3}\cos\pi - -\frac{2}{3}\cos 0\right]$

$= \left[\frac{2}{3} + \frac{2}{3}\right]$

$= \frac{4}{3}$ units^2

c) $\lim_{x \to 0} \frac{\sin x}{2x}$

$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{2}$

$= \frac{1}{2}$

QUESTION 5

a) i) $u = -x \quad v = \cos x$

$u' = -1 \quad v' = -\sin x$

$\therefore \frac{d}{dx}(\sin x - x\cos x)$

$= \cos x - \cos x + x\sin x$

$= x\sin x$

ii) $\int_0^{\frac{\pi}{2}} x\sin x \, dx$

$= \left[\sin x - x\cos x\right]_0^{\pi/2}$

$= \left(\sin\frac{\pi}{2} - \frac{\pi}{2}\cos\frac{\pi}{2}\right) - (0)$

$= 1$

b) $y = x\ln x$

i) $x > 0$

ii) $u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$\frac{dy}{dx} = \ln x + 1$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

st pts $y' = 0 \quad \ln x + 1 = 0$

$\log_e x = -1$

$\therefore x = e^{-1}$

at $\left(\frac{1}{e}, \frac{-1}{e}\right)$ $y'' > 0 \therefore$ min

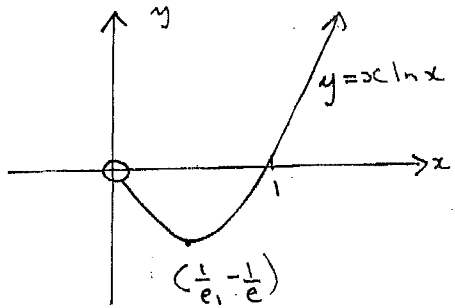
if $x = \frac{1}{e}$ $y = \frac{1}{e}\ln e^{-1}$

$y = -\frac{1}{e}$

iii) pt inf $y'' = 0$

since $\frac{1}{x} \neq 0 \therefore$ no pt inf.

iv)



$y = 0 \qquad x \cdot \ln x = 0$

$x \neq 0 \quad \ln x = 0$

$\log_e x = 0$

$e^0 = x$

$\therefore x = 1$

cut x axis at $x = 1$