Student's name: ____________ Student's number: __________ Teacher's name: _________

Presbyterian Ladies' College, Sydney
Mathematics Department

HSC Assessment Task 2, 2008
Instruction, Notification and Reporting Sheet

Course: Mathematics Extension 1
Topics: Integration using substitution
Estimation of roots
Exponential and Logarithmic Functions
Date: Week 13A, Friday May 9, period 2
Time allowed: 50 minutes
Weighting: 20%

The outcomes being assessed are printed overleaf.

Instructions:
- Approved calculators may be used.
- Write your name and number on this question booklet.
- Write your student number on every page you hand in.
- All questions may be attempted.
- Start each Section on a new sheet of paper. The Sections will be collected separately.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- The question booklet will be collected with your answers.
- Your answers will be collected in 4 separately stapled bundles. **BRING A STAPLER.**

Marks awarded:

Section	Mark
1	/15
2	/15
Total	/30

Teacher's comment:

Student's comment:

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Section 1 (15 marks) Start a new sheet of writing paper	Marks
1. Solve the equation $7^x = 10$, correct to 3 significant figures.	/2
2. For $x^3 + x^2 - 1 = 0$ (i) Show that a root exists between $x = 0.7$ and $x = 0.8$. (ii) Use **one application** of halving the interval to find a shorter interval that contains the root.	/3
3. Differentiate $\log_5(7-4x)$.	/2
4. Use the substitution $u = 4 - x^2$ to find $\int x\sqrt{4-x^2}\,dx$.	/3
5. The equation $e^x - 4x - 8 = 0$ has a root **close** to $x = 3$. Find an approximation for the root by using Newton's method **once** and starting with an approximation of $x = 3$. Give your answer correct to 4 decimal places.	/3
6. Find the value of $\ln\left(\frac{\sqrt[3]{a}}{b}\right)$, given that $\ln a = 0.86$ and $\ln b = 0.42$. Give your answer correct to two decimal places.	/2

End of Section 1

Section 2 (15 marks) Start a new sheet of writing paper	Marks
1. Determine the following integrals. (i) $\int \frac{x+1}{x^2+2x-5}\,dx$ (ii) $\int_1^{e^3} \frac{5}{2x}\,dx$	/4
2. Use the substitution $x = 1 - u^2$ to find $\int_0^1 x\sqrt{1-x}\,dx$. $(u > 0)$	/4
3. For the function $y = 2 + \log_e(x+1)$ (i) Draw a neat **sketch** of the function, clearly showing the y intercept and the vertical asymptote. (ii) Determine the **area** bound by the curve, the y-axis, and the lines y = 0 and y = 3. Give your answer correct to two decimal places.	/4
4. (i) Show that $\frac{d}{dx}\left(\sqrt{x}\ln x\right) = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$. (ii) Hence, or otherwise, determine $\int \frac{\ln x}{\sqrt{x}}\,dx$.	/3

End of Section 2

Academic Year	Y12	Calendar Year	2008
Course	Ext 1	Name of task/exam	Task2

Section 1

1. $7^x = 10$

$\ln 7^x = \ln 10$

$x \ln 7 = \ln 10$

$x = \dfrac{\ln 10}{\ln 7}$

$x = 1.18$

2. (i) $f(x) = x^3 + x^2 - 1$

$f(0.7) = -0.167$

$f(0.8) = 0.152$

$f(x)$ is continuous for all x.

$f(0.7)$ and $f(0.8)$ are opposite in sign

$\therefore$ A root exists between $x = 0.7$ and $x = 0.8$

Note: $f(x)$ is cts in the vicinity of the root also acceptable

(ii) $f(0.75) = -0.015625$

$f(0.8) > 0$ and

$f(0.75) < 0$

$\therefore$ A root exists between $x = 0.75$ and $x = 0.8$.

3. $\log_5 (7-4x) = \dfrac{\ln(7-4x)}{\ln 5}$

$\dfrac{d}{dx}\left(\dfrac{\ln(7-4x)}{\ln 5}\right)$

$= \dfrac{1}{\ln 5} \times \dfrac{1}{(7-4x)} \times -4$

$= \dfrac{-4}{\ln 5\,(7-4x)}$

PLC Sydney Maths Department Ver 1
Solutions for exams and assessment tasks

Academic Year	Y12	Calendar Year	2008
Course	Ext 1	Name of task/exam	Task 2

4. $u = 4 - x^2$

$\dfrac{du}{dx} = -2x$

$\dfrac{du}{-2} = x\,dx$

$\int x\sqrt{4-x^2}\,dx$

$= \int u^{\frac{1}{2}} \dfrac{du}{-2}$

$= -\frac{1}{2} \int u^{\frac{1}{2}} du$

$= -\frac{1}{2} \times \dfrac{2u^{\frac{3}{2}}}{3} + c$

$= \dfrac{-u^{\frac{3}{2}}}{3} + c$

$= \dfrac{-(4-x^2)^{\frac{3}{2}}}{3} + c$

5. $f(x) = e^x - 4\cancel{x}x - 8$

$f'(x) = e^x - 4$

$a_1 = a - \dfrac{f(a)}{f'(a)}$

$a_1 = 3 - \dfrac{e^3 - 20}{e^3 - 4}$

$\therefore a_1 \doteqdot 2.9947$

6. $\ln\left(\dfrac{\sqrt[3]{a}}{b}\right)$

$= \ln\left(\dfrac{a^{\frac{1}{3}}}{b}\right)$

$= \ln a^{\frac{1}{3}} - \ln b$

$= \frac{1}{3}\ln a - \ln b$

$= \frac{1}{3} \times 0.86 - 0.42$

$= -0.1\dot{3}$

Solutions for exams and assessment tasks

Academic Year	Y12	Calendar Year	2008
Course	Ext1	Name of task/exam	Task2

Section 2

1.(i) $\int \frac{x+1}{x^2+2x-5}\,dx$

$= \frac{1}{2}\int \frac{2(x+1)}{x^2+2x-5}\,dx$

$= \frac{1}{2}\ln(x^2+2x-5)+c$ 1

(ii) $\int_1^{e^3} \frac{5}{2x}\,dx$

$= \frac{5}{2}\int_1^{e^3} \frac{1}{x}\,dx$

$= \frac{5}{2}\left[\ln x\right]_1^{e^3}$

$= \frac{5}{2}\left(\ln e^3 - \ln 1\right)$

$= \frac{5}{2}(3-0)$

$= \frac{15}{2}$

2. $\boxed{x = 1-u^2}$

$\frac{dx}{du} = -2u$

$\boxed{dx = -2u\,du}$

$x = 1-u^2$

$\boxed{u^2 = 1-x}$

when $x=0,\ u=1$
$x=1,\ u=0$

$\int_0^1 x\sqrt{1-x}\,dx$

$= \int_1^0 (1-u^2)\sqrt{u^2}\times(-2u)\,du$

$= \int_1^0 (1-u^2)\times(-2u^2)\,du$

$= \int_1^0 (-2u^2+2u^4)\,du$

$= 2\int_1^0 (u^4-u^2)\,du$

$= 2\left[\frac{u^5}{5}-\frac{u^3}{3}\right]_1^0$

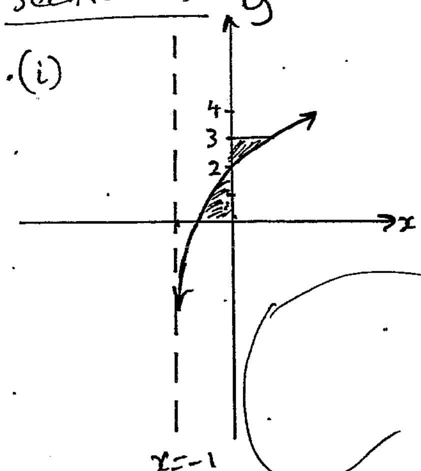
$= 2\left[0-\left(-\frac{2}{15}\right)\right] = \frac{4}{15}$

Solutions for exams and assessment tasks

Academic Year	Y12	Calendar Year	2008
Course	Ext 1	Name of task/exam	Task 2

Section 2

3.(i)



$y = 2+\ln(x+1)$

(ii) $y = 2+\ln(x+1)$

$y-2 = \ln(x+1)$

$e^{y-2} = x+1$

$\boxed{x = e^{y-2}-1}$

$A_1 = \left|\int_0^2 \left(e^{y-2}-1\right)dy\right|$

$= \left|\left[e^{y-2}-y\right]_0^2\right|$

$= \left|1-2-(e^{-2})\right|$

$= \left|-1-\frac{1}{e^2}\right|$

$= \left|-\left(1+\frac{1}{e^2}\right)\right|$

$= 1+\frac{1}{e^2}$

$A_2 = \int_2^3 \left(e^{y-2}-1\right)dy$

$= \left[e^{y-2}-y\right]_2^3$

$= e-3-(1-2)$

$= e-2$

$\therefore$ Total Area $=$

$= 1+\frac{1}{e^2}+e-2$

$= \frac{1}{e^2}+e-1$

$= \underline{1.85}$ (2dp)

Academic Year		Calendar Year	
Course		Name of task/exam	

4 (i) $\frac{d}{dx}\left(\sqrt{x}\ln x\right)$

$= \frac{d}{dx}\left(x^{\frac{1}{2}}\ln x\right)$

$= \frac{1}{2}x^{-\frac{1}{2}}\ln x + \frac{1}{x^1}\times x^{\frac{1}{2}}$

OR show that $x^{\frac{1}{2}} \div x^1 = x^{-\frac{1}{2}}$

$= \frac{1}{2x^{\frac{1}{2}}}\ln x + \frac{1}{x^{\frac{1}{2}}}$

$= \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$

(ii) $\int\left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx = \sqrt{x}\ln x + C$

$\frac{1}{2}\int \frac{\ln x}{\sqrt{x}}dx + \int \frac{1}{\sqrt{x}}dx = \sqrt{x}\ln x + C$

splitting integral

$\frac{1}{2}\int \frac{\ln x}{\sqrt{x}}dx = \sqrt{x}\ln x - \int \frac{1}{\sqrt{x}}dx + C$

$\frac{1}{2}\int \frac{\ln x}{\sqrt{x}}dx = \sqrt{x}\ln x - 2x^{\frac{1}{2}} + C$

$\int \frac{\ln x}{\sqrt{x}}dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

Multiplying by 2