Our Lady of the Sacred Heart College
Kensington

2015

YEAR 12

HALF YEARLY EXAMINATION

Student Number: ________________

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 The value of $\dfrac{5.79+0.55}{\sqrt{4.32-3.28}}$ is closest to:

(A) 4
(B) 6
(C) 9
(D) 10

2 What are the values of p and q given $\left(3\sqrt{12}+\sqrt{75}\right)\left(2+\sqrt{48}\right)=p+q\sqrt{3}$?

(A) $p=132$ and $q=15$
(B) $p=396$ and $q=15$
(C) $p=132$ and $q=22$
(D) $p=396$ and $q=22$

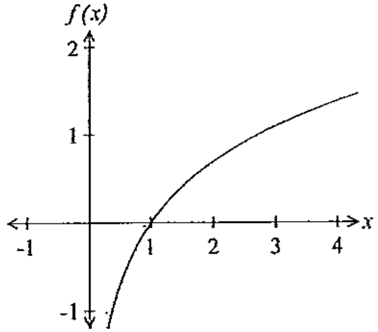
3 The line $6x-ky=8$ passes through the point (3,2). What is the value of k?

(A) -13
(B) -5
(C) 5
(D) 15

4 What is the value of $\int_0^1 (6x^2-4)dx$?

(A) -2
(B) -1
(C) 0
(D) 1

5



Which of the following properties matches the above graph?

(A) $f'(x)>0$ and $f''(x)<0$
(B) $f'(x)>0$ and $f''(x)>0$
(C) $f'(x)<0$ and $f''(x)<0$
(D) $f'(x)>0$ and $f''(x)>0$

6 What are the values of x for which $|4-3x|<13$?

(A) $x<-3$ or $x<\dfrac{17}{3}$

(B) $x>-3$ or $x>\dfrac{17}{3}$

(C) $x>-3$ or $x<\dfrac{17}{3}$

(D) $x<-3$ or $x>\dfrac{17}{3}$

7 What is the simultaneous solution to the equations $2x+y=7$ and $x-2y=1$?

(A) $x=3$ and $y=1$
(B) $x=-1$ and $y=9$
(C) $x=2$ and $y=3$
(D) $x=5$ and $y=1$

8 The second term of an arithmetic series is 39 and the sixth term is 19.
What is the sum of the first ten terms?

(A) 64
(B) 215
(C) 290
(D) 400

9 What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1, -3)$?

(A) $x + 2y - 7 = 0$
(B) $x - 2y - 7 = 0$
(C) $2x - y - 5 = 0$
(D) $2x + y + 5 = 0$

10 An area is bounded by the curve $y = \frac{2}{3}\sqrt{9 - x^2}$, the coordinate axes and the line $x = 2$.

What is an approximation for this area using the trapezoidal rule and three function values?

(A) 1.82
(B) 2.69
(C) 3.63
(D) 7.26

Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Simplify: $\frac{5}{m-2} - \frac{2}{m-3}$ **2**

(b) Given that $\log_k 5 = 0.627$ and $\log_k 2 = 0.270$, determine the value of

(i) $\log_k 10$ **1**

(ii) $\log_k 25$ **1**

(c) Differentiate with respect to x:

(i) $(3x^2 - 7)^5$ **2**

(ii) xe^{2x} **2**

(d) Find an expression for each of the following integrals.

(i) $\int (2x+3)^{10} dx$ **1**

(ii) $\int \frac{6}{x^3} dx$ **1**

(e) The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 - 4$ **2**

The curve passes through (1, 8). Determine the equation of the curve.

(f) The roots of the equation $2x^2 - x - 15 = 0$ are α and β. Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

Question 12 (15 marks) **Marks**

(a) Solve, giving your answer(s) in exact form: $2x^2 - 5x - 4 = 0$ **2**

(b) One root of the quadratic equation $4x^2 - 24x + k = 0$ is twice the other root.
Determine the value of k. **2**

(c) (i) Sketch the graphs of $y = 8 - x^2$ and $y = x^2$ on the same number plane. **1**

(ii) Find the points of intersection of the curves. **2**

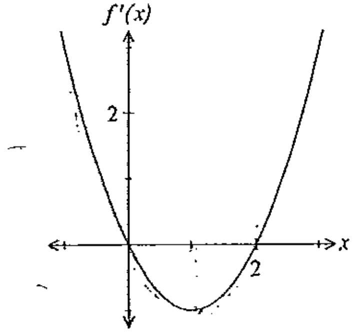
(iii) Calculate the area bounded by the graphs of $y = 8 - x^2$ and $y = x^2$. **2**

(d) Steven's father invested $1000 on each of his birthdays. When Steven reached 21 years of age he would receive the total investment. The first deposit of $1000 was made on Steven's first birthday and the last was made on his 21st birthday. The money was invested at 7% p.a. interest, compounded annually.

(i) What is the value of the first deposit after 21 years? Answer to the nearest dollar. **1**

(ii) How much does Steven receive after 21 years? Answer to the nearest dollar. **2**

(e) The curve $y = f(x)$ has a gradient function $f'(x)$ as shown below.

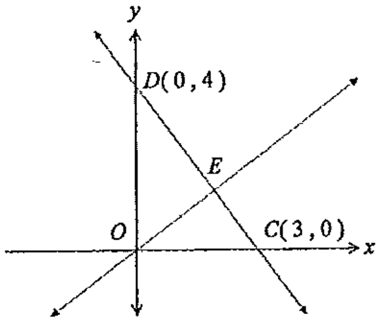


(i) For what values of x is the function $f(x)$ decreasing? **1**

(ii) Where does the curve $y = f(x)$ have a minimum turning point? **2**

Question 13 (15 marks) **Marks**

(a)



The coordinates of O, D and C are (0,0), (0,4) and (3,0) respectively. Point E lies on CD. Copy the diagram onto your workbook.

(i) Show that the equation of CD is $4x+3y-12=0$ — 1

(ii) Equation OE is $3x-4y=0$. Explain why OE is perpendicular to CD. — 2

(iii) Prove that ΔDOE is similar to ΔOCE. — 2

(iv) Show that $\dfrac{OE}{DE}=\dfrac{CE}{OE}=\dfrac{3}{4}$. — 1

(v) Find the ratio of the areas of triangles DOE and OCE. — 1

(b) Find the equation of the tangent to the curve $y=e^x+1$ at the point $(1, e+1)$. — 2

(c) The equation of a parabola is given by $y=x^2-2x+5$.

(i) Find the coordinates of its vertex. — 2

(ii) What is its focal length? — 1

(iii) Find the equation of the normal at the point $P(2,5)$. — 2

(iv) For what values of x is the parabola concave upwards? — 1

Question 14 (15 marks) **Marks**

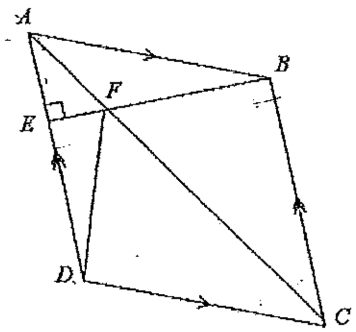
(a) Alex and Bella leave from point O at the same time. Alex travels at 20 km/h along a straight road in the direction 085° T. Bella travels at 25 km/h along another straight road in the direction 340° T.

Draw a diagram to represent this information.

(i) Show that $\angle AOB$ is 105° where $\angle AOB$ is the angle between the directions taken by Alex and Bella. — 1

(ii) Find the distance Alex and Bella are apart to the nearest kilometre after two hours. — 2

(b)



ABCD is a rhombus, BE is perpendicular to AD and intersects AC at F.

Copy the diagram onto your workbook.

(i) Explain why $\angle BCA = \angle DCA$. — 1

(ii) Prove that the triangles BFC and DFC are congruent. — 3

(iii) Show that $\angle FBC$ is a right angle. — 1

(iv) Hence or otherwise find the size of $\angle FDC$. — 1

(c) Solve the equation $(\cos x+2)(2\cos x+1)=0$ in the domain $0 \le x \le 360$. — 2

(d) If $A = \sin\beta$ express $1+\cot^2\beta$ in terms of A. — 2

(e) The exterior angle of a regular polygon is 18^0.

(i) What is the size of each interior angle? — 1

(ii) How many sides does the polygon have? — 1

Question 15 (15marks) **Marks**

(a) Cylindrical pipes are stacked in layers, where each layer contains one pipe less that the layer below. There are four pipes in the top layer, five pipes in the next layer, and so on. There are n layers altogether.

(i) Write down the number of pipes in the bottom layer. **2**

(ii) Show that there are $\frac{1}{2}n(n+7)$ pipes in the stack. **2**

(b) Let $f(x)=(x-3)^{-2}$ be a function defined for $4 \le x \le 6$.

(i) Copy the following table of values into your writing booklet and evaluate the missing values of $f(x)$. Express each value correct to 3 significant figures. **2**

x	4	4.5	5	5.5	6
$f(x)$					

(ii) Use the above five values of the function and Simpson's rule to find an approximate value for $\int_4^6 (x-3)^{-2}\,dx$. **2**

(c) The area enclosed by the curve $y=x^4$, the y-axis and the line $y=4$ is rotated about the y-axis. Find the volume of the solid of revolution. **2**

(d) Rose borrows \$50,000 to purchase furniture for her small business. The interest is calculated monthly at a rate of 2% per month. She intends to repay the loan with interest in two annual instalments of \$$M$ at the end of the first and second years.

(i) Write an expression involving M for the total amount owed by Rose after 12 months, just after the first instalment of \$$M$ has been paid. **1**

(ii) Show that $M=\dfrac{\$50{,}000\times 1.02^{24}}{1.02^{12}+1}$. **2**

(iii) What will be the total amount of interest paid on this loan? **2**

Question 16 (15 marks) **Marks**

(a) Consider the infinite series $\frac{x}{3}+\frac{2x^2}{9}+\frac{4x^3}{27}+\dots$

(i) Show that it is a geometric series. **1**

(ii) Find the values of x such that the series has a limiting sum. **1**

(iii) What is the limiting sum in terms of x? **1**

(b) A function $f(x)$ is defined by $f(x)=7+4x^3-3x^4$.

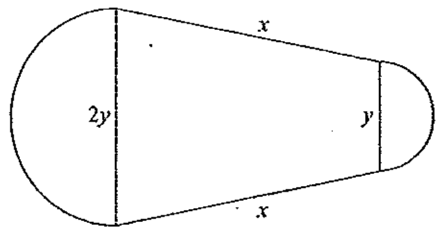
(i) Find the coordinates of the stationary points for the curve $y=f(x)$. **2**

(ii) Find all values of x for which $f''(x)=0$. **1**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the graph of $y=f(x)$ for the domain $-1 \le x \le 2$. **2**

(c) A V8 supercars racetrack consists of two semicircular curves and two straights. The dimensions of the racetrack are shown below. The total length of the racetrack is 4.8 km.



(i) Let x km represent the length of the straight and y km represent the diameter of the smaller semicircle. Show that $y=\dfrac{9.6-4x}{3\pi}$. **2**

(ii) The average speed of a V8 supercar on this racetrack is dependent on the length of the straight. It is given by: **3**

$$S=200-\left(\frac{x^3}{27}+\frac{\pi}{6}y\right)$$

What is the length of the straight that maximizes the speed?

End of paper

Our Lady of the Sacred Heart College Kensington

Year 12 Mathematics 2015 Half Yearly Section I - Answer Sheet

Student Name/Number ______________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ~~●~~ C ○ D ○

	A	B	C	D	
1.	○	●	○	○	✓
2.	○	○	●	○	✓
3.	○	○	●	○	✓
4.	●	○	○	○	✓
5.	●	○	○	○	✓
6.	○	○	●	○	✓
7.	●	○	○	○	✓
8.	○	●	○	○	✓
9.	○	●	○	○	✓
10.	○	○	●	○	✓

10

(Q11)

a. $\frac{5}{(m-2)} - \frac{2}{(m-3)}$

$\frac{5m-15-(2m-4)}{(m-2)(m-3)}$

$\frac{5m-15-2m+4}{m^2-5m+6}$

$\frac{3m-11}{(m-2)(m-3)}$ ✓ 2

b. $\log_k 10 = \log_k 5 + \log_k 2$

$= 0.627 + 0.270$

$= 0.897$ ✓ 1

(ii) $\log_k 25 = \log_k 5^2$

$= 2\log_k 5$

$= 2 \times 0.627$

$= 1.254$ ✓ 1

c. $(3x^2-7)^5$

$\frac{d}{dx}(3x^2-7)^5$

$5(3x^2-7)^4 \cdot 6x$

$30x(3x^2-7)^4$ ✓ 2

(ii) xe^{2x}

$\frac{d}{dx} xe^{2x}$ $u = x$, $u' = 1$ $v = e^{2x}$, $v' = 2e^{2x}$

$vu' + uv'$

$e^{2x} + 2xe^{2x}$ ✓ 2

$e^{2x}(1+2x)$

(Q11)

$\int (2x+10)^{10} \, dx$

$\dfrac{(2x+10)^{11}}{11 \cdot 2} = \dfrac{(2x+10)^{11}}{22} + C$ ✓ 1

$\int \dfrac{6}{x^3} \, dx.$

$\int 6x^{-3}.$ $u = 6.$

$6 \cdot \dfrac{x^{-2}}{-2}$

$-3x^{-2} + C.$ ✓ 1

e. gradient $= 6x^2 - 4$

passes $(1, 8)$.

$m = 2.$

$\int 6x^2 - 4.$

$y = 2x^3 - 4x + C.$ $y = 2x^3 - 4x + 10.$

$y = 2x^3 - 4x + C.$ ✓ 2

$8 = 2 - 4 + C.$

$8 + 2 = C.$

$C = 10.$

(Q11)

$2x^2 - x - 15 = 0$

$\alpha + \beta = -\dfrac{b}{a}$

$= \dfrac{-(-1)}{2}$

$= \dfrac{1}{2}.$ ✓ 1

$(\alpha+\beta)(\alpha+\beta)$

$\alpha^2 + 2\alpha\beta + \beta^2$

$\alpha\beta = \dfrac{c}{a}.$

$= \dfrac{-15}{2}$

$= -7\frac{1}{2}$ ✓ 1

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{1}{2}\right)^2 - 2\left(-\frac{15}{2}\right).$

$= \frac{1}{4} + 15$

$= 15\frac{1}{4}$ ✓ 1

(Q12)

a. $2x^2-5x-4$.

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{5\pm\sqrt{25-4\cdot 2\cdot -4}}{4}$

$= \frac{5\pm\sqrt{25+32}}{4}$ ✓ (2)

$= \frac{5+\sqrt{57}}{4}$ / $\frac{5-\sqrt{57}}{4}$

$= 3.14$ / -0.63.

b. α, 2α.

Value of k =

$\alpha + 2\alpha = \frac{-b}{a}$

$= \frac{24}{4}$

$3\alpha = 6$.

$\alpha = 2$.

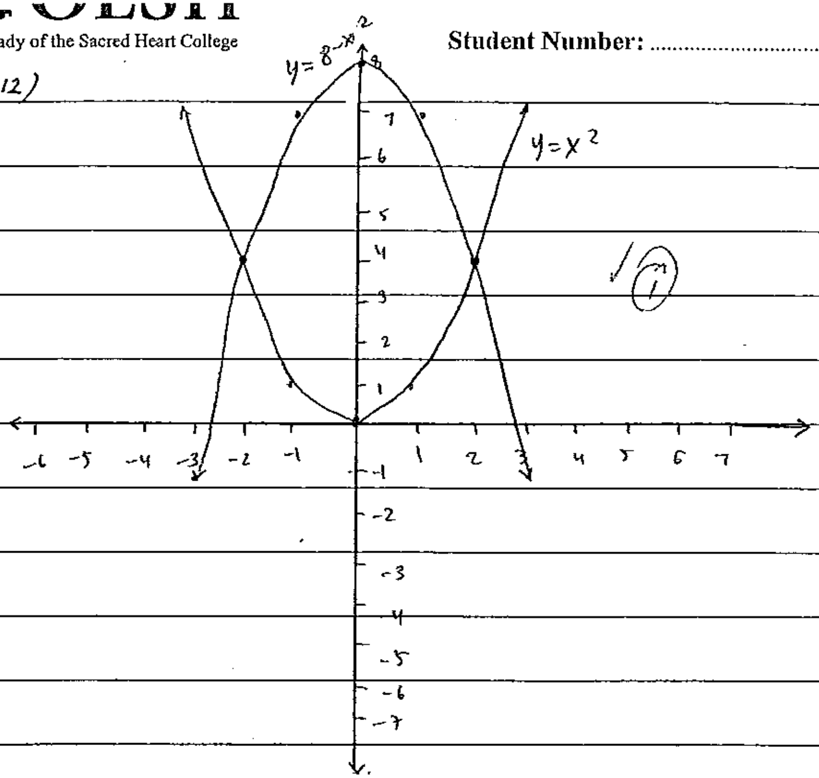
$\alpha \cdot 2\alpha = \frac{k}{4}$.

$2 \cdot 4 = \frac{k}{4}$

$32 = k$ ✓ (2)

(Q12)

c.



✓ (1)

(i) $y = 8-x^2$ & $y = x^2$

$x^2 = 8-y$ $x = 2$, $y = 4$.

$x = 2/-2$, $y = 4$

$x = 3/-3$, $y = -1$

(ii) Pt of intersection.

$x^2 = 8-x^2$

$2x^2 = 8$.

$x^2 = 4$ ✓ (2)

$x = \pm 2$.

$(2,4)$ $(-2,4)$

(iii) bounded $y = 8-x^2$ $y=x^2$

$2\int_0^3 (8-x^2) - x^2$

$2\int_0^3 (8-x^2-x^2)$

$2\int_0^3 (8-2x^2)$ (1)

$2\left[8x - \frac{2}{3}x^3\right]_0^3$.

$2[24-18]$

$2(6)$

12. units^2

(Q12)

(ii) $A_1 = 1000$

$A_2 = 1000(1.07) + 1000$

$A_3 = 1000(1.07)^2 + 1000(1.07) + 1000$.

$\vdots$

$A_{21} = 1000(1.07)^{20} + 1000(1.07)^{19} + 1000(1.07)^{18} + \dots + 1000$.

$= 1000\,\underbrace{(1 + 1.07 + 1.07^2 + 1.07^3 + 1.07^4 + \dots + 1.07^{20})}_{S_n}$.

$= \dfrac{a(r^n-1)}{r-1}$

$= \dfrac{1(1.07^{21}-1)}{0.07}$

$= 44.9$.

= total when he ~~ss~~ get money = \$44865 ✓ (2)

(i) $1000(1.07)^{20}$

\$ 3870 ✓ (1)

(Q12)

e. value of x function f(x) decreasing.

decrease $= 0 < x < 2$ ✓ (1), 0 & 2 stationary point

(ii) 0 & 2 Stationary point

↓

$2 > x > 0$ = decrease

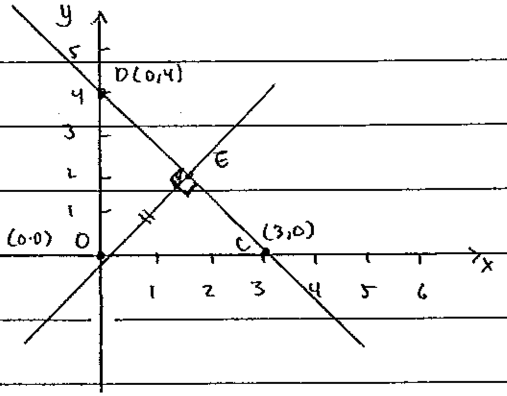
$x < 0$ = Increase

$x > 2$ = increase.

min turning point is where decrease then increase

Therefore when $x = 2$ is the minimum turning point ✓ (2)

(Q13)



(i) eq. of CD is $4x+3y-12=0$.

$\frac{y_2-y_1}{x_2-x_1} = \frac{4-0}{-3} = -\frac{4}{3}$

$(y-y_1) = m(x-x_1)$

$(y-0) = -\frac{4}{3}(x-3)$ ✓ 1

$3y = -4x+12$.

$4x+3y-12=0$.

(ii) eq. of OE is $3x-4y=0$.

$m_{OE} = \frac{3}{4}$ ✓ 1 $\quad m_{CD} = -\frac{4}{3}$ $\quad \therefore m_{OE} \times m_{CD}$

$(y-y_1) = m(x-x_1)$. $\quad = \frac{3}{4} \times -\frac{4}{3} = -1$

$y = \frac{3}{4}x$. $\quad \therefore OE \perp CD$

$4y = 3x$

$3x-4y=0$.

point of E = $4x+3y-12 \mid \times 3$

$3x-4y \mid \times 4$ ✗

$12x+9y-36$

$12x-16y$

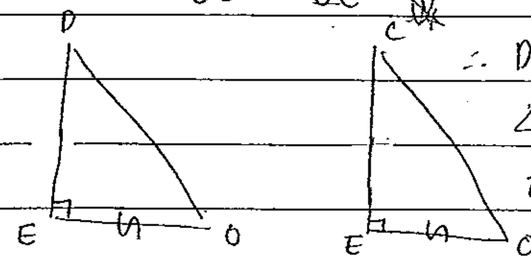
$25y-36$

$y = \frac{36}{25}$ $\quad x = \frac{48}{25}$

(Q13)

(iii) Similar $\angle CEO = \angle DEO = 90°$ ✓ 1

$DE = DE$



$\therefore DE = CE$ in proportion

$\angle DEO = \angle CEO$

$EO = EO$ ✗

$DE = CE$ in proportion.

(iv) $E\left(\frac{48}{25}, \frac{36}{25}\right)$ $\quad C(3,0)$, $D(0,4)$.

~~$\frac{1}{DE} = \frac{CE}{1} = \frac{3}{4}$~~

~~$\frac{1}{DE} = \frac{CE}{1} = \frac{3}{4}$~~

~~$\frac{CE}{DE} = \frac{3}{4}$~~

$CE = \sqrt{\left(3-\frac{48}{25}\right)^2 + \left(0-\frac{36}{25}\right)^2}$ $\quad DE = \sqrt{\left(0-\frac{48}{25}\right)^2 + \left(4-\frac{36}{25}\right)^2}$

$= \frac{9}{5}$ $\quad = \frac{16}{5}$.

$\frac{CE}{DE} = \frac{3}{4}$

$\frac{\frac{9}{5}}{\frac{16}{5}} = \frac{3}{4}$. 1 $\quad OE = \sqrt{\left(\frac{48}{25}\right)^2 + \left(\frac{36}{25}\right)^2} = \frac{12}{5}$

$\frac{9}{5} \times \frac{5}{16} = \frac{3}{4}$

$\frac{OE}{DE} = \frac{CE}{OE} = \frac{3}{4}$.

$\frac{\frac{12}{5}}{\frac{16}{5}} = \frac{\frac{9}{5}}{\frac{12}{5}} = \frac{12}{5} \times \frac{5}{16} = \frac{9}{5} \times \frac{5}{12}$

$\frac{3}{4} = \frac{3}{4}$ ✓

Q(v) at the back

(QB)

b. $y = e^x + 1$

$\frac{dy}{dx} = e^x$

$m = e$.

$(y - y_1) = e(x - x_1)$

$(y - (e+1)) = e(x-1)$ ✓ 2

$y - \cancel{e} - 1 = ex - \cancel{e}$

$y - 1 = ex$

$y - ex - 1 = 0$.

c. $y = x^2 - 2x + 5$.

$\cancel{x^2 - 2x = -5 + y}$ $x^2 = 4ay$.

$(x-1)^2 + 4 = y$ $(x-h)^2 = 4a(y-k)$

$(x-1)^2 = (y-4)$ $(x-1)(x-1)$

$V = (1, 4)$ ✓ 2 $x^2 - 2x + 1 + 4$

(ii) focal length.

$a = \frac{1}{4}$ ✓ 1

(iii) normal at $P(2.5)$.

$\frac{dy}{dx} = 2x - 2$. ✓ 2 $(y-5) = -\frac{1}{2}(x-2)$

$m = 2$. $-2y + 10 = x - 2$

$m_{normal} = -\frac{1}{2}$ eq. $x + 2y - 12 = 0$.

(iv) when value of x is positive ✓ 1

(QB)

a (V) ratio area $\triangle DOE$ & $\triangle OCE$

$\frac{1}{2} \cdot b.h$: $\frac{1}{2} bh$.

$\frac{1}{2} DE \cdot OE$: $\frac{1}{2} \cdot CE \cdot OE$

$\frac{1}{\cancel{2}} \cdot \frac{16}{5} \cdot \frac{\cancel{12}6}{5}$: $\frac{1}{\cancel{2}} \cdot \frac{9}{5} \cdot \frac{\cancel{12}6}{5}$

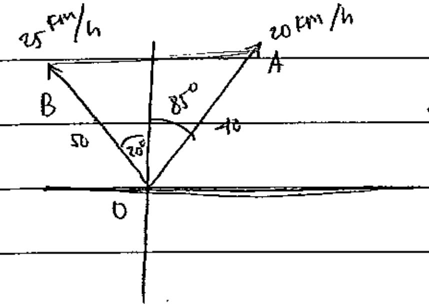
$\frac{96}{25}$: $\frac{54}{25}$

$96 : 54$.

$48 : 27$

$16 : 9$ ✓ 1

(Q14)



$\angle AOB = 85° + 20°$

$= 105°$ ✓ (1)

(ii) 2 hours.

A = 40 , B = 50.

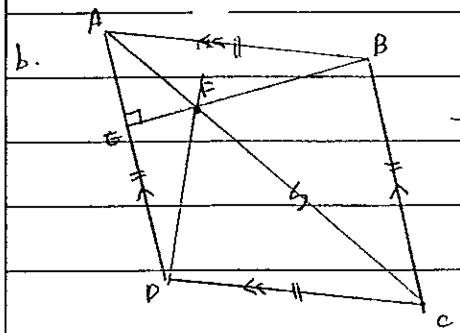
$c^2 = 40^2 + 50^2 (+) 2.40.50. \cos 105°$ (1)

$c^2 = 1600 + 2500 + 4000 \cos 105°$

$c^2 = 4100 + 4000 \cos 105°$

c = 55 km.

b.



$\angle BCA = \angle DCA$

properties of rhombus.

– because diagonal bisects the angle equally. ✓ (1)

(ii) Congruent

Therefore SAS. ✓ (3)

DC = BC (given) sides of rhombus are equal need to set out a bit better.

$\angle ACD = \angle ACB$ = same θ (above)

CF = CF

(Q14)

(iii) $\angle BFC = \angle AFE$ opposite angle.

$\angle BCF = \angle FAE$ properties of rhombus diagonal bisect angle.

$\Delta AFE = 180$

$\Delta BFC = 180$

$\Delta AFE = 90$ (given) $+ \angle FAE + \angle AFE$

$= 180.$

$= 90° = \angle FAE + \angle AFE$ ✗

Therefore $90° = \angle BCF + \angle BFC$.

$\underbrace{\angle BCF + \angle BFC}_{90} + \angle FBC = 180$

$90 + \angle FBC = 180$

$\angle FBC = 90.$

(iv) ΔFDC is congruent to ΔFBC:

$\angle BCF = \angle FCD$ just reasons? (1)

DC = BC.

FC = FC

Therefore $\angle FDC = \angle FBC$.

90°.

(Q14)

c. $(\cos x + 2)(2\cos x + 1) = 0$.

$\cos x = -2$ (no solution)

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

sin	(+)
tan	cos.

180 — 360

$x = 60°$ ✓ (2)

$(180 - 60) = 120°$

$(180 + 60) = 240°$

d. $A = \sin\beta$ express $1 + \cot^2\beta$

$\cos^2 x + \sin^2 x = 1$ $\cos^2\beta = 1 - \sin^2\beta$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$

$A = \sin\beta$.

$1 + \cot^2\beta = \operatorname{cosec}^2\beta$. ✓ (1)

✗ $\sin^2\beta$

$\sin^2\beta + + \cos^2\beta = 1$ ✗

$A^2 + \cos^2\beta = 1$

$A^2 + \cos^2\beta = 1$.

$A^2 + \cos^2\beta = 1$.

	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

(Q14)

e. size each interior angle.

$n = 360 / 18$.

$= 20$.

$\frac{(20-2) \times 180}{20} = 162°$ ✓ (1)

$(4-2) \times 180$.

(ii) $n = 20$. ✓ (1)

$= 360/18$

(Q15)

$T_1 = 4.$

$T_2 = 5 \qquad d = 1.$

(i) $T_n = a + (n-1)d$ ✓ (2)

$T_n = 4 + (n-1) \quad = ?$

(ii) $S_n = \frac{n}{2}(a + l)$

$= \frac{n}{2}(4 + (4 + n - 1))$

$= \frac{n}{2}(7 + n)$ ✓ (2)

$= \frac{1}{2}n(n+7)$

b. $f(x) = (x-3)^{-2}$

x	4	4.5	5	5.5	6
f(x)	1.00	0.445	0.25	0.16	0.111

✓ (2)

1 2 3 4 5

(iii) Simpson's rule

$\int_4^6 (x-3)^{-2}\,dx \doteqdot \frac{h}{3}\left(\cancel{f1 + last + 4evens + 2odds}\right)$

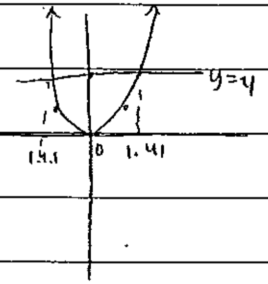
$\frac{0.5}{3}\left(1 + 0.111 + 4(0.445 + 0.16) + 2(0.25)\right)$

$\doteqdot 0.672$ ✓ (2)

(Q15)

(c) $y = x^4. \qquad x^2 = \sqrt{y}$

$y = 4.$



~~When $y = 4$~~

~~$x = 1.41$~~

~~$\pi \int x^2\,dy$~~

~~$\pi \int \sqrt{y}\,dy$~~

~~$\pi \left[\frac{2}{3} y^{\frac{3}{2}}\right]_0^{1.41}$~~

~~$= 1.116$ unit^3 x 2~~

~~$= \pi 2.23$ unit^3~~

~~Vol = 2.82 x 4~~

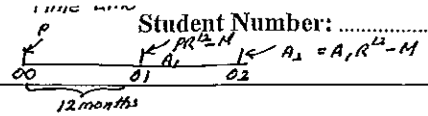
$\pi \int_0^4 \sqrt{y}$ ✓ (2)

$\pi \left[\frac{2}{3} y^{\frac{3}{2}}\right]_0^4$

$= \pi\left[\frac{2}{3}\, 4^{\frac{3}{2}} - 0\right]$

$= 16.75$ unit^3

(Q15)



$A_1 = 50\,000$ $\quad P = 50\,000$

$A_2 = 50\,000 \times (1.02)^{12} - M$ ✓ 1 $\quad R = 1.02$, $n = 12$

(i) $A_1 = PR^{12} - M = 50000(1.02)^{12} - M$

(ii) $A_2 = A_1R^{12} - M$

$= PR^{24} - MR^{12} - M$

$= PR^{24} - M(R^{12}+1)$

$= 50\,000(1.02)^{24} - M(1.02^{12}+1)$

Let $A_2 = 0$

$\therefore M(1.02^{12}+1) = 50000(1.02)^{24}$

$\therefore M = \dfrac{50000 \times 1.02^{24}}{1.02^{12}+1}$ as reqd.

(iii) $M = \$35\,455.60$

$\therefore$ Total interest paid

$= \$35\,455.60 \times 2 - \$50\,000$

$= \$20\,911.20$

(Q16)

(i) $\dfrac{\frac{2x^2}{9}}{\frac{x}{3}} = \dfrac{\frac{4x^3}{27}}{\frac{2x^2}{9}}$

$\dfrac{2x^2}{9} \times \dfrac{3}{x} = \dfrac{4x^3}{27} \times \dfrac{9}{2x^2}$ ✓ 1

$\dfrac{2x}{3} = \dfrac{2x}{3}$ ✓ geometrical P..

(ii) $-1 < r < 1$ keep going x

$\dfrac{x}{3} = ar^{n-1}$ $\qquad \dfrac{2x^2}{9} = ar^1$

$a = \dfrac{x}{3}$ $\qquad \dfrac{2x^2}{9} = \dfrac{x}{3}r$.

$6x^2 = 9xr$.

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{\frac{x}{3}}{1 - \frac{2}{3}x}$ ✓ 1

(Q16)

b. $f(x) = 7 + 4x^3 - 3x^4$ For $-1 \le x \le 2$

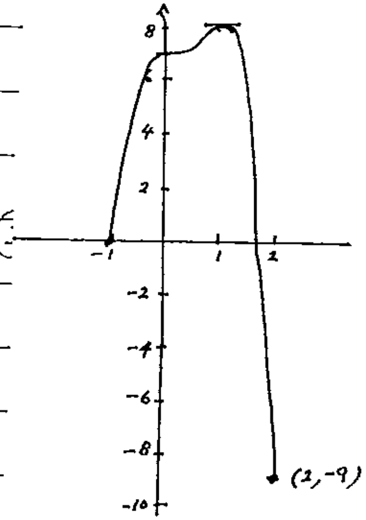
$f'(x) = 12x^2 - 12x^3$ $0 \le y \le -9$

$0 = 12x^2 - 12x^3$

$0 = 12x^2(1-x)$

$x = 0$ / $x = 1$

$(0,7)$ $(1,8)$



(ii) $f''(x) = 24x - 36x^2$

$0 = 24x - 36x^2$

$0 = 6x(4-6x)$

$= 12x(2-3x)$.

$x = 0$, $x = \frac{2}{3}$ (1) $-2 = -3x$

$y = 7$ $y = 7.6 \approx \frac{205}{27}$ $\frac{2}{3} = x$

(iii) nature

$x = 0$

-0.5	0	0.5
+	0	+

= ~~horizontal point~~ point of inflexion.

$x = 1$ (2)

0.5	1	1.5
+	0	-

$f''(x) = 24(1) - 36(1)$

$= -8$. therefore max turning pts

iv) Diagram

c. πd.

$\frac{1}{2}\pi \cdot 2y + \frac{1}{2}\pi y$.

$y\pi + \frac{y\pi}{2}$.

$\frac{2y\pi + y\pi}{2} = \frac{3y\pi}{2}$.

$4.8\text{ km} = \frac{3y\pi}{2} + 2x$. (2)

$4.8\text{ km} = \frac{3y\pi + 4x}{2}$

$9.6\text{ km} = 3y\pi + 4x$

$\frac{9.6 - 4x}{3\pi} = y$

(ii) $S = 200 - \left(\frac{x^3}{27} + \frac{\cancel{\pi}}{6}\left(\frac{9.6-4x}{3\cancel{\pi}}\right)\right)$

$= 200 - \left(\frac{x^3}{27} + \frac{9.6-4x^2}{18}\right)$

$\frac{dS}{dx} = -\left(\frac{\cancel{3}x^2}{\cancel{27}9} + \frac{-\cancel{4}x^2}{\cancel{18}9}\right) = 0$

$\frac{x^2}{9} = \frac{2}{9}$

$\therefore x = \pm\sqrt{2}$ but $x > 0$

Also $\frac{d^2S}{dx^2} = -\left(\frac{2x}{9}\right) < 0$ $\therefore$ Max speed occurs

at $x = \sqrt{2}$ *