OUR LADY OF THE SACRED HEART COLLEGE
KENSINGTON

STUDENT – NAME / NUMBER

MATHEMATICS TEACHER .

Year 11

Mathematics
Extension 1
Assessment 3

2008

Time allowed: **45 minutes**

Assessed Outcomes

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Directions to Candidates

- Attempt all questions
- START EACH QUESTION ON A NEW PAGE
- Show all necessary working on the paper
- Calculators may be used
- Good Luck!!

Question 1 (11 marks)

a) Divide the interval AB externally in the ratio $4:3$, where A is the point $(2,-1)$ and B is $(1,-3)$.

2 marks

b) Write the general form for the equation of a line with an angle of inclination of 135°, cutting through the point A(0,3) . 2 marks

c) Triangle ABC has vertices A(1,0), B(6,5) and C(0,3)

i) Show that ΔABC is right- angled. 2 marks

ii) Find the equation of AB 2 marks

d) Find the equation of the line through the intersection of the lines $x-y-3=0$ and $3x+y-5=0$ and the point (0,-2) **without** finding the point of intersection of the lines.

3 marks

Question 2: (8 marks)

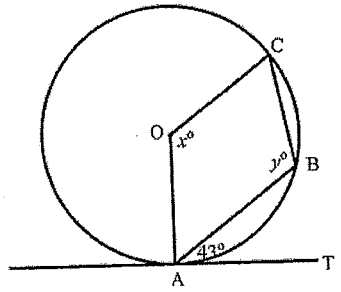
a) Show that the acute angle θ between the straight lines $y=x+2$ and $y=mx+c$ is given by $\tan\theta=\dfrac{m-1}{m+1}$. 2 marks

b) The poing P (-2,5) divides the interval joining A (-4,1) and B (x,y) externally in the ratio 2:3. Find the coordinates of point B. 2 marks

c) A line passing through (7,5) makes a right angle with the line $4x-y=6$ at the point P. Find the coordinates of P. 4 marks

Question 3: (11 marks)

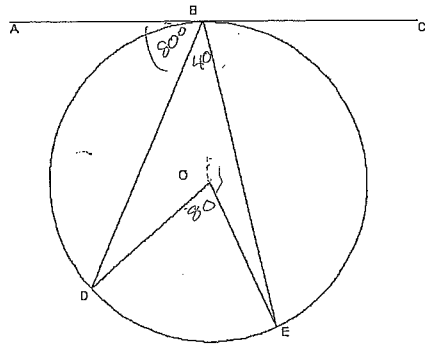
a)



AT is a tangent to a circle center O. OC is *parallel* to AB and angle BAD is 43°. Let angle AOC be $x°$ and angle ABC be $y°$. Copy the diagram and find the values of x and y, giving reasons.

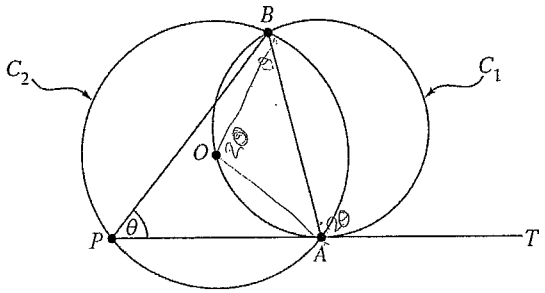
4 marks

b) In the diagram below, O is the centre of the circle, AC is a tangent at B and D and E are points on the circumference. If $\angle ABD = 80^\circ$ and $\angle DBE = 40^\circ$, find the size of $\angle BEO$, giving reasons



3 marks

c) Two circles, C_1 and C_2, intersect at points A and B. Circle C_1 passes through the centre O of circle C_2. The point P lies on circle C_2 so that the line PAT is tangent to circle C_1 at point A. Let $APB = \theta$



d) Copy or trace the diagram into your writing booklet.

(i) Find AOB in terms of θ. Give a reason for your answer. 1 mark

(ii) Explain why $TAB = 2\theta$. 1 mark

(iii) Deduce that $PA = BA$. 2 marks

question 1

1) ratio becomes $4:-3$ A $(2,-1)$ B $(1,-3)$

$(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$(x,y) = \left(\frac{4(1) + -3(2)}{4-3}, \frac{4(-3) - 3(-1)}{4-3}\right)$

$(x,y) = (-2, -9)$

b) A $(0,3)$ $m = -1$

$y - y_1 = m(x - x_1)$

$y - 3 = x$

$x + y - 3 = 0$

c) i)

B $(6,5)$

C $(0,3)$

A $(1,0)$

$m_{BC} = \frac{5-3}{6-0}$

m of CB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-5}{0-6}$

$= \frac{-2}{-6}$

$= \frac{2}{6}$

m of AC $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-0}{0-1}$

you must prove it

Question 1 cont

ii) A (x_1, y_1) (1, 0) B (x_2, y_2) (6, 5)

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y = \frac{5 - 0}{6 - 1}(x - 1)$

$y = \frac{5}{5}(x - 1)$

$y = 1(x - 1)$

$y = x - 1$ ✓

(2)

d) $x - y - 3 = 0$, $3x + y - 5 = 0$

(0)

$(x - y - 3) - \frac{1}{7}(y + 3x - 5) = 0$

$7x - 7y - 21 - y - 3x + 5 = 0$

$4x - 8y - 16 = 0$

$x - 2y - 4 = 0$

(5)

Question 2

a) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \times m_2}\right|$ $y = x + 2$ $y = mx + c$

$m_1 = 1$ ✓ $m_2 = 1$ ✗

$= \left|\frac{1 - 1}{1 + :}\right|$ $m_1 = 1$ $m_2 = m$

$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

$\frac{m - 1}{1 + m}$

(1)

b) $(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$ ratio 2 : 3 (m n)

A (x_1, y_1) (−4, 1) B (x_2, y_2) (x, y)

$P(-2, 5) = \left(\frac{2(x) + 3(-4)}{2 - 3}, \frac{2(y) + 3(1)}{2 - 3}\right)$

$\frac{-2 = 2x + 12}{-1}$ $\frac{5 = 2y - 3}{-1}$

$2 = 2x + 12$ $-5 = 2y - 3$

$-10 = 2x$ ✓ $-2 = 2y$

$-5 = x$ ✓ $4 = y$ ✗ $y = -1$

∴ B(−5, 4)

(1)

c) eqn of point (x_1, y_1) (7, 5)

$4x - y = 6$

$m_1 = 4$ $m_2 = -4$ ✗

$\frac{-1}{4}$

$y - y_1 = m(x - x_1)$

$y - 5 = -4(x - 7)$ ✓

$y - 5 = -4x + 28$ ✓

$4x - 28 + y - 5 = 0$

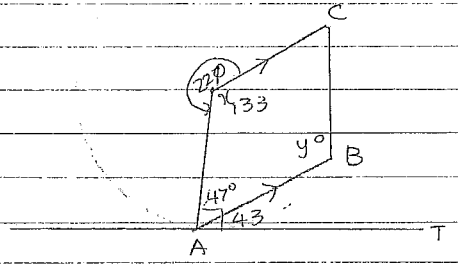
$4x - 33 + y = 0$ ✓

(3)

Q2 — (5)

Question 3

a)



OA is a radius.

$\therefore \angle OAT = 90 - 43°$ (radius and tangent are ⊥)

$\angle OAT = 47°$

$\therefore \angle x = 180 - 47$ (cointerior ∠s)

$= 133°$

reflex $\angle COA = 360 - 133$ (sum of revolution ∠)

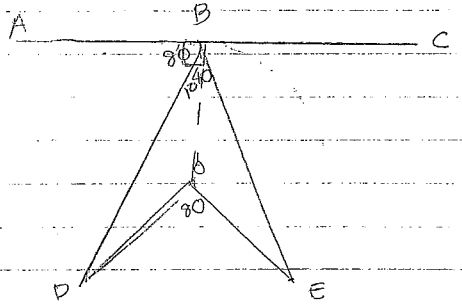
$= 227°$

$\therefore \angle y = \frac{227}{2}$ (∠ at centre = 2x ∠ at circumference)

$= 113.5°$

4

b)



$\angle DBO = 90° - 80°$ (radius is ⊥ to tangent ABC)

$\therefore \angle DBO = 10°$

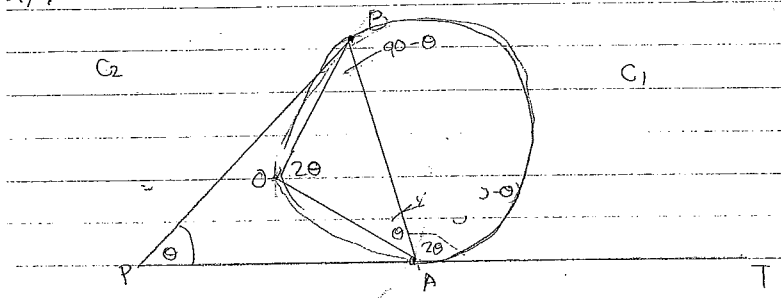
$\therefore \angle OBE = 30°$

$\therefore \angle BEO = 30°$ (base ∠s of isosceles △)

3

Question 3 cont

i) ii)



i) $\angle AOB = 2\theta$ (∠ at centre = 2x ∠ at circumference)

ii) $\angle TAB = 2\theta$ (2 internal ∠'s of △ added = external ∠)

iii) $\angle TAB = 2\theta$

$\angle BPA = \theta$

$\therefore \angle PBA = 2\theta - \theta$ (2 internal ∠'s of △ added = external ∠)

$= \theta$

$\therefore PA = BA$ (base ∠'s of isosceles △ are =, $\angle BPA = \angle PBA$)

4

Excellent work