REGNET CHRISTUS

OLSH COLLEGE

STUDENT – NAME / NUMBER. _____

MATHEMATICS TEACHER _____

2010

Preliminary – Assessment 3

Extension 1 Mathematics

Time allowed: **50 minutes**

Assessed Outcomes:

H5 applies appropriate techniques from the study of trigonometry to solve problems

PE3 solves problems involving trigonometry

Directions to Candidates

- Questions are of equal value – 11 marks
- Total mark is 33
- Show all working
- READ the questions carefully
- Board approved calculators may be used as well as mathematical templates

Question One: 11 marks

(a) Simplify

$\tan A \sin A + \cos A$ **2 marks**

(b) If $\cos\theta = 0.6$ and θ is acute, find $\sin\theta$ without the use of a calculator **2 marks**

(c) Solve

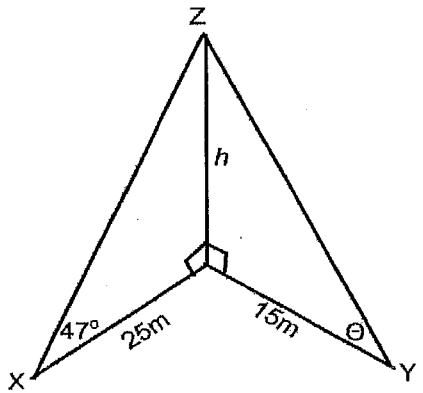
(i) $2\cos 2\theta + 1 = 0$ for $0 \le \theta \le 360^0$ **2 marks**

(ii) $3\sin^2 x - 4\sin x = 4$ for $0 \le \theta \le 360^0$ **3 marks**

(d) Sketch the curve $y = 2 + \sin\theta$ between $0 \le \vartheta \le 360$ **2 marks**

Question Two: 11 marks

(a) From point X, 25m due south of the base of a tower, the angle of elevation is 47°. Point Y is 15m due east of the tower

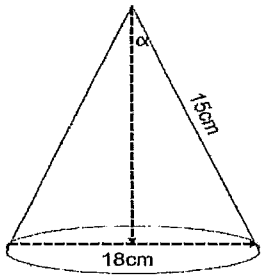


Find:

(i) the height, h of the tower, correct to 1 decimal place. **1 mark**

(ii) the angle of elevation of the tower from point Y. (nearest degree) **1 mark**

(b) A cone has a base diameter of 18cm and a slant height of 15cm. Find the vertical angle at the top of the cone.(to the nearest minute)



2 marks

(c) A plane is flying due east at 600km/h at a constant altitude. From an observation point P on the ground, the plane is sighted on a bearing of 320°. One minute later, the bearing of the plane is 75° and its angle of elevation is 25°.

(i) Draw a diagram to represent the given information **1 mark**

(ii) Show that the altitude, h metres of the plane is given by $h = \dfrac{10000 \sin 50 \tan 25}{\sin 65}$

3 marks

(iii) Find, correct to the nearest degree, the angle of elevation of the plane from P when it was first sighted **3 marks**

Question Three: (11 marks)

(a) From a point A, the angle of elevation of the top of a tower due north of it is 20°. From B, due east of the tower, the angle of elevation is 18°. A and B are 100m apart.

(i) Draw a labelled diagram illustrating the information above **1 mark**

(ii) Show that the height h of the tower is given by **3 marks**

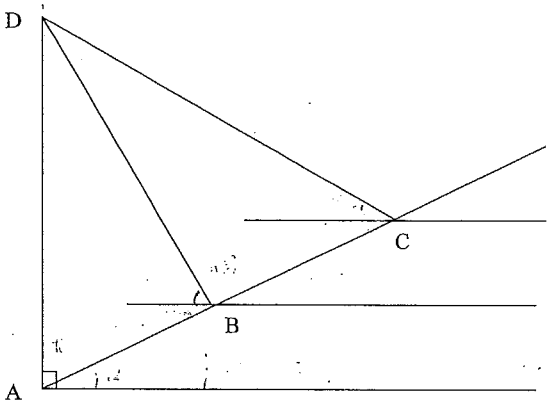
$$h = \frac{100}{\sqrt{(\tan^2 72 + \tan^2 70)}}$$

(iii) Hence find the value of h **1 mark**

(b)

A, B and C are three points, in order, on a straight road that runs up a hillside. The hillside makes a constant angle of 12^0 with the horizontal. A building of height AD stands at A. From B and C, the top of the building has elevations of 40^0 and 15^0 respectively above the horizontal. BC is 200 *m*.

(i) Copy and complete the following diagram showing all the above information. **1 mark**



(ii) Show that $DB = \dfrac{200\sin 27^0}{\sin 25^0}$ **2 mark**

(iii) Show that $AD = \dfrac{200\sin 27^0 \sin 52^0}{\sin 25^0 \sin 78^0}$ **2 marks**

(iv) Find AD correct to one decimal place **1 mark**

Question one:

a) $\tan A \sin A + \cos A$

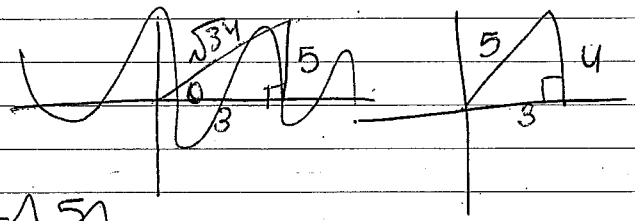
$\frac{\sin A}{\cos A} \times \sin A + \frac{\cos^2 A}{\cos A}$ ✓

$\frac{\sin^2 A + \cos^2 A}{\cos A}$ (2)

$= \frac{1}{\cos A} = \sec A$ ✓

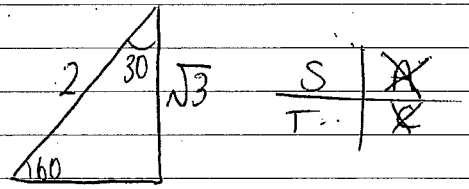
b) $\cos \theta = 0.6$. θ is acute

$\cos \theta = \frac{3}{5}$ ✓



~~sin θ = √5~~

$\sin \theta = \frac{4}{5}$ ✓ (2)



S	~~A~~
T	~~C~~

c) i) $2\cos 2\theta + 1 = 0$

$\cos 2\theta = -\frac{1}{2}$ ✓ (1)

reference 60

$\theta = 60$

$\cos 2\theta = 120°, 240°, 480, 600$

$\theta = 60°, 120°, 240°, 300°$ ✓ (1)

iii) $3\sin^2 x - 4\sin x = 4$

let $\sin x = m$

$3m^2 - 4m = 4 = 0$ ✓ (1)

$(3m - 2)(3m \quad) = 0$

3

$3m^2 - 4m = 4$

$m(3m - 4) = 4$

~~$m = 4$~~

~~2 m~~

let $m = \sin x$

$3m^2 - 4m - 4 = 0$

$(3m + 2)(m - 2) = 0$

$m = -\frac{2}{3}, 2$

$\sin x = -\frac{2}{3}$ or $\sin x = 2$

NO solution

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$4 \pm \sqrt{(-4)^2 - 4 \times 3 \times -4}$

2×3

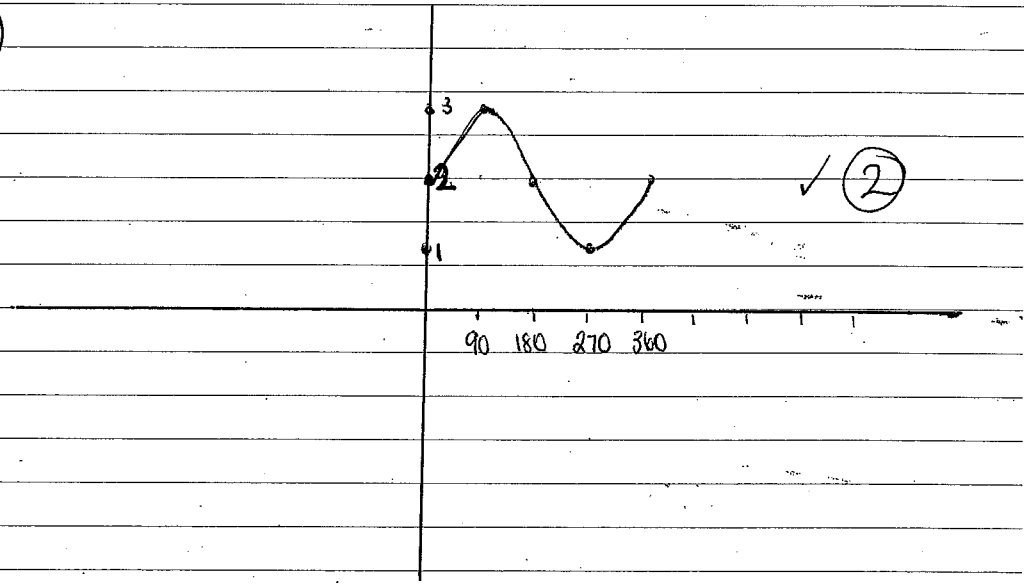
$\frac{4 \pm \sqrt{16 + 48}}{6}$

acute $x = 41° 49'$

$x = 180 +$ ____

$x = 360 -$ ____

d)



✓ (2)

Question 2:

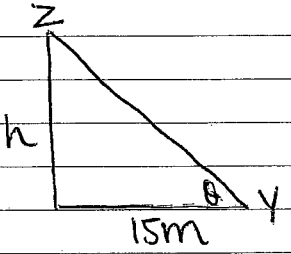
a)

i) $\tan 47 = \frac{h}{25}$

$h = 25 \tan 47$

$h = 26.8\text{m}$ ✓ (1)

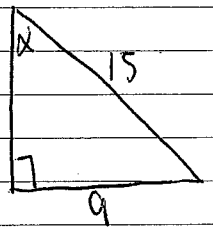
ii)



$\tan\theta = \frac{26.8}{15}$

$\theta = 60°46'$ ✓ (1)

b)



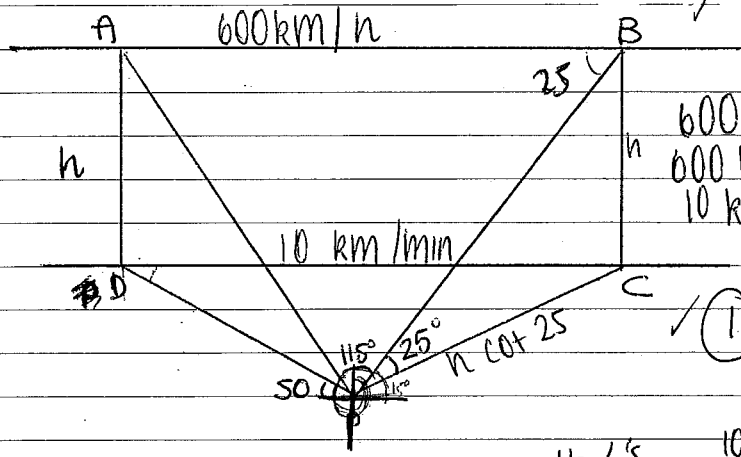
diameter = 18
radius = 9

$\sin\alpha = \frac{9}{15}$ ✓

$\alpha = 36°52'$ ✓ (2)

c)

i)

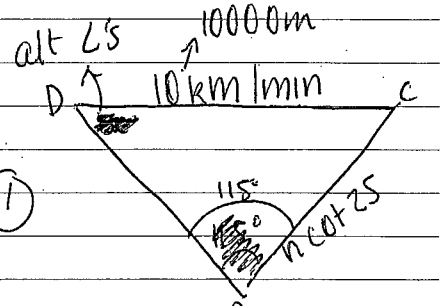


600 km/h
600 km/60 mins
10 km/min

✓ (1)

ii) $\frac{PC}{} = \tan 25 = \frac{h}{PC}$

$PC = h \cot 25$ ✓ (1)



$\frac{PB}{} = \sin 25 = \frac{h}{PB}$

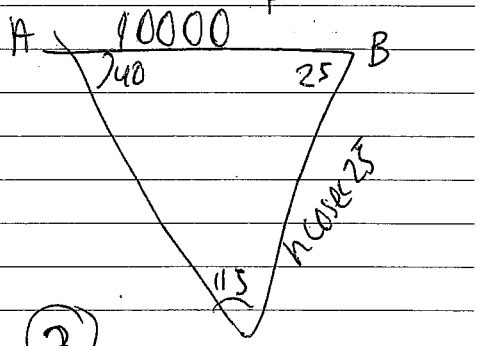
$PB = h \operatorname{cosec} 25$

$\frac{10000}{\sin 115}$ ✓ $= \frac{h \operatorname{cosec} 25}{\sin 40}$

$\frac{10\,000 \sin 40}{\sin 115} = h \operatorname{cosec} 25$

$\frac{10\,000 \sin 40}{\sin 115} = \frac{h}{\sin 25}$

$\frac{10000 \sin 50 \sin 25}{\sin 65} = h$ ✓



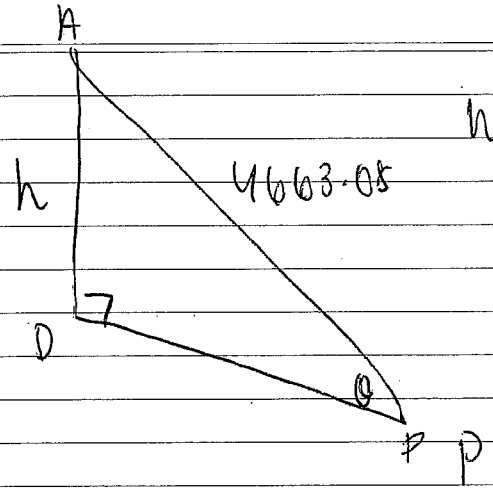
(2)

P

$\sin 40 = \sin 50$ (complements)

$\sin 115 = \sin 65$ ✓ (obtuse = acute)

PTO ⇒



$h = 3572.12$

$\frac{10000}{\sin 115} = \frac{AP}{\sin 25} \sin 15.$

$AP = 4663.08$

$\sin\theta = \frac{3572.12 x}{4663.08 x}$ (1)

$\theta = 50°$

$\frac{DB}{\sin 15} = \frac{10000}{\sin 115}$

$DP = \frac{10000 \times \sin 15}{\sin 115}$

$\tan\theta = \frac{h}{DP} = \frac{10000 \sin 50 \tan 25}{\sin 65} \times \frac{\sin 115}{10000 \sin 15}$

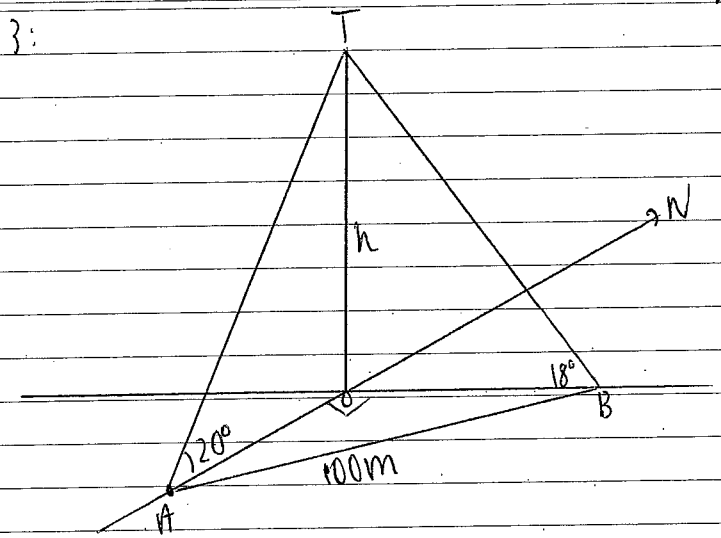
$= \frac{\sin 50 \tan 25}{}$ $\theta = 54$

(10)

Question 3:

a)

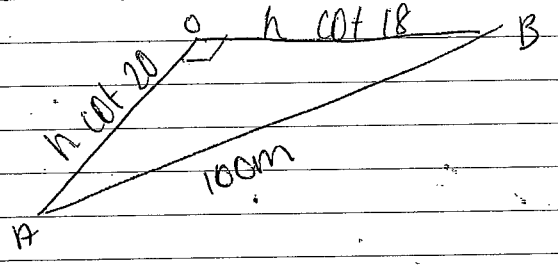
i)



(1) ✓

i) $\tan 20 = \frac{h}{AO}$ $\tan 18 = \frac{h}{OB}$

$AO = h \cot 20$ $OB = h \cot 18$ ✓ (1)



$100^2 = h^2 \cot^2 20 + h^2 \cot^2 18$

$100^2 = h^2 (\cot^2 20 + \cot^2 18)$

$h^2 = \frac{100^2}{\cot^2 20 + \cot^2 18}$ (1)

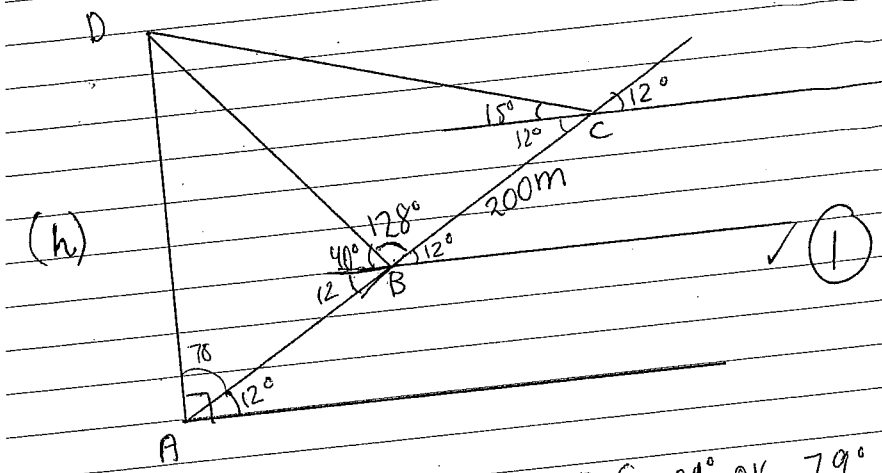
$h = \frac{100}{\sqrt{\tan^2 70 + \tan^2 72}}$

$\frac{1}{\tan 20} = \tan 70$ ✓ (1)

$\frac{1}{\tan 18} = \tan 72$

iii) $h = 24.2$ m ✓ (1)

(h)



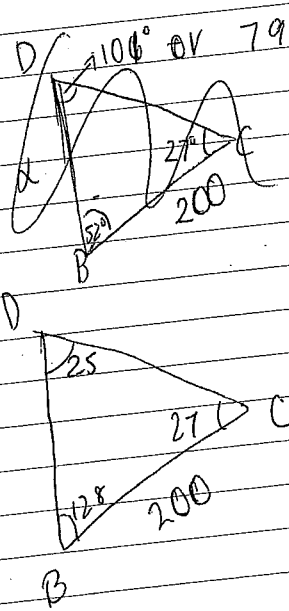
✓ (1)

ii) $DB = \frac{200 \sin 27}{\sin 25}$

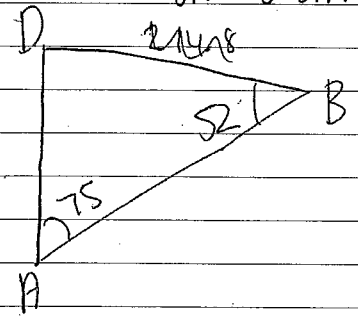
$\frac{200}{\sin 25} = \frac{DB}{\sin 27}$ ✓ (1)

$\frac{200 \sin 27}{\sin 25} = DB$ ✓ (1)

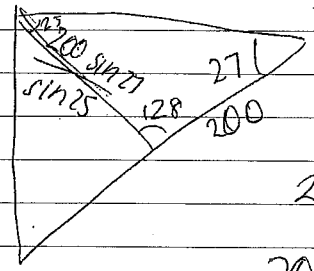
$DB = 214.8 \text{ m}$



iii) $AD = \frac{200 \sin 27 \sin 52}{\sin 25 \sin 78}$



$\frac{214.8}{\sin 75}$



$\frac{200 \sin 27}{\sin 25} \div \sin 27 = \frac{200}{\sin 25}$ ✓

$\frac{200 \sin 27}{\sin 25} \times \frac{1}{\sin 27} = \frac{200}{\sin 25}$

~~$200 \sin 27 \times \sin 27^{-1}$~~

iv) $AD = 173.1 \text{ m}$ ✓ (1)

$\frac{AD}{\sin 52} = \frac{200 \sin 27}{\sin 27} \div \sin 75$

$\frac{AD}{\sin 52} = \frac{200 \sin 25 \sin 75}{\sin 25}$ (1)

$AD = \frac{200 \sin 25 \sin 75 \sin 52}{\sin 25}$

$AD = 200 \sin 75 \sin 52$