OLSH

REGNET CHRISTUS

PRELIMINARY FINAL 2009

Student Number: ______________________________

Mathematics - Extension 1

General Instructions

- Reading Time - 5 minutes.
- Working Time – 1 hour.
- Write using a blue or black pen.
- Board approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question in a new booklet

Very good paper. Excellent knowledge on what you attempted.

Total marks (42)

- Attempt Questions 1 - 3
- All questions are of equal value.

$\frac{}{14} + \frac{}{14} + \frac{}{14} = \frac{}{42}$

Question 1 **(14 Marks)** **Marks**

(a) Find the acute angle between the lines $y = 5 - 3x$ and $2x - 3y - 4 = 0$ **3**

(b) Solve the inequality $\frac{2x+1}{x-2} > 1$ **3**

(c) Find the coordinates of the point that divides the interval joining A (7, 1) and B (0, -6) internally in the ratio 4 : 3. **2**

(d) Evaluate $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$ **2**

(e) If $a = \sqrt{3} + \sqrt{2}$, find the value of $a + \frac{1}{a}$ **2**

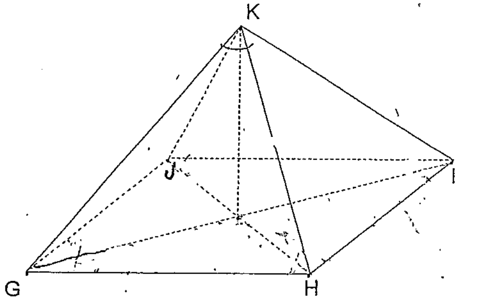
(f) Given a, b and c are three consecutive integers, show that $abc + b = b^3$ **2**

Question 2 on page 3

Question 2 (14 Marks) **Marks**

(a) The figure shown below is a rectangular pyramid with base GHIJ and apex K. L is the point of intersection of GI and HJ.

GH = 20 m, HI = 15 m and $\angle KHL = 48°$



(i) Find the height of the pyramid KL. **2**

(ii) Find the size of $\angle GKH$ to the nearest minute. **2**

(b) The graph $y = P(x) = x^4 + x - 3$ crosses the x- axis between $x = 1$ and $x = 2$

(i) Show that $P(1)$ and $P(2)$ have different signs. **2**

(ii) Use the 'halving the interval' method to find the first approximation **1**

(iii) Then apply Newton's method once to give a better approximation **3**

(c) Find the value of x and y if $\frac{4^x}{16} = 8^{x+y}$ and $2^{2x+y} = 128$ **4**

Question 3 is on page 4

OLSH College Kensington Preliminary Final 2009 Mathematics – Extension 1

Question 3 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) Differentiate the following expressions. There is no need to simplify to a single fraction.

(i) $(x^2+3)\sqrt{2x^3-5}$ **2**

(ii) $\frac{x\sqrt{x}-x^2}{\sqrt{x+2}}$ **3**

(b) For $P(x) = 4x^3 - 2x^2 + 3x - 2$ which has roots α, β and γ, find

(i) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **3**

(c) Express $P(x) = x^4 - 3x^3 - 2x^2 - 5x$ in the form $P(x) = (x^2 - 4x).Q(x) + R(x)$ where the degree of $R(x) \le 1$. **3**

(d) Given that $g(x) = x^4 - kx^3 - 2x + 33$ has a factor of $(x-3)$, find the value of k. **2**

END OF PAPER

Question 1

a. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$y = 5 - 3x$

$y = -3x + 5$

$m_1 = -3$

$2x - 3y - 4 = 0$

$\frac{2x}{3} - \frac{4}{3} = \frac{3y}{3}$

$m_2 = \frac{2}{3}$

$\tan\theta = \left|\frac{-3 - \frac{2}{3}}{1 + (-3)(\frac{2}{3})}\right|$

$= \left|\frac{-3\frac{2}{3}}{-1}\right|$

$\tan\theta = \left|3\frac{2}{3}\right|$

$\theta = 74°44'41.57$ ✓

$\theta = 75°$

3/3

b. $\frac{2x+1}{x-2} > \cancel{0}\ 1$

transcription error

$x - 2 > 0$

$\boxed{x > 2}$ ✓

$\frac{2x+1}{x-2} > 0$

$2x + 1 > 0$

$2x > -1$

$\boxed{x > \frac{-1}{2}}$

$-\frac{1}{2}$ 2

$x > 2$ ✓

$x - 2 < 0$

$\boxed{x < 2}$

$\frac{2x+1}{x-2} > 0$

$2x + 1 < 0$

$2x < -$

$\boxed{x < \frac{-1}{2}}$

$-\frac{1}{2}$ 2

$x < \frac{-1}{2}$ ✓

$x > 2,\quad x < \frac{-1}{2}$

$\frac{2x+1}{x-2} - 1 > 0$

$\frac{2x+1-x+2}{x-2} > 0$ ✓

$\frac{x+3}{x-2} > 0$

$x+3 > 0$ AND $x-2 > 0$

$\Rightarrow x > 2$

OR

$x+3 < 0$ AND $x-2 < 0$

$\Rightarrow x < -3$

3/3

Q1

c. $A(7, 1)$ $B(0, -6)$

$4 : 3$

$m \quad n$

$= \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$

$= \frac{(0)(4) + (7)(3)}{4+3}, \frac{(-6)(4) + (1)(3)}{4+3}$

$= \frac{0 + 21}{7}, \frac{-24 + 3}{7}$

$= \frac{21}{7}, \frac{-21}{7}$

$= (3, -3)$

2/2

d. $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{(x+2)\cancel{(x-2)}}$

$\lim_{x \to 2} \frac{(x-3)}{(x+2)}$

$y \to \frac{-1}{4}$

~~$(2)^2 - 5(2) + 6$~~

~~$(2)^2 - 4$~~

~~$\frac{4 - 10 + 6}{4 - 4}$~~

2/2

Q1

$a = (\sqrt{3} + \sqrt{2})$

$a + \frac{1}{a}$

$= \frac{(\sqrt{3}+\sqrt{2})}{1} + \frac{1}{(\sqrt{3}+\sqrt{2})}$

$= \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}) + 1}{(\sqrt{3}+\sqrt{2})}$

$= 3 + \sqrt{6} + \sqrt{6} + 2 + 1$

$= \frac{6+2\sqrt{6}}{\sqrt{3}+\sqrt{2}} \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \frac{6(\sqrt{3}-\sqrt{2}) + 2\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$

$= \frac{6\sqrt{3} - 6\sqrt{2} + 2\sqrt{18} - 2\sqrt{12}}{1}$

$= 6\sqrt{3} - 6\sqrt{2} + 6\sqrt{2} - 4\sqrt{3}$

$= 2\sqrt{3}$

SIMPLER

OR $\Rightarrow \frac{1}{a} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \sqrt{3} - \sqrt{2}$

$\therefore\ a + \frac{1}{a} = (\sqrt{3}+\sqrt{2}) + (\sqrt{3}-\sqrt{2})$

$= 2\sqrt{3}$

2/2

Q1

f. Let $b = x$:

$a = x-1$

$c = x+1$

$(x-1)(x)(x+1) = (x^2-x)(x+1)$

$= x^3 + x^2 - x^2 - x$

$(x-1)(x)(x+1) = x^3 - x$

$(a \times b \times c) = b^3 - b$

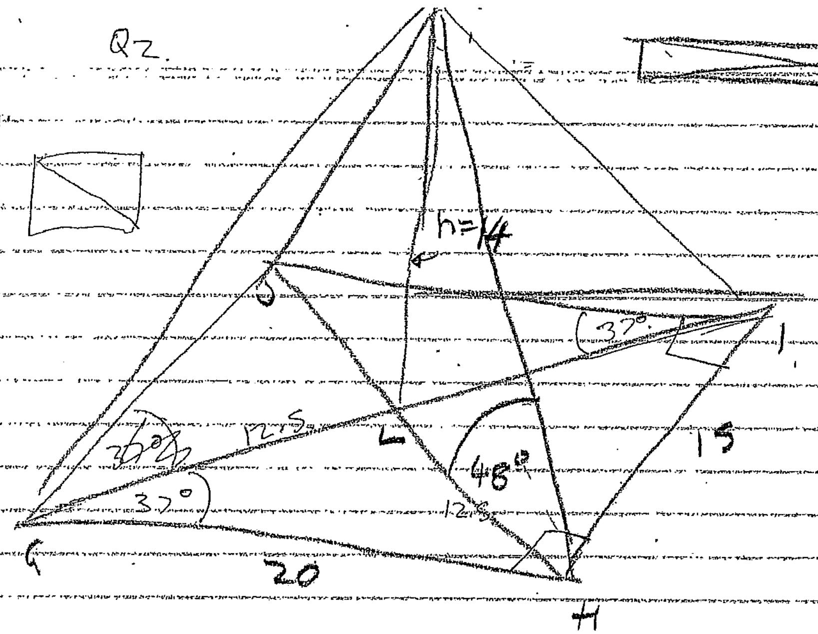
$\therefore\ a \times b \times c + b = b^3$ as req.

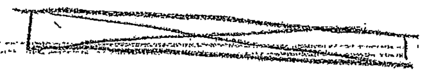
2/2

14/14

Q2.

a i.





~~[illegible]~~ $\tan IGH = \frac{15}{20}$

$IGH = 36°52'11.63 \rightarrow GI^2 = 15^2 + 20^2$

$= 25$

$GL = \frac{1}{2} \times 25$ (diagonals of rectangle bisect)

$= 12.5$

$c^2 = a^2 + b^2 - 2ab \cos C$

In $\triangle GLH$

$c^2 = (12.5)^2 + (20^2) - 2(12.5)(20)\cos 36°52'$

$c^2 = 156.23$

$c = 12.4993$

$= 12.5$ ✓

In $\triangle LKH$,

$\tan 48 = \frac{h}{12.5}$

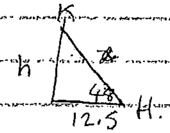
$h = 13.88190409$ ✓

$= 14m$

2/2

Q2.

a ii.



$\cos 48 = \frac{12.5}{KH}$

$KH = \frac{12.5}{\cos 48}$

$= 18.68$

$= 18.7m$

$\angle KGL$

$\tan KGL = \frac{KL}{GL}$

$= \frac{14}{12.5}$

$KGL = 48°14'$

$\cos 48°14' = \frac{12.5}{GK}$

$GK = 18.76832438$

$= 18.8$

$c^2 = a^2 + b^2 - 2ab\cos C$ ~~In △GKL~~

$20^2 = (18.8^2) + (18.7^2) - 2(18.8)(18.7)\cos GKL$

$20^2 = 703.13 +$ " $-$ " etc. ~~didn't finish~~

$\Rightarrow \cos\theta = 0{\cdot}4311$

$\theta = 64° 28'$ ✓✓

2

Q2

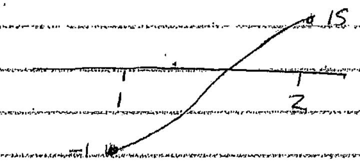
b, i) $y = P(x) = x^4 + x - 3$ between $x = 1$, $x = 2$

$P(1) = (1)^4 + (1) - 3$

$= -1$

$P(2) = (2)^4 + (2) - 3$

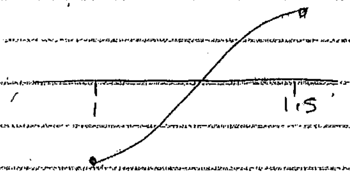
$= 15$



$P(1)$ & $P(2)$ have different signs, as shown above. 2/2

ii) $\frac{1+2}{2} = 1.5$ ✓ 1/1

$P(1.5) = 3.5625$



iii) $x_2 = x_1 - \frac{P(x)}{P'(x)}$

$P(x) = x^4 + x - 3$ ~~$P(1.5)$~~

$P'(x) = 4x^3 + 1$

$x_2 = x_1 - \frac{P(1.5)}{P'(1.5)}$ $\quad P(1.5) = (1.5)^4 + (1.5) - 3$

$= 3.5625$

$= 1.5 - \frac{3.5625}{14.5}$ $\quad P'(1.5) = 4(1.5)^3 + 1$

$= 14.5$ 3/3

$x_2 = 1.254310345$

$= 1.25$

(c) $\frac{4^x}{4^2} = (2^3)^{x+y}$

$(2^2)^{x-2} = 2^{3x+3y}$

$\Rightarrow 2x - 4 = 3x + 3y$

$\Rightarrow x + 3y + 4 = 0$ — (1)

$2^{2x+y} = 2^7$

$\Rightarrow 2x + y - 7 = 0$ — (2)

Solve simultaneously

$x = 5$, $y = -3$

(9) /14.

Q3.

a i) let $(x^2+3)\sqrt{2x^3-5} = f(x)$ $\quad \frac{dy}{dx} = vu' + uv'$ ✓

$f(x) = \underbrace{(x^2+3)}_{u}\ \underbrace{(2x^3-5)^{\frac{1}{2}}}_{v}$

$v = (2x^3-5)^{\frac{1}{2}}$ $\quad u = (x^2+3)$

$u' = 2x$ ✓ $\quad v' = \frac{1}{2}\cdot(2x^3-5)^{-\frac{1}{2}} \cdot 6x^2$ ✓

$= 3x^2 (2x^3-5)^{-\frac{1}{2}}$

$= \frac{3x^2}{\sqrt{2x^3-5}}$ 2/2

$f'(x) = 2x(2x^3-5)^{\frac{1}{2}} + (x^2+3)\cdot\left(\frac{3x^2}{\sqrt{2x^3-5}}\right)$

$= 2x\sqrt{2x^3-5} + \left(\frac{3x^4 + 9x^2}{\sqrt{2x^3-5}}\right)$

ii) let $\frac{x\sqrt{x} - x^2}{\sqrt{x+2}} = f(x)$

$f'(x) = \frac{vu' - uv'}{v^2}$ $\quad v = x\sqrt{x} - x^2$

$u' = \frac{1}{2}(x+2)^{-\frac{1}{2}}$

$u = \sqrt{x+2}$

$v' = \left(1 \cdot \frac{1}{2}x^{-\frac{1}{2}}\right) - 2x$ NO 2/3

$f'(x) = \frac{(x\sqrt{x} - x^2)\left(\frac{1}{2}(x+2)^{-\frac{1}{2}}\right) - (\sqrt{x+2})\left(\frac{1}{2}x^{-\frac{1}{2}} - 2x\right)}{(x\sqrt{x} - x^2)^2}$

$f(x) = \frac{x^{3/2} - x^2}{\sqrt{x+2}}$ $\begin{matrix}\leftarrow u \\ \leftarrow v\end{matrix}$

$f'(x) = \frac{\sqrt{x+2}\cdot\left(\frac{3}{2}x^{1/2} - 2x\right) - \left(x^{3/2} - x^2\right)\cdot\frac{1}{2}(x+2)^{-1/2}}{(x+2)}$

$= \frac{(x+2)\left(\frac{3}{2}\sqrt{x} - 2x\right) - \left(x^{3/2} - x^2\right)}{(x+2)\sqrt{x+2}}$

$= \frac{\frac{3}{2}x^{3/2} - 2x^2 + 3x^{1/2} - 4x - x^{3/2} + x^2}{(x+2)^{3/2}}$

$= \frac{\frac{1}{2}x^{3/2} + 3x^{1/2} - x^2 - 4x}{(x+2)^{3/2}}$

Q3

b) $P(x) = 4x^3 - 2x^2 + 3x - 2.$

i) $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

$= \frac{3}{4}$

1/1

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \left(\frac{1}{\alpha} \frac{\times \beta\gamma}{\times \beta\gamma}\right) + \left(\frac{1}{\beta} \frac{\times \alpha\gamma}{\times \alpha\gamma}\right) + \left(\frac{1}{\gamma} \frac{\times \beta\alpha}{\times \beta\alpha}\right)$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{3/4}{1/2} = 1\frac{1}{2}.$

3/3

$\alpha\beta\gamma = \frac{-d}{a}$

$= \frac{2}{4}$

$= \frac{1}{2}.$

d) $g(x) = x^4 - kx^3 - 2x + 33.$

$g(3) = (3)^4 - k(3)^3 - 2(3) + 33$

$0 = 81 - 27k - 6 + 33$

$= 108 - 27k$

$27k = 108$

$k = 4.$

2/2

~~$x(x^3 ... 2x^2 ... 5) = x^3(x-3) = x(2x-5)$~~

long division.

10

T4