

Student Number: _____



OUR LADY OF THE SACRED HEART COLLEGE

MATHEMATICS

H.S.C. ASSESSMENT TASK 2 (HALF – YEARLY)

Monday April 2, 2012.

Time allowed: 3 hours plus 5 minutes reading time

Weighting: 30%

Topics covered: Preliminary Work, Series and Sequences, Locus and Parabola, Calculus, Integration, The Quadratic Polynomial.

Section I: 10 multiple choice questions. (1 mark each)
Answer all questions on the answer sheet provided

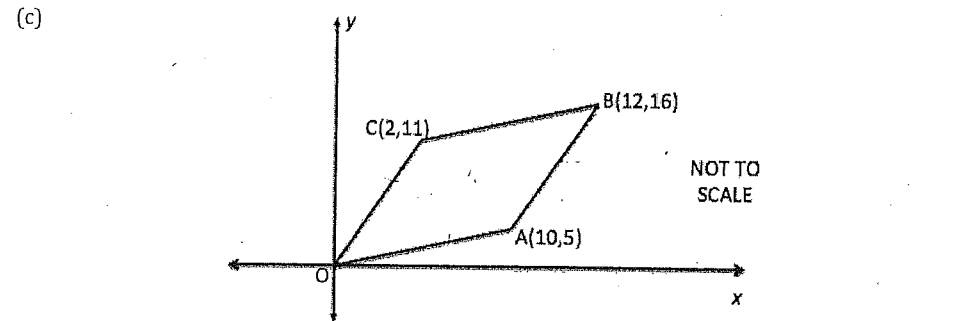
Section II: 6 questions (15 marks each)
Answer all questions in the answer booklets provided
Start each question in a new booklet
Show all necessary working

- Board approved calculators may be used in this test
- A standard integrals page has been attached to the back of the question paper.

SECTION B – QUESTION ONE (15 marks)

(a) What is the value of $\sqrt[3]{2a - b}$ if $a = 7.3$ and $b = 1.6$. Answer to 2 decimal places 1

(b) Rationalise and simplify $\frac{6}{\sqrt{3} - 1}$ 2



In the diagram, A, B and C are the points (10,5), (12,16) and (2,11) respectively.

Copy or trace the diagram into your writing booklet.

- (i) Find the distance AC 1
- (ii) Find the midpoint of AC 1
- (iii) Show that $OB \perp AC$ 2
- (iv) Find the midpoint of OB and hence explain why OACB is a rhombus. 2
- (v) Hence, or otherwise, find the area of OACB. 1

(d) Find the derivative of the function $f(x) = 2x^2 - 3x$ using first principles 3

(e) Simplify $(4e^2f^3)^4$ 1

(f) Solve the equation: $\frac{x+2}{3} + 7 = 10$ 1

SECTION B – QUESTION TWO (15 marks)

(a) In an isosceles triangle $\triangle PQR$, $PQ = QR$ and S is a point on QR such that $PS \perp QR$.

If $\angle PQR = 74^\circ 53'$ and $PQ = 85$ mm,

(i) Draw a diagram representing this information

1

(ii) Find the length of SR correct to 1 decimal place.

2

(b) Differentiate the following expressions

(i) $f(x) = (3x^2 + 7)^5$

1

(ii) $f(x) = \frac{1}{x^2}$

2

(iii) $f(x) = \frac{3x + 5}{2x - 7}$

2

(c) Find the primitive function of

(i) $\int \frac{1}{x^3} dx$

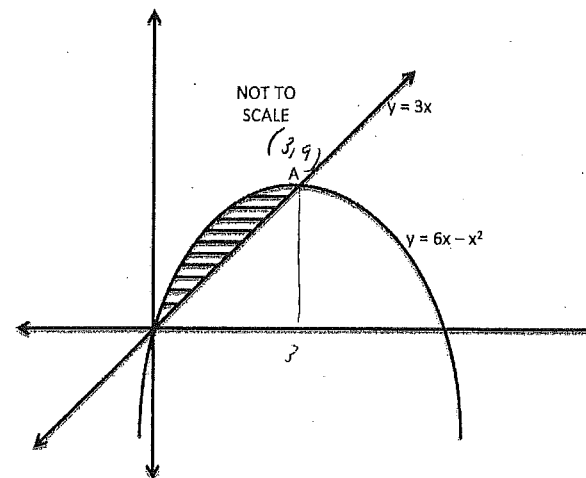
1

(ii) $\int \frac{4 + 3x^5}{x^2} dx$

2

Question 2 continued overleaf

(d) The graphs of $y = 6x - x^2$ and $y = 3x$ intersect at $(0,0)$ and the point A , as shown in the diagram.



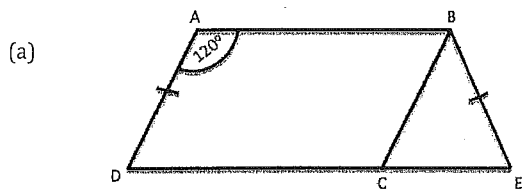
(i) Find the co-ordinates of A

1

(ii) Find the area of the shaded region bounded by $y = 6x - x^2$ and $y = 3x$

2

SECTION B – QUESTION THREE (15 marks)



This diagram shows a parallelogram ABCD with $\angle DAB = 120^\circ$. The line DC is produced to E so that $AD = BE$.

(a) Copy or trace the diagram into your writing booklet.

Prove that $\triangle BCE$ is equilateral.

2

(b) For the function $y = \frac{3}{\sqrt[3]{x^5}}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, writing both answers in surd form.

3

(c) If α and β are the roots of $2x^2 + 3x - 4 = 0$, find,

(i) $\alpha + \beta$

1

(ii) $\alpha\beta$

1

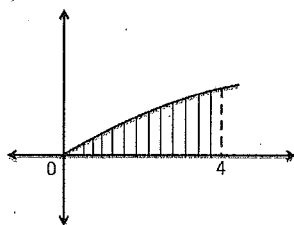
(iii) $(\alpha - 1)(\beta - 1)$

2

(d) The point P moves in such a way so that its distance from the point A (2,4) is always twice the distance from the point B (1,0). Show that the equation of this locus is $3x^2 + 3y^2 - 4x + 8y - 16 = 0$.

3

(e) This graph shows the curve $y = \sqrt{x}$ between the values of $x = 0$ and $x = 4$.



Show that the shaded area is equal to $5\frac{1}{3}$ units².

3

SECTION B – QUESTION FOUR (15 marks)

(a) Consider the graph $y = x^3 - 3x^2$.

(i) Find the co-ordinates of the stationary points of the curve and determine their nature.

3

(ii) Sketch the curve showing stationary points, points of inflexion and intercepts with both axes.

3

(b) For the quadratic equation $4x^2 + 3kx + k^2 = 0$,

(i) Find the discriminant in terms of k

1

(ii) Hence, or otherwise, state whether the roots of the quadratic are real or unreal whatever the value of k, giving reasons for your answer.

2

(c) Sketch the parabola $2y = x^2 + 6x + 5$, showing the vertex, focus and directrix.

3

(d) Given that $\frac{d^2y}{dx^2} = 12x - 2$ and that $\frac{dy}{dx} = 0$ at the point (1,4), find the expression for y in terms of x.

3

SECTION B – QUESTION FIVE (15 marks)

- (a) A store has cans of tomatoes stacked in a display which has 40 cans on the bottom row. Each row has two cans less than the row below it. The top row has 6 cans.
- (i) How many cans are in the 6th row from the bottom. 1
- (ii) Find an expression for the number of cans in the nth row. 2
- (iii) Using the formula for the sum of an arithmetic progression, find the total number of cans in the display. 2

(b) Solve the equation $x^4 - 13x^2 + 36 = 0$. 3

- (c) On a number plane, a circle has its centre O at the point (-4,2). The line $y = 3x + 4$ is a tangent to the circle.
- (i) Find the radius of the circle. 2
- (ii) Find the equation of the circle. 1

(d) (i) For the function $y = \frac{1}{10 + x^2}$, copy and complete the following table.

x	1	1.5	2	2.5	3
y					

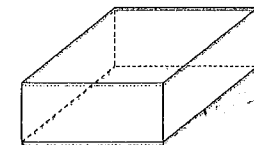
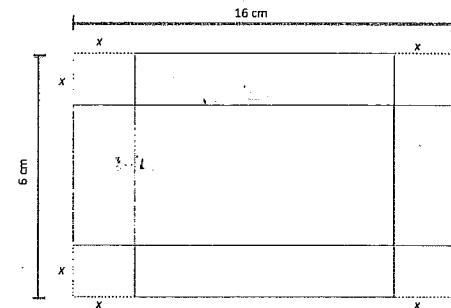
1

- (ii) Use Simpson's Rule and 5 function values to approximate the area under the curve $y = \frac{1}{10 + x^2}$ between the values $x = 1$ and $x = 3$. Use 2 decimal places throughout the working. 2

(e) Find the limiting sum of the Geometric Progression which begins 18, 12, 8, 1

SECTION B – QUESTION SIX (15 marks)

- (a) A box is formed from a rectangular piece of cardboard 16 cm long and 6 cm wide. A square of side x will be cut and removed from each corner so that the sides can be folded to form a box.



- (i) If x is the height of the finished box, what are the length and breadth of the finished box. 1
- (ii) Show that the volume V cm³ of the box is given by $V = 4x^3 - 44x^2 + 96x$. 1
- (iii) Show that the maximum volume of the box is achieved when $x = \frac{4}{3}$ cm. 3
- (iv) Find the maximum volume of the box correct to 2 decimal places. 1

Question Six continued overleaf

(b) A man borrows \$ 15000 to buy a car. He has to pay off the loan over 5 years in equal monthly repayments. If the interest is 15% p.a. and the interest is compounded monthly, and M is the size of each monthly repayment,

(i) Show that the amount owing after 1 month (A_1) is given by

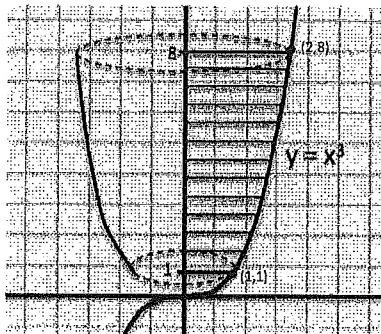
$$A_1 = 15000 \times 1.0125 - M \quad 1$$

(ii) Show that $A_3 = 15000 (1.0125)^3 - M (1.0125^2 + 1.0125 + 1)$ 1

(iii) Calculate the size of each monthly repayment 3

(iv) How much interest does the man pay over the life of the loan ? 1

(c) Here is a diagram of the curve $y = x^3$.



Find the volume of the solid generated when the area between $y = 1$ and $y = 8$ is rotated about the y-axis.

Answer in terms of π

3

Q1. a) $2\sqrt{2(7-3)} - 1.6$

$= 3\sqrt{13}$

$= 2.35$

b) $6(\sqrt{3}+1)$

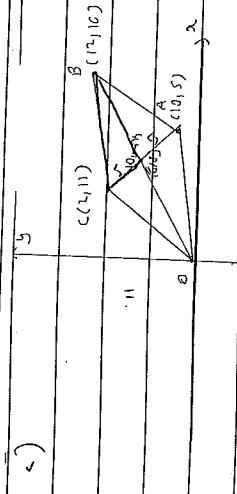
$\sqrt{3}-1 \sim (\sqrt{3}+1)$

$= 6\sqrt{3}+6$

$3-1$

$= \frac{3}{2}(\sqrt{3}+1)$

$= 3(\sqrt{3}+1)$ OR $3\sqrt{3}+3$



i) $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(10-2)^2 + (5-11)^2}$

$= \sqrt{64+36}$

$= 10$ units

-1-

Q1. ii) mid pt of AC

$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$\frac{10+2}{2}, \frac{5+11}{2}$

(6,8)

iii) for $OB \perp AC$, $m_1 \times m_2 = -1$

$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{11-5}{2-10} = 12-0$

$m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{11-5}{2-10} = 16-0$

$\frac{6}{-8} \cdot \frac{-4}{12} = 16 = 4$

$\frac{6}{-8} \cdot \frac{-4}{12} = 16 = 4$

$m_1 \times m_2 = -1$

~~$\frac{16}{-4} \times \frac{11-5}{2-10} = -1$~~

$\therefore OB \perp AC$

iv) mid pt of OB

$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$(\frac{12}{2}, \frac{16}{2})$

(6,8) -2-

Q1: OABC is a rhombus as diagonals meet at one point at right angles.

(v) Missing ~~part~~ part (y) is on a diff event

(vi) $D_{OB} = \sqrt{12^2+16^2}$

$= 20$ units

Area = $l \times s$ or A area

$= l \times 10 \times 5$

$= 200$ units²

-3-



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Student Number:

d) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = 2(x+h)^2 - 3(x+h)$

$= 2(x^2 + 2xh + h^2) - 3x - 3h$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h$

$\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - (2x^2 - 3x)}{h}$

$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h}$

$= \lim_{h \rightarrow 0} \frac{h(4x + 2h - 3)}{h}$

$= \lim_{h \rightarrow 0} 4x + 2h - 3$

~~$= 4x - 3$~~

$= 4x - 3$

-4-

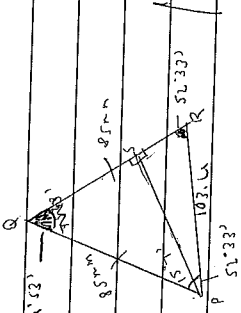
c) 256^3 cm^3

e) $\frac{2+7}{3} + 7 = 10$

$2+7 = 21 = 30$

$2+7 = 2 = 7$

82 a)



$\sin 76.53^\circ = \frac{85}{PR}$

$PR = \frac{85}{\sin 76.53^\circ}$

$PR = 85 \sin 76.53^\circ$

$= 103.6 \text{ mm}$

$\cos 52.33^\circ = \frac{SR}{PR}$

103.6

$\sin 103.6 \cos 52.33^\circ$

$= 62.94 \text{ mm}$

✓

b) i) $f(x) = 5(3x^2 + 7)^4 \times 6x$

$= 30x(3x^2 + 7)^4$

ii) $f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$= -\frac{2}{x^3}$

x^3

iii) $f'(x) = u'v - uv'$

$u = 3x+5$
 $u' = 3$
 $v = 2x-7$
 $v' = 2$

$= 3(2x-7) - 2(3x+5)$

$(2x-7)^2$

$= 6x-21-6x-10$

$(2x-7)^2$

$= -31$

$(2x-7)^2$

✓

c) i) $\int x^{-3} dx$

$= \frac{x^{-2}}{-2} + c$

$= \frac{1}{-2x^2} + c$

ii) $\int 4x^2 + 3x^3 dx$

$= \frac{4x^3}{3} + \frac{3x^4}{4} + c$

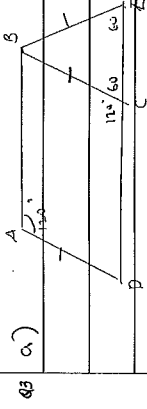
$= \frac{-4}{3} + \frac{3x^4}{4} + c$

✓

a) i) $3x = 6x - x^2$
 $-3x = -x^2$
 $x^2 - 3x = 0$
 $x(x-3) = 0$
 $x = 0, 3$

ii) $\lambda = 3$
 $y = 3(9)$
 $= 9$
 $A(3, 9)$

iii) $A = \int_0^9 6x - x^2 - 3x \, dx$
 $= \int_0^9 3x - x^2 \, dx$
 $= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^9$
 $= \left[\frac{27}{2} - 9 - 0 \right]$
 $= 4.5x^2$



$AD = BC$ (opposite sides are equal in parallelogram)
 $\angle DCB = 120^\circ$ (opposite angles are equal in parallelogram)
 $\angle BAE = 60^\circ$ (angles on a straight line)
 $\triangle BCE$ is isosceles, (base sides are equal in isosceles triangles)
 $\therefore \angle BEC = 60^\circ$ (base angles are equal in isosceles triangles)
 $120^\circ = \angle CBE + \angle BEC$ (exterior angle of triangle)
 $120^\circ = \angle CBE + 60^\circ$ (equival sum of interior opposite angles)
 $\angle CBE = 60^\circ$
 $\therefore \triangle BCE$ is equilateral (all angles are 60°)
 is equilateral triangle

b) $y = 3x - 5\sqrt{x}$

$\frac{dy}{dx} = -5 \cdot \frac{1}{2} x^{-\frac{1}{2}}$
 $\frac{dy}{dx} = -\frac{5}{2} x^{-\frac{1}{2}}$
 $= -\frac{5}{2} \cdot \frac{1}{\sqrt{x}}$
 $= -\frac{5}{2\sqrt{x}}$

c) $2x^2 + 3x - 4 = 0$

i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{-3}{2}$

ii) $\alpha\beta = \frac{-c}{a}$

$= \frac{-4}{2}$

$= -2$

iii) $(\alpha-1)(\beta-1)$

$= \alpha\beta - (\alpha+\beta) + 1$

$= -2 - \left(-\frac{3}{2}\right) + 1$

$= -\frac{1}{2}$

d) $A(2, 4)$
 $B(1, 0)$

$2PB^2 = PA^2$

$2x\sqrt{(x_1-2)^2 + (y_1-0)^2} = \sqrt{(x_1-2)^2 + (y_1-4)^2}$

$4[(2x-2)^2 + (y_1-0)^2] = (x_1-2)^2 + (y_1-4)^2$

$4[(x-1)^2 + (y-0)^2] = (x-2)^2 + (y-4)^2$

$4[x^2 - 2x + 1 + y^2] = x^2 - 4x + 4 + y^2 - 8y + 16$

$4x^2 - 8x + 4 + 4y^2 = x^2 - 4x + 4 + y^2 - 8y + 16$

$3x^2 + 3y^2 - 4x + 8y - 16 = 0$

$$\begin{aligned}
 c) \quad A &= \int_0^4 x^{1/2} dx \\
 &= \left[\frac{2}{3} x^{3/2} \right]_0^4 \\
 &= \frac{2}{3} \sqrt{16} - 0 \\
 &= \frac{2}{3} \cdot 4 = \frac{8}{3} \text{ units}^2
 \end{aligned}$$

$$\begin{aligned}
 a) \quad i) \quad y &= x^3 - 3x^2 \\
 y' &= 3x^2 - 6x \\
 y'' &= 6x - 6
 \end{aligned}$$

for station on $y, y' = 0$

$$\begin{aligned}
 3x^2 - 6x &= 0 \\
 3x^2 &= 6x \\
 x &= 2
 \end{aligned}$$

test $x=2$ into y''

$$\begin{aligned}
 6(2) - 6 &= 6 \\
 &> 0
 \end{aligned}$$

\therefore minimum

$$\begin{aligned}
 y &= 2^3 - 3(2)^2 \\
 &= -4
 \end{aligned}$$

~~(2, -4)~~ /

ii) ~~for turning point~~

for points of inflexion, $y'' = 0$

$$\begin{aligned}
 6x - 6 &= 0 \\
 6x &= 6 \\
 x &= 1
 \end{aligned}$$

$$\begin{aligned}
 y &= 1^3 - 3(1)^2 \\
 &= -2
 \end{aligned}$$

x	0	1	2	
y	-6	0	6	

~~graph~~

x intercepts, $y = 0$

$$\begin{aligned}
 0 &= x^3 - 3x^2 \\
 &= x^2(x - 3) \\
 &= 0 \quad | \quad x = 3
 \end{aligned}$$

y intercept, $x = 0$

$$\begin{aligned}
 0 &= 0^3 - 3(0)^2 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 a) \quad i) \quad y &= 3x^2 - 6x \\
 y' &= 6x - 6
 \end{aligned}$$

for station on $y, y' = 0$

$$\begin{aligned}
 6x - 6 &= 0 \\
 6x &= 6 \\
 x &= 1
 \end{aligned}$$

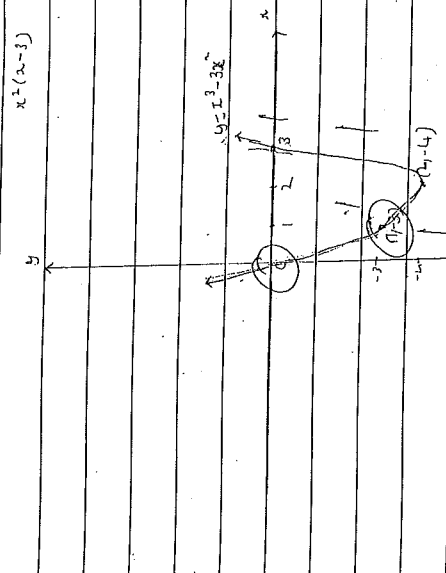
test $x=1$ into y''

$$\begin{aligned}
 6(1) - 6 &= 0 \\
 &= 0
 \end{aligned}$$

\therefore minimum

$$\begin{aligned}
 y &= 1^2 - 3(1)^2 \\
 &= -2
 \end{aligned}$$

~~(1, -2)~~ /



Transcription Error
Be careful.

b) $4x^2 + 3kx + k^2 = 0$

i) $\Delta = b^2 - 4ac$

$$\begin{aligned}
 &= (3k)^2 - 4(4)(k^2) \\
 &= 9k^2 - 16k^2 \\
 &= -7k^2
 \end{aligned}$$

ii) if $k > 0, \Delta < 0$
 \therefore the roots are unreal

c) $2y = x^2 + 6x + 5$

$2y + u = (x+3)^2$

When $y=0$

$2(0+2) = (x+3)^2$

$u = 2^2 + 6x + 9$

$u(\frac{1}{2})(y+2) = (x+3)^2$

$V(-3, -2)$

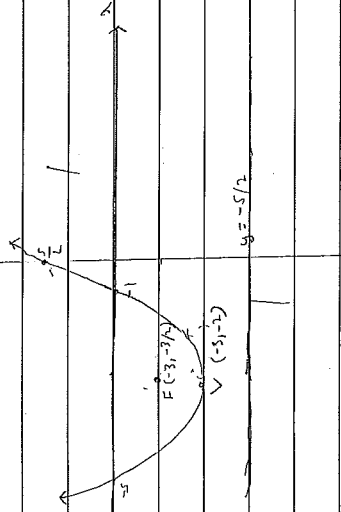
$F(-3, -2)$

Direction $y = -5/2$

When $x=0$

$2y = 5$

$y = 5/2$



d) $\frac{dy}{dx} = 12x - 2$

$\frac{dy}{dx} = 0 \Rightarrow (1, 1)$

$\frac{dy}{dx} = \int (12x - 2) dx$

$= \frac{12x^2}{2} - 2x + C$

$u = 6(1)^2 - 2(1) + C$

$C = 0$

$\frac{dy}{dx} = 6x^2 - 2x$ | \rightarrow Failed to find \int

OR $y = \int (6x^2 - 2x) dx$

$= \frac{6x^3}{3} - \frac{2x^2}{2} + C$

$= 2x^3 - x^2 + C$

$u = 2(1)^3 - 1^2 + C$

$3 = C$

$y = 2x^3 - x^2 + 3$

a) ~~$14 = 6 + (6 - 1)2$~~
 ~~$= 6 + 10$~~
 ~~$= 16$~~

i) ~~$T_{10} = 6 + 2(10)$~~
 ~~$= 60$~~

ii) ~~30~~

iii) ~~$T_n = 6 + (n-1)2$~~

$S_n = \frac{n}{2} [2a + (n-1)d]$

$= \frac{18}{2} [2(6) + 10(2)]$

$= 9 [12 + 34]$

$= 414$ same

ii) $T_n = 6 + (n-1)2$

~~$6 + 2n - 2$~~

~~$u + 2n$~~

b) $2^4 - 13a^2 + 36 = 0$

let $x^2 = k$

$k^2 - 13k + 36 = 0$

$(k - 4)(k - 9) = 0$

$k = 4, 9$

$u = x^2$

$9 = x^2$

$x = \pm 3$

$x = 4 - 1, 2, -3, 3$

d) i) $y = \frac{1}{10+2x^2}$

x	1	1.5	2	2.5	3
y	1/11	4/149	1/14	4/165	1/19

ii) $\frac{h}{3} [\text{first} + \text{last} + 4(\text{odd}) + 2(\text{even})]$

$= \frac{1}{3} \left[\frac{1}{11} + \frac{1}{19} + 4 \left(\frac{4}{149} + \frac{4}{165} \right) + 2 \left(\frac{1}{14} \right) \right]$

$= \frac{1}{6} \left[30/209 + 1624/3185 + 1/7 \right]$

$= 0.143180378$

$= 0.14$

e) $S_d = a$
 $1-r$

$= 18 = \frac{18}{1/3}$

$1 - \frac{1}{3} = \frac{2}{3}$

$= \frac{54}{2/3} = 81$

c) (i) Using pop. distance formula $d = \sqrt{(ax+by+c)^2 + (a^2+b^2)}$

$\therefore \text{rad} = \frac{|3(-4) - (-2) + 1|}{\sqrt{3^2 + (-4)^2}}$

$= \frac{10}{5} = 2$

(ii) Eqⁿ of the circle is $(x+a)^2 + (y-b)^2 = (r/a)^2$
 $(x+4)^2 + (y-2)^2 = 10$

a) i) $bx^2 + 16 - 2x, 6 - 2x$

ii) $V = x(6-2x)(16-2x)$
 $= x[96 - 12x - 32x + 4x^2]$
 $= 96x - 12x^2 - 32x^2 + 4x^3$
 $= 4x^3 - 44x^2 + 96x$

(iii) $V' = 12x^2 - 88x + 96$
 $V'' = 24x - 88$

For maximum, $V' = 0$
 $12x^2 - 88x + 96 = 0$

$4(3x^2 - 22x + 24) = 0$
 $3x^2 - 22x + 24 = 0$

$x = \frac{22 \pm \sqrt{484 - 4(3)(24)}}{6}$

$= \frac{22 \pm 14}{6}$
 $= 6, 4/3$

test $x = 6$ into y''

$24(6) - 88$

$= -56$

> 0

\therefore minimum

< 0

\therefore maximum

(iii) $x = 4/3$

$V = 4(4/3)^3 - 44(4/3)^2 + 96(4/3) = 59 \frac{2}{27}$

$= 9 \frac{13}{27} - 7 \frac{8}{9} + 12 \frac{8}{9} = 59.26 \text{ (approx.)}$

$= 48.30 \text{ cm}^3$

b) i) $A_1 = PR^1 - M$

$= 15000(1 + \frac{0.15}{12}) - M$

$= 15000 \times 1.0125 - M$

ii) $A_2 = (15000 \times 1.0125 - M) \times 1.0125 - M$

$= 15000 \times 1.0125^2 - M \times 1.0125 - M$

$A_3 = (15000 \times 1.0125^2 - M \times 1.0125 - M) \times 1.0125 - M$

$= 15000(1.0125)^3 - M(1.0125^2) - M(1.0125) - M$

$= 15000(1.0125)^3 - M(1.0125^2 + 1.0125 + 1)$

iii) $15000 \times (1.0125)^{60} - M(1 + 1.0125 + 1.0125^2 + \dots + 1.0125^{59})$

$M \left[\frac{1.0125^{60} - 1}{0.0125} \right] = 15000 \times 1.0125^{60}$

$M = \frac{15000 \times 1.0125^{60}}{1.0125^{60} - 1} \times 0.0125$

$= \frac{395,697}{1.107}$

$= 356,908$

Don't round off until the end!

$= \$356,911$

iv) $356,911 \times 60 = \$21,414,660$

c) $y = 2^3$

$y^{1/3} = 2$

$y^{2/3} = 2^2$

$V = \pi \int_1^8 y^{2/3} dy$

$= \pi \left[\frac{y^{5/3}}{5/3} \right]_1^8 = \frac{3\pi}{5} [8^{5/3} - 1]$

$= \frac{3\pi}{5} [10972.79 - 1] = \frac{3\pi}{5} \times 31$

$= \frac{3\pi}{5} [10922.79]$

$= 6553.4 \pi$ units³