Student's name ______ Student's number ______ Teacher's name ______

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2008

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

- Attempt questions 1-7
- All questions are of equal value

1	2	3	4	5	6	7	Total	Total
							/84	%

Question 1 (12 marks) Start a new sheet of writing paper.

Marks

(a) Evaluate, $\lim\limits_{x\to 0}\dfrac{3\sin\frac{x}{2}}{x}$, showing all working. **2**

(b) Find the coordinates of the point, P, that divides the interval AB internally in the ratio of 4:5 if $A(-2,3)$ and $B(1,0)$. **2**

(c) Find k if $x^{2k+3}=e^{9\ln x}$, where $x>0$. **2**

(d) $\int \cos^2 4x \ dx$ **3**

(e) Use the substitution $u=1+x^5$ to evaluate

$$\int_{-1}^{1} x^4\sqrt{1+x^5}\,dx$$ **3**

End of Question 1

Question 2 (12 marks) Start a new sheet of writing paper.

Marks

(a) Solve $\frac{2}{x-1} \geq \frac{3}{x}$, $x \neq 0$, $x \neq 1$ 3

(b) Given two roots of $2x^3 - kx + 8 = 0$ are equal, find k. 3

(c) Find the constant term in $(x^3 - \frac{1}{x})^8$ 3

(d) A particle is moving in a straight line with its acceleration as a function of x given by $\ddot{x} = -8x^3$. It is initially at the origin and is travelling with a velocity of $4m/s$.

i. Find the maximum speed of the particle. 1

ii. Show that $\dot{x} = 2\sqrt{4 - x^4}$ 2

End of Question 2

Question 3 (12 marks) Start a new sheet of writing paper.

Marks

(a) i. Show that $\cos 3x = 4\cos^3 x - 3\cos x$ 2

ii. Hence, or otherwise, find $\int \cos x \sin^2 x \, dx$ 2

(b) An oven has been heated to a constant temperature of $180°$C. A cake mixture, with a temperature of $20°$C is placed in the oven and after 15 minutes its temperature is measured at $100°$C. The heating rate is proportional to the difference between the cake temperature and the oven temperature.

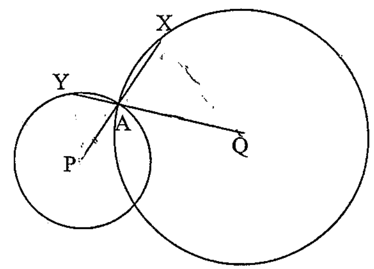
i. Show that the equation for the cake temperature is given by $T = 180 - 160e^{-0.046t}$. 2

ii. What will be the temperature of the cake after 30 minutes? 1

iii. How long will it take for the cake to reach $150°$C? 1

iv. What would be the limiting temperature which could be achieved by the cake? 1

(c)



P and Q are the centres of the circles in the diagram above. PAX and QAY are straight lines. If $\angle PAY = x$, prove that P, Q, X and Y are concyclic. 3

End of Question 3

Question 4 (12 marks) **Start a new sheet of writing paper.**

Marks

(a) The acute angle between the lines L_1 and L_2 is $\frac{\pi}{4}$ radians. The equation of L_1 is $y=3x-1$. The equation of L_2 is $y=mx+b$. Find the equation of the line L_2 if it passes through (-1,-4). **3**

(b) The equation $e^x-x-2=0$ has a root close to $x=1.2$. Use Newton's method **once** to find a better approximation to this root, correct to 2 decimal places. **2**

(c) i. Use mathematical induction to prove that for all positive integers n: **3**

$$4(1^3+2^3+3^3+...+n^3)=n^2(n+1)^2.$$

ii. Hence, or otherwise, find the value of : **2**

$$\lim_{n\to\infty}\left(\frac{1^3+2^3+3^3+...+n^3}{n^4}\right)$$

(d) Show that the chord of contact on the parabola $x^2=4ay$ is a focal chord if the external point lies on the directrix. The equation of the chord of contact is $xx_1=4a(y+y_1)$. Do NOT prove this result. **2**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper.**

Marks

(a) i. If $g(x)=e^{x+1}$, find $g^{-1}(x)$, the equation of the inverse of $g(x)$. **2**

ii. State the domain of $g^{-1}(x)$. **1**

iii. On a number plane, sketch the graphs of $y=g(x)$ and $y=g^{-1}(x)$, showing intercepts and at least one other point on each curve. **3**

iv. Using the graphs in **iii**, or otherwise, discuss the symmetry of the functions $y=g(x)$ and $y=g^{-1}(x)$. **1**

(b) Air is pumped into a spherical balloon at a constant rate of 12 cm^3/s. Find the rate of increase in its surface area when its radius is 8 *cm*. **3**

(c) Find the general solution of $\cos(2x-\frac{\pi}{4})=1$ **2**

End of Question 5

Question 6 (12 marks) Start a new sheet of writing paper. Marks

(a) Find the greatest coefficient in $(5+2x)^{12}$. 3

(b) An object is projected from the top of a vertical cliff 18 m above the horizontal ground at an angle θ where $\tan\theta = \frac{3}{4}$, with an initial speed of 25 m/s.

i. Show that the equations of motion of the object are: 4

$$x = 20t$$
$$y = -\frac{1}{2}gt^2 + 15t + 18$$

Neglecting air resistance, and taking g=9.8 m/s^2

ii. What is the greatest height reached by the particle (correct to 2 decimal places)? 2

iii. Find the distance from the base of the cliff to where the object hits the ground (correct to 2 decimal places). 3

End of Question 6

Question 7 (12 marks) Start a new sheet of writing paper. Marks

(a) A particle moves in a straight line and its position at time t is given by:
$x = 1 + \sqrt{3}\cos 2t - \sin 2t$

i. Show that $\sqrt{3}\cos 2t - \sin 2t = 2\cos(2t + \frac{\pi}{6})$. 2

ii. Show that the particle with equation $x = 1 + \sqrt{3}\cos 2t - \sin 2t$ is undergoing simple harmonic motion. 3

iii. Describe the motion of the particle including the centre, amplitude and period of motion. 2

iv. Find the first time the particle is at the origin (i.e. when $x = 0$). 2

(b) Given the fact that 3

$\int_0^{\pi} \sin mx \sin nx \, dx = 0$ and $\int_0^{\pi} \sin^2 mx \, dx = \frac{\pi}{2}$

if m and n are **any** unequal positive integers,

find the volume obtained when the area between the curve
$y = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$
and the x-axis from x=0 to $x = \pi$, is rotated about the x-axis.

End of Examination

Av . 43.8 / 84   Top : 80/84

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Course	Ext. 1.	Name of task/exam	Trial

Question 1 :

a) $\lim\limits_{x\to 0} \dfrac{3\sin\frac{x}{2}}{x} = 3\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{2}}{x}$

$= 3\times\frac{1}{2}\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{2}}{\frac{x}{2}}$

$= \frac{3}{2}\,(1)$

$= \frac{3}{2}$

b) $A\overset{x_1,\ y_1}{(-2,3)}$ $B\overset{x_2\ y_2}{(1,0)}$ $\overset{m\ n}{4:5}$

$x = \dfrac{mx_2+nx_1}{m+n}$, $y = \dfrac{my_2+ny_1}{m+n}$

$x = \dfrac{4(1)+5(-2)}{9}$, $y = \dfrac{4(0)+5(3)}{9}$

$x = -\frac{6}{9}$, $y = \frac{15}{9}$

$P\left(-\frac{2}{3}, \frac{5}{3}\right)$

c) $x^{2k+3} = e^{9\ln x}$

$x^{2k+3} = e^{\ln x^9}$

$\therefore 2k+3 = 9$

$2k = 6$

$k = 3$

d) $\int \cos^2 4x\, dx$ $\cos 8x = 2\cos^2 4x - 1$
$\frac{1}{2}(\cos 8x + 1) = \cos^2 4x$

$= \int\left(\frac{1}{2}\cos 8x + \frac{1}{2}\right)dx$

$= \frac{1}{2}\dfrac{\sin 8x}{8} + \frac{1}{2}x + c$

$= \frac{1}{16}\sin 8x + \frac{1}{2}x + c$

e) $u = 1+x^5$ when $x=-1$ $u=0$

$\dfrac{du}{dx} = 5x^4$ when $x=1$ $u=2$.

$du = 5x^4\,dx$

$\therefore \int_{-1}^{1} x^4\sqrt{1+x^5}\,dx = \frac{1}{5}\int_{-1}^{1} 5x^4(1+x^5)^{\frac{1}{2}}dx$

$= \frac{1}{5}\int_0^2 u^{\frac{1}{2}}\,du$

$= \frac{1}{5}\left[\dfrac{2u^{3/2}}{3}\right]_0^2$

$= \frac{2}{15}\left[u^{3/2}\right]_0^2$

$= \frac{2}{15}\left[2^{3/2}\right]$

$= \frac{4\sqrt{2}}{15}$

Question 2 :

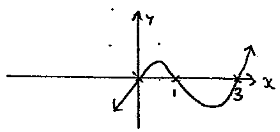
a) $\dfrac{2}{x-1} \geq \dfrac{3}{x}$, $x\neq 0,\ x\neq 1$

$x^2(x-1)\,2 \geq 3(x-1)^2x$

$3(x-1)^2x - 2(x-1)x^2 \leq 0$

$x(x-1)\left[3(x-1)-2x\right] \leq 0$

$x(x-1)(x-3) \leq 0$



$x<0,\quad 1<x\leq 3$

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OR : $\dfrac{2}{x-1} \geq \dfrac{3}{x}$, $x\neq 0,\ x\neq 1$

Critical points : $x=0,\ x=1$

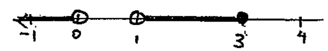
$\dfrac{2}{x-1} = \dfrac{3}{x}$

$2x = 3x-3$

$-x = -3$

$x = 3$

$\therefore$ Look at intervals



Test : $x = \frac{1}{2}$

$\dfrac{2}{\frac{1}{2}-1} \geq \dfrac{3}{\frac{1}{2}}$

$-4 \not\geq 6$

Test : $x = 2$

$\dfrac{2}{2-1} \geq \dfrac{3}{2}$

$2 \geq \frac{3}{2}$ true

Test : $x = 4$

$\frac{2}{3} \not\geq \frac{3}{4}$

Test : $x = -1$

$\dfrac{2}{-2} \geq \dfrac{3}{-1}$

$-1 \geq -3$ true

$\therefore\ x<0,\quad 1<x\leq 3$

$2x^3 - kx + 8 = 0$

b) Let the roots be α, α and β.

Sum of roots one at a time

$\alpha+\alpha+\beta = -\frac{b}{a}$

$2\alpha+\beta = 0$

$\therefore \beta = -2\alpha$

Sum of roots two at a time

$\alpha^2+\alpha\beta+\alpha\beta = -\frac{k}{2}$

$\alpha^2+2\alpha\beta = -\frac{k}{2}$

$\alpha(\alpha+2\beta) = -\frac{k}{2}$

$\alpha(\alpha+2[-2\alpha]) = -\frac{k}{2}$

$\alpha(\alpha-4\alpha) = -\frac{k}{2}$

$\alpha(-3\alpha) = -\frac{k}{2}$

$3\alpha^2 = \frac{k}{2}$

$\alpha^2 = k/6$

Product of roots

$\alpha^2\beta = -4$

$\alpha^2(-2\alpha) = -4$

$-2\alpha^3 = -4$

$\alpha^3 = 2$

$\alpha = \sqrt[3]{2}$

$\therefore k = 6\alpha^2$

$= 6\left(2^{\frac{1}{3}}\right)^2$

$= 6\left(2^{2/3}\right)$ or $\dfrac{12}{\sqrt[3]{2}}$

c) $T_{k+1} = {}^8C_k\,(x^3)^k\left(-\frac{1}{x}\right)^{8-k}$

$= {}^8C_k\, x^{3k}(-1)^{8-k}x^{k-8}$

$= {}^8C_k\,(-1)^{8-k}x^{4k-8}$

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for constant term: $4k - 8 = 0$

$\therefore k = 2.$

$\therefore$ Constant term is ${}^8C_2 \, (-1)^6$

$= 28$

d) $\ddot{x} = -8x^3$

when $t = 0$, $x = 0$, $v = 4$ m/s

i max speed is when $\ddot{x} = 0$

i.e. $0 = -8x^3$

$x = 0.$

So when particle is at origin it has max speed.

$\therefore$ max speed = 4 m/s.

ii $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -8x^3$

$\frac{1}{2}v^2 = -8\int x^3\,dx$

$\frac{1}{2}v^2 = -8\,\frac{x^4}{4} + c$

$\frac{1}{2}v^2 = -2x^4 + c$

when $x = 0$ $v = 4$

$8 = 0 + c$

$\therefore \frac{1}{2}v^2 = -2x^4 + 8$

$v^2 = -4x^4 + 16$

$v^2 = 4(4 - x^4)$

$v = \pm\sqrt{4(4 - x^4)}$

$= \pm 2\left(\sqrt{4 - x^4}\right)$

Since $v = 4$ when $x = 0$

$\therefore v = 2\sqrt{4 - x^4}$

Question 3:

a) i Show $\cos 3x = 4\cos^3 x - 3\cos x$

LHS $= \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$

$=$ RHS

ii $\int \cos x \sin^2 x \, dx$

Method 1:

$\int \cos x (1 - \cos^2 x)\,dx$

$= \int (\cos x - \cos^3 x)\,dx$

$= \int \left(\cos x - \left[\frac{1}{4}(\cos 3x + 3\cos x)\right]\right)dx$

$= \int \left(\cos x - \frac{1}{4}\cos 3x - \frac{3}{4}\cos x\right)dx$

$= \int \left(\frac{1}{4}\cos x - \frac{1}{4}\cos 3x\right)dx$

$= \frac{1}{4}\sin x - \frac{1}{12}\sin 3x + c.$

Method 2:

$\int f'(x)\,[f(x)]^n\,dx$

$= \frac{[f(x)]^{n+1}}{n+1} + c$

$\therefore \int \cos x\,(\sin x)^2\,dx$

$= \frac{\sin^3 x}{3} + c$

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Method 3 Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\therefore \int \cos x \sin^2 x\,dx = \int u^2\,du$

$= \frac{u^3}{3} + c$

$= \frac{\sin^3 x}{3} + c.$

Note: $\frac{1}{4}\sin x - \frac{1}{12}\sin 3x$

$= \frac{1}{4}\sin x - \frac{1}{12}\left[\sin(2x + x)\right]$

$= \frac{1}{4}\sin x - \frac{1}{12}\left[\sin 2x \cos x + \cos 2x \sin x\right]$

$= \frac{1}{4}\sin x - \frac{1}{12}\left[2\sin x \cos^2 x + (1 - 2\sin^2 x)\sin x\right]$

$= \frac{1}{4}\sin x - \frac{1}{12}\left[2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x\right]$

$= \frac{1}{4}\sin x - \frac{2}{12}\sin x + \frac{2}{12}\sin^3 x - \frac{1}{12}\sin x + \frac{2}{12}\sin^3 x$

$= \frac{1}{3}\sin^3 x$

b) i $\frac{dT}{dt} = k(N - P)$

$T = P + Ae^{kt}$

$P = 180$

$t = 0$ $T = 20$

$t = 15$ $T = 100$

when $t = 0$ $T = 20$

$20 = 180 + Ae^0$

$A = -160$

$T = 180 - 160e^{kt}$

when $t = 15$ $T = 100$

$100 = 180 - 160e^{15k}$

$\therefore -80 = -160e^{15k}$

$\frac{1}{2} = e^{15k}$

$\ln\frac{1}{2} = 15k$

$k = \frac{1}{15}\ln\frac{1}{2}$

$\therefore k = -0.0462098\ldots$

$\therefore T = 180 - 160e^{-0.046t}$

ii when $t = 30$ $T = ?$

$T = 180 - 160e^{-0.046 \times 30}$

$T = 140$

$\therefore$ temp will be 140°C

iii $T = 150$ $t = ?$

$150 = 180 - 160e^{-0.046\ldots t}$

$-30 = -160e^{-0.046\ldots t}$

$\frac{-30}{-160} = e^{-0.046\ldots t}$

$\ln\frac{3}{16} = -0.046\ldots t$

$t = 36$ mins 13.5 sec.

iv $T = 180 - 160e^{-0.046t}$

as $t \to \infty$

limiting temp is 180°C.

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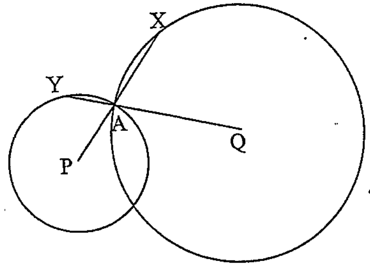
5



If $\angle PAY = x$

then $\angle QAX = x$ (vertically opposite angles are equal)

Join PY and QX

$PA = PY$ (equal radii)

$QA = QX$ (equal radii)

$\therefore \angle PYA = \angle PAY = x$ (angles opposite equal sides are equal)

and

$\angle QAX = \angle QXA$ (angles opposite equal sides are equal)

$\therefore \angle PYA = \angle QXA$

$\therefore PQXY$ are concyclic as angles subtended on the same side of chord, PQ, are equal.

OR If $\angle PAY = x$

then $\angle QAX = x$ (vertically opposite angles are equal)

Join PY and QX

$PA = PY$ (equal radii)

$QA = QX$ (equal radii)

$\therefore \angle PYA = \angle PAY = x$ (angles opposite equal sides are equal)

and $\angle QAX = \angle QXA = x$ (similarly)

$\therefore \triangle APY \ ||| \ \triangle AQX$ (equiangular)

$\therefore$ In similar triangles, corresponding sides are in the same ratio

i.e. $\dfrac{PA}{QA} = \dfrac{YA}{XA}$

$\therefore PA \times XA = YA \times QA$

Since the product of the intercepts on intersecting intervals is equal then the endpoints of the intervals are concyclic

Question 4:

a $y = 3x - 1 \qquad m_1 = 3$

$y = mx + b \qquad m_2 = m$

$\therefore \tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan \dfrac{\pi}{4} = \left| \dfrac{3 - m}{1 + m(3)} \right|$

$\pm 1 = \dfrac{3 - m}{1 + 3m}$

$1 + 3m = 3 - m \qquad\qquad -1 - 3m = 3 - m$

$4m = 2 \qquad\qquad -4 = 2m$

$m = \frac{1}{2} \qquad\qquad m = -2$

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$\therefore$ eqn of line

$y + 4 = \frac{1}{2}(x + 1) \qquad\qquad y + 4 = -2(x + 1)$

$2y + 8 = x + 1 \qquad\qquad y + 4 = -2x - 2$

$x - 2y - 7 = 0 \qquad\qquad 2x + y + 6 = 0$

OR $y = \frac{1}{2}x - 3\frac{1}{2} \qquad\qquad$ OR $y = -2x - 6$

b. Let $P(x) = e^x - x - 2$

$P(1.2) = 0.1201169\ldots$

$P'(x) = e^x - 1$

$P'(1.2) = 2.3201169\ldots$

$\therefore x_2 = x_1 - \dfrac{P(x_1)}{P'(x_1)}$

$= 1.2 - \dfrac{0.1201169\ldots}{2.3201169\ldots}$

$= 1.148\ldots$

$= 1.15$

$\therefore$ a better approximation is 1.15 and $P(x)$ is continuous.

c i

Prove

$4(1^3 + 2^3 + 3^3 + \ldots + n^3) = n^2(n+1)^2$

Step 1 : prove true for $n = 1$

$LHS = 4(1^3) \qquad\qquad RHS = 1^2(1+1)^2$

$= 4 \qquad\qquad = 1(4)$

$\qquad\qquad = 4$

$\therefore$ true for $n = 1$

Step 2 : assume true for $n = k$

i.e. $4(1^3 + 2^3 + 3^3 + \ldots k^3) = k^2(k+1)^2$

Step 3 : prove true for $n = k+1$

i.e. prove

$4(1^3 + 2^3 + \ldots k^3 + (k+1)^3) = (k+1)^2(k+2)^2$

$LHS = 4(1^3 + 2^3 + \ldots k^3) + 4(k+1)^3$

$= k^2(k+1)^2 + 4(k+1)^3$ from assumpt.

$= (k+1)^2[k^2 + 4(k+1)]$

$= (k+1)^2[k^2 + 4k + 4]$

$= (k+1)^2(k+2)^2$

$= RHS$

$\therefore$ true

$\therefore$ By the principle of mathematical induction it is true for all positive integers n.

ii $\displaystyle \lim_{x \to \infty} \left(\frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4} \right)$

$\displaystyle = \lim_{x \to \infty} \frac{\frac{n^2(n+1)^2}{4}}{n^4}$

$\displaystyle = \lim_{x \to \infty} \frac{n^2(n+1)^2}{4n^4}$

$\displaystyle = \lim_{x \to \infty} \frac{n^2(n^2 + 2n + 1)}{4n^4}$

$\displaystyle = \lim_{x \to \infty} \frac{n^4 + 2n^3 + n^2}{4n^4}$

$= \dfrac{1}{4}$

d. $xx_1 = 4a(y+y_1)$

If external point lies on directrix, coordinates would be $(x_1, -a)$

$\therefore xx_1 = 4a(y + -a)$

$xx_1 = 4a(y-a)$

If a focal chord $(0, a)$ satisfies

LHS $= 0(x_1)$ $= 0$ RHS $= 4a(a-a)$ $= 0$

$\therefore$ LHS = RHS

$\therefore xx_1 = 4a(y-a)$ is a focal chord.

$\therefore$ If external point lies on directrix, the chord of contact is a focal chord.

Question 5:

a i $g(x) = e^{x+1}$

i.e. $y = e^{x+1}$

inverse: $x = e^{y+1}$

$\ln x = y+1$

$y = (\ln x) - 1$

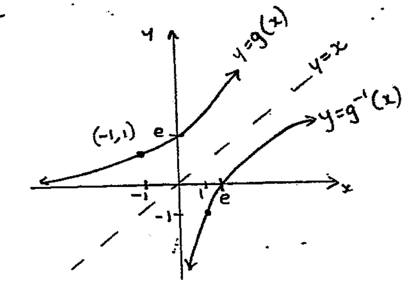
$\therefore g^{-1}(x) = -1 + \ln x$

ii domain of $g^{-1}(x)$ = range of $g(x)$

$\therefore$ range of $g(x)$ is $y > 0$

$\therefore$ domain of $g^{-1}(x)$: $x > 0$

iii



iv The graphs are symmetrical about the line $y = x$.

b $V_{sphere} = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = 12$ cm^3/s

$SA_{sphere} = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$\frac{dS}{dt} = ?$ when $r = 8$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dV}{dt} \times \frac{dr}{dV}$

$\frac{dS}{dt} = 8\pi r \times 12 \times \frac{1}{4\pi r^2}$

$= \frac{24}{r}$

when $r = 8$

$\frac{dS}{dt} = 3$ cm^2/s

PLC Sydney Maths Department

Ver 1

Solutions for exams and assessment tasks

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OR

$\frac{dV}{dt} = 12$ $\frac{dS}{dt} = ?$ when $r = 8$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$12 = 4\pi r^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{12}{4\pi r^2}$

$\frac{dr}{dt} = \frac{3}{\pi r^2}$

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$\frac{dS}{dt} = \frac{8\cancel{\pi} r}{1} \times \frac{3}{\cancel{\pi} r^2}$

$\frac{dS}{dt} = \frac{24}{r}$

when $r = 8$

$\frac{dS}{dt} = \frac{24}{8}$

$\frac{dS}{dt} = 3$ cm^2/sec

c $\cos\left(2x - \frac{\pi}{4}\right) = 1$

$2x - \frac{\pi}{4} = 2\pi n$

$2x = 2\pi n + \frac{\pi}{4}$

$x = \pi n + \frac{\pi}{8}$, n integer

Question 6:

a $(5 + 2x)^{12}$

$T_{k+1} = {}^{12}C_k (5)^k (2x)^{12-k}$

$T_k = {}^{12}C_{k-1} (5)^{k-1} (2x)^{12-(k-1)}$

Coeff:

$\frac{T_{k+1}}{T_k} = \frac{{}^{12}C_k\, 5^k\, 2^{12-k}}{{}^{12}C_{k-1}\, 5^{k-1}\, 2^{13-k}}$

$= \dfrac{\frac{12!}{k!\,(12-k)!}\, 5^k\, 2^{12-k}}{\frac{12!}{(k-1)!\,(13-k)!}\, 5^{k-1}\, 2^{13-k}}$

$\frac{T_{k+1}}{T_k} = \frac{\cancel{12!}}{k!\,\cancel{(12-k)!}} 5^k \cancel{2^{12-k}} \times \frac{\cancel{(k-1)!}\,(13-k)!}{\cancel{5^{k-1}}\, \cancel{2^{13-k}}\, \cancel{12!}}$

$= 13 - k \times \frac{5}{2}$

$= \frac{65 - 5k}{2k}$

for greatest coefficient $\frac{T_{k+1}}{T_k} > 1$

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$\therefore \frac{65-5k}{2k} > 1$

$65-5k > 2k$

$-7k > -65$

$k < \frac{65}{7}$

$k < 9\frac{2}{7}$

$\therefore k = 9.$

$\therefore$ Greatest Coefficient is

${}^{12}C_9\, 5^9\, 2^3 = 3437500000$

OR : $(5+2x)^{12}$

$T_{k+1} = {}^{12}C_k\,(2x)^k\,(5)^{12-k}$

$T_k = {}^{12}C_{k-1}\,(2x)^{k-1}\,5^{12-(k-1)}$

Coeff :

$\frac{T_{k+1}}{T_k} = \frac{{}^{12}C_k\, 2^k\, 5^{12-k}}{{}^{12}C_{k-1}\, 2^{k-1}\, 5^{13-k}}$

$= \frac{12-k+1}{k} \times \frac{2}{5}$

$= \frac{(13-k)2}{5k}$

for greatest coefficient $\frac{T_{k+1}}{T_k} > 1$

$\therefore \frac{2(13-k)}{5k} > 1$

$26-2k > 5k$

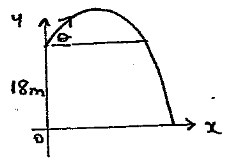
$26 > 7k$

$k < 3\frac{5}{7}$

$\therefore$ Greatest coefficient is

${}^{12}C_3\, 2^3\, 5^9 = 3437500000.$

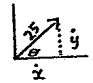
b)



$\tan\theta = \frac{3}{4}$

$v = 25\text{ m/s}.$

i Initially



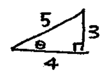
$\cos\theta = \frac{\dot{x}}{25}$

$\dot{x} = 25\cos\theta$

$\sin\theta = \frac{\dot{y}}{25}$

$\dot{y} = 25\sin\theta$

we know $\tan\theta = \frac{3}{4}$



$\therefore \cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$\therefore \dot{x} = 25 \times \frac{4}{5} = 20$

$\dot{y} = 25 \times \frac{3}{5} = 15.$

$\ddot{x} = 0$

$\dot{x} = \int dt$

$\dot{x} = C_1$

$20 = C_1$

$\therefore \dot{x} = 20$

$x = \int 20\,dt$

$x = 20t + C_2$

when $x=0$ $t=0$

$\therefore C_2 = 0$

$\therefore x = 20t$

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$\ddot{y} = -g$

$\dot{y} = \int -g\,dt$

$\dot{y} = -gt + C_3$

when $t=0$ $\dot{y}=15$

$15 = C_3$

$\therefore \dot{y} = -gt + 15$

$y = \int(-gt+15)\,dt$

$y = -\frac{1}{2}gt^2 + 15t + C_4$

when $t=0$ $y=18$

$18 = C_4$

$\therefore y = -\frac{1}{2}gt^2 + 15t + 18$

ii greatest height is when

$\dot{y} = 0$

$\dot{y} = -gt + 15$

$0 = -9.8t + 15$

$9.8t = 15$

$t = \frac{15}{9.8}$

$= 1.5306....$

when $t = 1.53....$

$y = ?$

$y = -\frac{1}{2}(9.8)(1.53...)^2 + 15(1.53...) + 18$

$= 29.47959....$

$\therefore$ greatest height is 29.48 m.

iii when $y=0$ $t=?$

$0 = -\frac{1}{2}gt^2 + 15t + 18$

$t = \frac{-15 \pm \sqrt{225 - 4(-\frac{1}{2}g)(18)}}{2(-\frac{1}{2}g)}$

$t = 3.9834....$ $t > 0$

$\therefore x = 20t$

$\therefore x = 20(3.9834...)$

$x = 79.668....$

$\therefore$ distance is 79.67 m

Question 7 :

a) i $\sqrt{3}\cos 2t - \sin 2t = R\cos(2t+\alpha)$

$\sqrt{3}\cos 2t - \sin 2t = R\cos 2t\cos\alpha - R\sin 2t\sin\alpha$

$\sqrt{3} = R\cos\alpha$ ①

$1 = R\sin\alpha$ ②

$4 = R^2(\cos^2\alpha + \sin^2\alpha)$ ①² + ②²

$R = 2$ $R > 0$

$\tan\alpha = \frac{1}{\sqrt{3}}$ ② ÷ ①

$\alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\cos 2t - \sin 2t = 2\cos(2t + \frac{\pi}{6})$

OR

RHS $= 2\cos(2t + \frac{\pi}{6})$

$= 2[\cos 2t\cos\frac{\pi}{6} - \sin 2t\sin\frac{\pi}{6}]$

$= 2[\cos 2t(\frac{\sqrt{3}}{2}) - \sin 2t(\frac{1}{2})]$

$= \sqrt{3}\cos 2t - \sin 2t$

$=$ LHS

PLC Sydney Maths Department

Solutions for exams and assessment tasks

Ver 1

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ii $x = 1 + \sqrt{3}\cos 2t - \sin 2t$

$x = 1 + 2\cos\left(2t + \frac{\pi}{6}\right)$

$x - 1 = 2\cos\left(2t + \frac{\pi}{6}\right)$

$\dot{x} = -4\sin\left(2t + \frac{\pi}{6}\right)$

$\ddot{x} = -8\cos\left(2t + \frac{\pi}{6}\right)$

$\ddot{x} = -4\left[2\cos\left(2t + \frac{\pi}{6}\right)\right]$

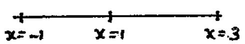
$\ddot{x} = -4\left[x - 1\right]$

$\therefore$ SHM.

iii Centre is at $x = 1$

amplitude is 2

period is π sec.



iv $x = 1 + 2\cos\left(2t + \frac{\pi}{6}\right)$

when $x = 0$

$-1 = 2\cos\left(2t + \frac{\pi}{6}\right)$

$-\frac{1}{2} = \cos\left(2t + \frac{\pi}{6}\right)$

$2t + \frac{\pi}{6} = \frac{2\pi}{3}, \ldots$

$2t = \frac{2\pi}{3} - \frac{\pi}{6}$

$2t = \frac{\pi}{2}$

$t = \frac{\pi}{4}$ sec

b. $V = \pi\int_0^{\pi}\left[\left(\sin x + \frac{1}{3}\sin 3x\right) + \frac{1}{5}\sin 5x\right]^2 dx$

$= \pi\int_0^{\pi}\left[\left(\sin x + \frac{1}{3}\sin 3x\right)^2 + \frac{2}{5}\sin 5x\left(\sin x + \frac{1}{3}\sin 3x\right) + \frac{1}{25}\sin^2 5x\right] dx$

$= \pi\int_0^{\pi}\left(\sin^2 x + \frac{2}{3}\sin x\sin 3x + \frac{1}{9}\sin^2 3x + \frac{2}{5}\sin x\sin 5x + \frac{2}{15}\sin 5x\sin 3x + \frac{1}{25}\sin^2 5x\right) dx$

Since $\int_0^{\pi}\sin mx\sin nx\,dx = 0$

$\therefore V = \pi\int_0^{\pi}\left(\sin^2 x + 0 + \frac{1}{9}\sin^2 3x + 0 + 0 + \frac{1}{25}\sin^2 5x\right) dx$

Since $\int_0^{\pi}\sin^2 mx\,dx = \frac{\pi}{2}$

$\therefore V = \pi\left[\frac{\pi}{2} + \frac{1}{9}\left(\frac{\pi}{2}\right) + \frac{1}{25}\left(\frac{\pi}{2}\right)\right]$

$= \pi^2\left[\frac{1}{2} + \frac{1}{18} + \frac{1}{50}\right]$

$= \frac{259\pi^2}{450}$ units ^{3}