PARTIAL FRACTIONS

To separate an algebraic fraction ${}^{A(x)}/_{B(x)}$ into 2 or more 'partial fractions', we follow these steps:-

1^{st} If degree A(x) > degree B(x) => then divide A(x) by B(x) and separate the remainder.

2^{nd} Factorise B(x) into a product of linear and irreducible quadratic factors.

3^{rd} Rewrite ${}^{A(x)}/_{B(x)}$ as the sum of partial fractions over these factors (denominators), where . .

(a) for each linear factor $(x - x_1)$ include $\frac{a}{x-x_1}$ in the sum.

(b) for each irreducible quadratic factor (mx^2+nx+p), include $\frac{ax+b}{mx^2+nx+p}$ in the sum.

(c) for each repeated linear factor, eg. $(x - x_1)^3$, include $\frac{a}{x-x_1}$ and $\frac{b}{(x-x_1)^2}$ and $\frac{c}{(x-x_1)^3}$ in the sum.

Answers:

1) $1+\frac{4}{x-2}-\frac{3}{x}$ 2) $2-\frac{2}{x+2}+\frac{1}{x-2}$ 3) $x+\frac{1}{x}-\frac{3}{x-5}$ 4) $3-\frac{1}{x}+\frac{3}{x^2}-\frac{4}{x+1}$ 5) $3+\frac{2x}{x^2+4}-\frac{1}{x}$

EXAMPLES INVOLVING IMPROPER RATIONAL FUNCTIONS

Separate the following into Partial Fractions:-

1. $\dfrac{x^2 - x + 6}{x^2 - 2x}$

2. $\dfrac{2x^2 - x - 2}{x^2 - 4}$

3. $\dfrac{x^3 - 5x^2 - 2x - 5}{x^2 - 5x}$

4. $\dfrac{3x^3 - 2x^2 + 2x + 3}{x^3 + x^2}$

5. $\dfrac{3x^3 + x^2 + 12x - 4}{x^3 + 4x}$