Parametric Name: ____________________ Total: /10

Two points are defined by $x = 2\cos t$ and $y = \cos 2t$

1. Show that these points lie on a parabolic arc.

2. Sketch the arc, showing its end points, focus and directrix.

The chord PQ of the parabola $x^2 = 4y$ subtend a right angle at the origin O. If the co-ordinates of P and Q are $(2t,t^2)$ and $2s,s^2)$ respectively

3. Write down the gradients of PO and QO.

4. Show that $ts = -4$.

5. Express the coordinates of Q in terms of t.

6. Write down the co-ordinates of the mid-point M of PQ.

7. Determine the cartesian equation of the locus of M, the midpoint of PQ.

PARAMETRICS

① $\cos 2t = 2\cos^2 t - 1$

$y = 2\left(\frac{x}{2}\right)^2 - 1$

$= 2\left(\frac{x^2}{4}\right) - 1$

$= \frac{x^2}{2} - 1$ ✓

$2y + 2 = x^2$

$2(y+1) = x^2$ ✓

$2 = 4a$

$\boxed{a = \frac{1}{2}}$

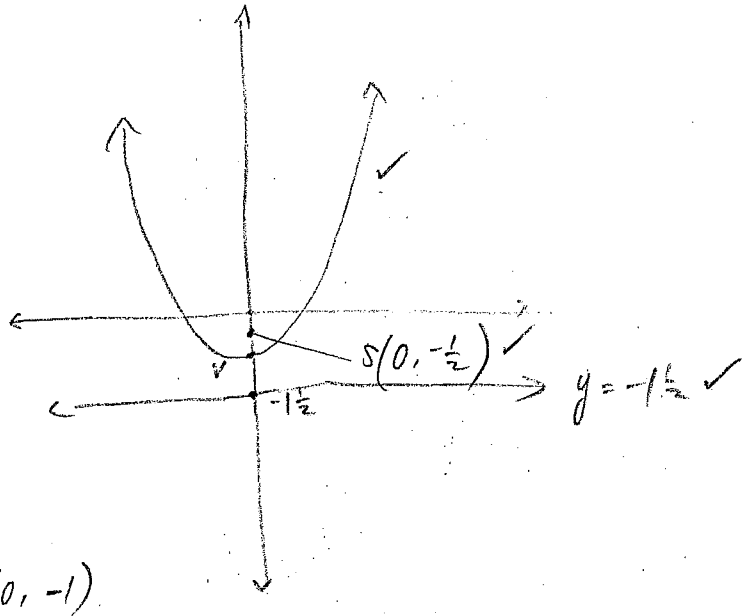
vertex $(0, -1)$

$-1 + \frac{1}{2} = -\frac{1}{2}$

focus $(0, -\frac{1}{2})$

directrix $y = -1\frac{1}{2}$

②



③ $PO = \frac{t^2}{2t}$ $\quad QO = \frac{s^2}{2s}$

$= \frac{t}{2}$ ✓ $\quad = \frac{s}{2}$ ✓

④ $\frac{t}{2} \times \frac{s}{2} = -1$

$\boxed{ts = -4}$ ✓

⑤ $P(2t, t^2)$ $\quad Q(2s, s^2)$ ①

since $ts = -4$

$s = \frac{-4}{t}$

$Q = \left(\frac{-8}{t}, \frac{16}{t^2}\right)$ ✓

⑥ $P(2t, t^2)$ $\quad Q\left(\frac{-8}{t}, \frac{16}{t^2}\right)$

$\frac{2t - \frac{8}{t}}{2}$, $\frac{t^2 + \frac{16}{t^2}}{2}$

$\frac{2t^2 - 8}{t} \times \frac{1}{2}$ $\quad \frac{\frac{t^4 + 16}{t^2}}{\frac{2}{1}}$

$\frac{\cancel{2}(t^2 - 4)}{\cancel{2}t}$

$\frac{t^2 - 4}{t}$ ✓ $\quad \frac{t^4 + 16}{2t^2}$

$x^2 = \frac{t^4 - 8t^2 + 16}{t^2}$ $\quad y = \frac{t^4 + 16}{2t^2}$ ✓

$x^2 = \frac{y2t^2 - 8t^2}{t^2}$ ✓ $\quad y2t^2 = t^4 + 16$

$= \frac{\cancel{t^2}(2y - 4)}{\cancel{t^2}}$ $\quad \boxed{x^2 = 2y - 4}$ ✓