REGNET CHRISTUS

2012
HIGHER SCHOOL CERTIFICATE
ASSESSMENT 3

Mathematics

General Instructions

- Working Time - 45 mins.
- Write using a blue or black pen.
- Approved calculators may be used..
- All necessary working should be shown for every question.
- Begin each section in a new booklet
- Remember to use the standard integrals sheet

Total marks (**36**)

- Section 1 – Exponential and Logarithmic Functions (18 marks)
- Section 2 – Trigonometric Functions (18 marks).

SECTION 1 - EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Q.1 Evaluate $3e^{2.7}$ correct to 3 decimal places **1**

(a) 3
(b) 288.950
(c) 44.639
(d) 1.098

Q.2 If $\log_3 7 = y$, then which expression gives the value of $\log_3 49$. **1**

(a) 2y
(b) y^2
(c) y + 3
(d) y - 5

Q.3 Calculate **2**

$$\int_0^{\ln 2} e^{2x}\,dx$$

Q.4 Find the derivative of the expression $\frac{\ln x}{x}$ **2**

Q.5 If $y = 4e^{-x} + 5e^{-3x}$, show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$ **3**

Q.6 Find the primitive function of

(i) e^{4x+3} **1**

(ii) $\frac{4x}{x^2+1}$ **2**

Q.7 Show that the area under the curve $y = \frac{x}{x^2+1}$ between the values of x = 2 and x = 4 is equal to $\frac{1}{2}\ln\left(\frac{17}{5}\right)$ **3**

Q.8 For the curve $y = e^x - 2x$,

(i) Find the equation of the tangent to the curve at the point where x = 1 **2**

(ii) Show that this tangent passes through the origin. **1**

SECTION 2 - TRIGONOMETRIC FUNCTIONS

Q.1 What is the period and amplitude of the curve y = cos 2x **1**

(a) Period = 2π , Amplitude = 2
(b) Period = π, Amplitude = 2
(c) Period = 2π, Amplitude = 1
(d) Period = π, Amplitude = 1

Q.2 In which quadrant does the angle $\left(\frac{7\pi}{6}\right)$ radians lie in ? **1**

(a) 1st
(b) 2nd
(c) 3rd
(d) 4th

Q.3 Find the derivatives of :

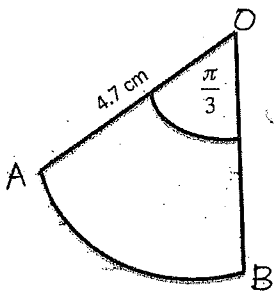
(i) cos $2t$ **1**

(ii) $x \tan x$ **2**

Q.4 Calculate

$$\int_{\frac{\pi}{3}}^{\pi} \cos\frac{1}{2}x\, dx$$ **2**

Q.5



Find the perimeter of the sector OAB, given that $\angle AOB = \frac{\pi}{3}$ *Radians* and OA = OB = 4.7 cm. **2**

Answer to 3 significant figures

Q.6 Find the equation of the tangent to the curve y = 2cos x at the point $\left(\frac{\pi}{6}, \sqrt{3}\right)$ **3**

Q.7 Use Simpson's Rule and 5 function values to estimate the area under the curve y = x sin x, between the values of x = 0 and x = π. **3**

Use 2 decimal places throughout your calculations.

Q.8 The region under the curve y = tan x between the values of $x = \frac{\pi}{3}$ and $x = \frac{\pi}{4}$ is rotated about the x-axis. **3**

Use the identity $\tan^2 x = \sec^2 x - 1$ to calculate the volume of the solid formed.

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1 c) .

2 a) .

3. $\int_0^{\ln 2} e^{2x}\,dx$

$= \left[\frac{e^{2x}}{2}\right]_0^{\ln 2}$.

$= \frac{e^{2\ln 2}}{2} - \frac{e^{2(0)}}{2}$.

$= 2 - \frac{1}{2} = \frac{3}{2}$.

4. $\frac{\ln x^{u}}{x^{v}}\,dx \quad \frac{u'v - v'u}{v^2}$ $\quad u = \ln x \quad v = x$ $\quad u' = \frac{1}{x} \quad v' = 1$

$= \frac{1-\ln x}{x^2}$.

5. $y = 4e^{-x} + 5e^{-3x}$.

$\cancel{\frac{dy}{dx} = -4e^{-x}+}$

$\frac{dy}{dx} = -4e^{-x} - 15e^{-3x}$

$\frac{d^2y}{dx^2} = 4e^{-x} + 45e^{-3x}$.

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 4e^{-x} + 45e^{-3x} + 4(-4e^{-x} - 15e^{-3x})$
$+ 3(4e^{-x} + 5e^{-3x})$

$= \cancel{4e^{-x}} + \cancel{45e^{-3x}} - \cancel{16e^{-x}} - \cancel{60e^{-3x}} + \cancel{12e^{-x}} + \cancel{15e^{-3x}}$

$= 0$.

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6 i) $\int e^{4x+3}\,dx$

$= \frac{e^{4x+3}}{4} + c$.

ii) $\int \frac{4x}{x^2+1}\,dx$. $\quad f(x) = x^2+1$
$f'(x) = 2x$.

$= 2\int \frac{2x}{x^2+1}\,dx$.

$= 2\ln(x^2+1) + c$.

7. $A = \int_2^4 \frac{x}{x^2+1}\,dx$ $\quad f(x) = x^2+1$
$f'(x) = 2x$.

$= \frac{1}{2}\int_2^4 \frac{2x}{x^2+1}\,dx$.

$= \frac{1}{2}\left[\ln(x^2+1)\right]_2^4$

$= \frac{1}{2}\{\ln(4^2+1) - \ln(2^2+1)\}$.

$= \frac{1}{2}(\ln 17 - \ln 5)$.

$= \frac{1}{2}(\ln\frac{17}{5})$.

$= \frac{1}{2}\ln(\frac{17}{5})$

8i) $y = e^x - 2x$. where $x = 1$

$y = e^1 - 2(1)$

$= e - 2$.

$m = \frac{dy}{dx}$

$= e^x - 2$ where $x = 1$..

$= e^1 - 2$.

$= e - 2$.

eqn' =

$y - y_1 = m(x - x_1)$

$y - e + 2 = (e-2)(x-1)$.

$y - \cancel{e+2} = ex - \cancel{e} - 2x + \cancel{2}$.

$y = ex - 2x$.

$y = x(e-2)$.

ii) when $x = 0$.

$y = 0(e-2)$

$= 0$

$\therefore (0,0)$

$\therefore$ tangent passed through origin.

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SECTION 2

1) d

2) c

3) i) $\frac{d}{dx} \cos 2t$

$= -2\sin 2t$

ii) $\frac{d}{dx}\, x \tan x$ $u = x$ $v = \tan x$ $u' = 1$ $v' = \sec^2 x$

$= u'v + uv'$

$= 1(\tan x) + x\sec^2 x$

$= \tan x + x\sec^2 x$

4) $\int_{\pi/3}^{\pi} \cos \frac{1}{2} x \, dx$

$= \frac{1}{2}\left[\sin \frac{1}{2} x\right]_{\pi/3}^{\pi}$

$= \frac{1}{2}\left[\sin \frac{\pi}{2} - \sin \frac{\pi}{6}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{2}\right]$

$= \frac{1}{2}\left(\frac{1}{2}\right)$

$= \frac{1}{4}$

$l = r\theta$

5) $AB = 4.7 \times \frac{\pi}{3}$

$= 1\,{}^{17}/_{30}\,\pi$

$P = 2r + l$

$= 2(4.7) + 1\frac{17}{30}\pi$

$= 9.4 + 1\frac{17}{30}\pi$

$= 14.32182349$

$= 14.3$ (3 sig figs)

6) $y = 2\cos x$

$\frac{dy}{dx} = -2\sin x$

$m = -2\sin\frac{\pi}{6}$

$= -2(\frac{1}{2})$

$= -1$

$y - y_1 = m(x - x_1)$

$y - \sqrt{3} = -1(x - \frac{\pi}{6})$

$y - \sqrt{3} = -x + \frac{\pi}{6}$

$x + y - \frac{\pi}{6} - \sqrt{3} = 0$

7) $\frac{h}{3}[\text{1st} + \text{last} + 2(\text{odd}) + 4(\text{even})]$

$y = x\sin x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}} = \frac{\pi}{4\sqrt{2}}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}\cdot\frac{1}{\sqrt{2}} = \frac{3\pi}{4\sqrt{2}}$	0

~~$= \frac{\pi/4}{3}[0 + 0 + 4(\frac{\pi}{4\sqrt{2}} + \frac{3\pi}{4\sqrt{2}}) + 2(\frac{\pi}{2})]$~~

~~$= \frac{\pi}{12}[\frac{\pi}{\sqrt{2}} + \frac{3\pi}{\sqrt{2}} + \pi]$~~

~~$= \frac{\pi}{12}[\frac{4\pi}{\sqrt{2}} + \pi]$~~

~~$= \frac{4\pi^2}{12\sqrt{2}} + \frac{\pi^2}{12}$~~

~~$= 2.326288067 + 0.822467033$~~

~~$= 2.326 + 0.823$~~

~~$= 3.149$~~

~~$= 3.15$~~

$= \frac{\pi/4}{3} \times [0 + 0 + [2(\frac{\pi}{4\sqrt{2}} + \frac{3\pi}{4\sqrt{2}}) + 4(\frac{\pi}{2})]]$

$= \pi/12 \times (\frac{\pi}{2\sqrt{2}} + \frac{3\pi}{2\sqrt{2}} + 2\pi)$

$= \pi/12 \times (\frac{4\pi}{2\sqrt{2}} + 2\pi)$

$= \frac{\pi}{12} \times (\frac{2\pi}{\sqrt{2}} + 2\pi)$

$= 2.8080781$

$= 2.81\,u^2$

8) $y^2 = \tan^2 x$

$= \sec^2 x - 1$

$V = \pi \int_{\pi/4}^{\pi/3} \sec^2 x - 1 \quad dx$

$= \pi \left[\tan x - x \right]_{\pi/4}^{\pi/3}$

$= \pi \left[\tan \frac{\pi}{3} - \frac{\pi}{3} \right] - \left[\tan \frac{\pi}{4} - \frac{\pi}{4} \right]$

$= \pi \left[\sqrt{3} - \frac{\pi}{3} - 1 + \frac{\pi}{4} \right]$

$= \pi \left[-\frac{\pi}{12} + \sqrt{3} - 1 \right]$