PENDLE HILL HIGH SCHOOL

YEAR 12

HALF YEARLY EXAM

MATHEMATICS (2 Unit)

March 2015

TIME ALLOWED:- 2 hour + 5 mins reading

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen.
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the front of this paper
- In Questions 6 – 10, show relevant mathematical reasoning and/or calculations

Total Marks 65
Section I
5marks
Attempt Questions 1 – 5; circle your answer on the question paper

Section II
60 marks
Attempt Questions 6 - 10; begin each question on a new sheet of paper

Section I

Multiple Choice (Q1 – 5) **5 Marks**

Circle the correct answer

1. What is the value of $\pi^2/6$, correct to 3 significant figures?

(A) 1.64

(B) 1.65

(C) 1.644

(D) 1.645

2. Which equation represents the line perpendicular to $2x - 3y = 8$, passing through the point (2, 0)?

(A) $3x + 2y = 4$

(B) $3x + 2y = 6$

(C) $3x - 2y = -4$

(D) $3x - 2y = 6$

3. Which expression is a factorisation of $8x^3 + 27$?

(A) $(2x-3)(4x^2+12x-9)$

(B) $(2x+3)(4x^2-12x+9)$

(C) $(2x-3)(4x^2+6x-9)$

(D) $(2x+3)(4x^2-6x+9)$

4. How many solutions of the equation $(\sin x-1)(\tan x+2) = 0$ lie between 0 and 2π?

(A) 1

(B) 2

(C) 3

(D) 4

5. A parabola has focus (5, 0) and directrix $x = 1$.
What is the equation of the parabola?

(A) $y^2 = 16(x - 5)$

(B) $y^2 = 8(x - 3)$

(C) $y^2 = -16(x - 5)$

(D) $y^2 = -8(x - 3)$

Section II

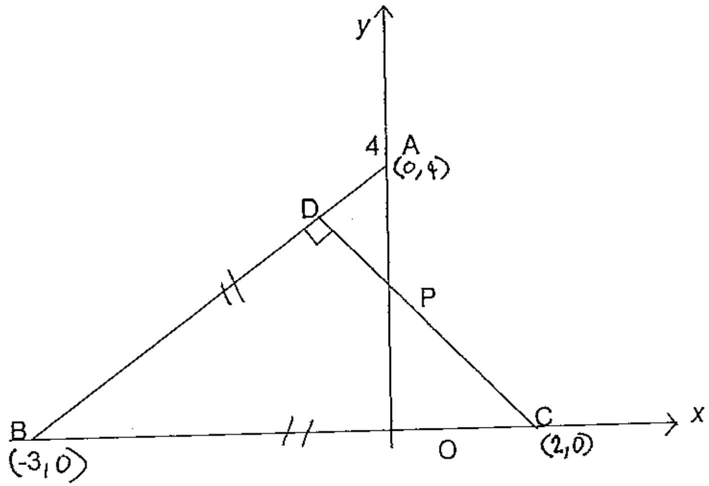
Question 6 *Begin a new sheet of paper.* **Marks**

a) Solve $|x - 5| = 6$ **2**

b) Solve $2 - 3p < 7$ **2**

c) Find the primitive of $\frac{1}{3e^x}$ **2**

d)



In the diagram AB = BC and CD is perpendicular to AB.
CD intersects the y-axis at P.

Copy the diagram onto your answer sheet.

i) Find the length of AB. **1**

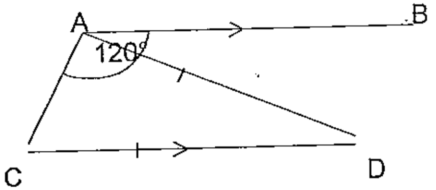
ii) Hence show the co-ordinates of C are (2,0) **1**

iii) Show the equation of CD is $3x + 4y = 6$ **3**

iv) Show the co-ordinates of P are (0,1½) **1**

Question 7 *Begin a new sheet of paper.*

a) NOT TO SCALE 3



In the diagram, AB ∥ CD, AD = CD and ∠BAC = 120°.
Copy the diagram onto your answer sheet.

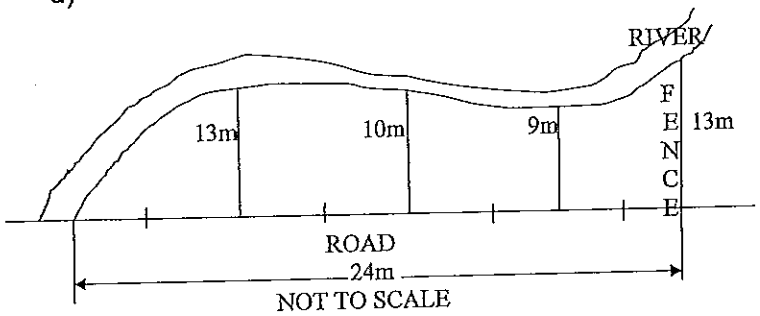
i) Explain why ∠ACD = 60°.

ii) Show that Δ ADC is equilateral, giving reasons.

b) Find $\lim_{x \to 4} \frac{x-4}{x^2-16}$ 1

c) The focal chord that cuts the parabola $x^2 = -6y$ at (6, -6) cuts the parabola again at X. 4
Find the co-ordinates of X.

d) 4



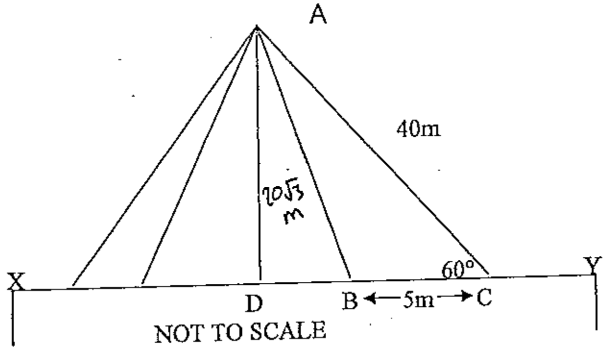
Wasteland bordering a river bank and a straight road was fenced off and used as a recreational park. Perpendicular distances from the road to the river bank are shown on the diagram.

Use Simpson's Rule, with 5 function values, to approximate the area of the recreational park. 4 Sub.

Question 8 *Begin a new sheet of paper.* **Marks**

a) Find the equation of the tangent to the curve $y = x^3 - 2x^2$ at the point (1,-1) 3

b) 4



A horizontal bridge was built between points X and Y. cables were used to support the bridge as shown in the diagram above. The distance between cables AB and AC was 5 metres. Cable AC was 40 metres long and ∠ACB = 60°.

i) Show that the height of A above the horizontal bridge is $20\sqrt{3}$ metres.

ii) Use the cosine rule to show the exact length of the cable AB is $5\sqrt{57}$ metres.

c) Simplify $\sec x \cot x$ 1

d) Find $\int \frac{2x}{x^2+5} dx$ 1

e) The region bounded by the curve $y = x^3$, the y-axis and the line $y = 8$ is rotated about the y-axis. 3

Find the volume of the solid formed.

Question 9 *Begin a new sheet of paper.* **Marks**

a) Differentiate $\frac{x}{e^{2x}}$ **2**

b) The area of a sector is $\frac{3\pi}{10}$ cm² and the arc length cut off by the sector is $\frac{\pi}{5}$ cm. Find the angle subtended at the centre and find the length of the radius. **2**

c) Consider the curve given by the equation **8**

$$y = 9x(x-2)^2$$

i) Find the co-ordinates of the stationary points and determine their nature

ii) Find the co-ordinates of any points of inflexion

i) Sketch the curve in the domain $-1 \le x \le 3$

iv) What is the maximum value of $9x(x-2)^2$ in the domain $-1 \le x \le 3$?

Q 10 overleaf

Question 10 *Begin a new sheet of paper.* **Marks**

a) Differentiate $\ln(x^2 - 9)$ **2**

b) Rationalise the denominator: **2**

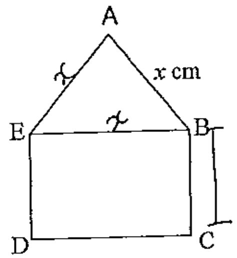
$$\frac{2\sqrt{3}}{\sqrt{5}+2\sqrt{6}}$$

c) For the sequence 1, 4, 7, 10 ...

i) Find the value of the 79th term **1**

ii) Calculate the sum of the first 25 terms **1**

d) **6**



NOT TO SCALE

ABCDE is a pentagon of fixed perimeter P cm. In the figure, triangle ABE is equilateral and BCDE is a rectangle. The length of AB is x cm.

i) Show that the length of BC is $\frac{P-3x}{2}$ cm.

ii) Show that the area of the pentagon is given by:

$$A = \frac{1}{4}[2Px - (6-\sqrt{3})x^2] \text{ cm}^2$$

iii) Find the value of $\frac{P}{x}$ for which the area of the pentagon is a maximum.

End of Examination

Year 12 Mathematics Half Yearly Exam March 2015

Section 1 Multiple Choice

1. A
2. B
3. D
4. ~~B~~ C
5. B

6. a) $|x-5| = 6$

$x - 5 = 6$ $\quad$ $-(x-5) = 6$

$\underline{\underline{x = 11}}$ $\quad$ $-x + 5 = 6$ $\quad$ 2

$\underline{\underline{-1 = x}}$

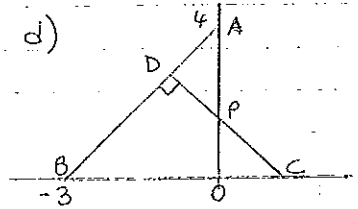
b) $2 - 3p < 7$

$-3p < 5$ $\quad$ 2

$\underline{\underline{p > -5/3}}$

c) $\frac{1}{3}\int e^{-x}\,dx = -\frac{1}{3}e^{-x} + c = -\frac{1}{3e^{x}} + c$ $\quad$ 2

d)



i) $OB^2 + OA^2 = AB^2$

$3^2 + 4^2 = AB^2$

$5 = AB$ $\quad$ 1

ii) Given $BC = AB = 5$ $\quad$ SHOW

$CO + OB = BC$

$CO + 3 = 5$

$\Rightarrow CO = 2$ $\quad \Rightarrow (2, 0)$ $\quad$ 1

6 d) (cont.)

iii) eq^n CD is $3x + 4y = 6$ $\quad$ SHOW

Gradient $AB = \frac{4}{3}$

CD is $\perp \Rightarrow$ gradient $-\frac{3}{4}$

Point C $(2, 0)$

Point/gradient $y - y_1 = m(x - x_1)$

$y - 0 = -\frac{3}{4}(x - 2)$ $\quad$ 3

$\Rightarrow 3x + 4y = 6$

iv) $x = 0$ $\quad$ SHOW

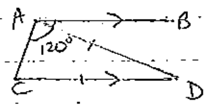
sub in $3x + 4y = 6$

$3 \times 0 + 4y = 6$

$y = 3/2$ $\Rightarrow P(0, 1\frac{1}{2})$ $\quad$ 1

7. $\quad$ 3

a) i)



EXPLAIN

cointerior angles $\quad$ 1

$\angle ACD = 60°$ because $\angle BAC + \angle ACD$ supp. on $\parallel^l$ lines

ii) $\triangle ADC$ is equilateral. $\quad$ SHOW

$\angle DAC = \angle ACD = 60°$ $\frac{1}{2}$ base angles of isosceles $\triangle$ $\frac{1}{2}$

$\angle ADC = 180 - 2\times 60 = 60°$ $\frac{1}{2}$ $\quad$ $\triangle$ sum. $\frac{1}{2}$ $\quad$ 2

All 3 angles equal $\Rightarrow$ equilateral $\triangle$ $\frac{1}{2}$

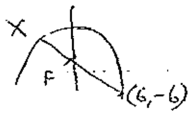
b) $\lim_{x \to 4} \frac{x-4}{x^2-16} = \frac{\cancel{(x-4)}\,1}{\cancel{(x-4)}(x+4)} = \frac{1}{8}$ $\quad$ 1

7 (cont.)

c) $F(0,a)$ $\quad x^2 = 4ay$ $\quad x^2 = -6y \Rightarrow 4a = -6$ $\quad a = -1.5$ OR $F(0,-1.5)$ (1)

$P(6,-6)$ and $F(0,^{-}1.5)$ $\quad m = (-1.5 - ^{-}6)/(0-6) \Rightarrow m = ^{-}3/4$ (1)

$y - ^{-}6 = -\frac{3}{4}(x-6) \Rightarrow 3x + 4y + 6 = 0.$



Solve $x^2 = -6y$ } ①

$3x + 4y + 6 = 0$ } ② simultaneously

from ① $\quad \frac{x^2}{-6} = y$ ③

4

sub in ② $\quad 3x + 4\frac{x^2}{-6} + 6 = 0.$

$\times -6 \qquad -18x + 4x^2 - 36 = 0.$

$\div 2 \qquad 2x^2 - 9x - 18 = 0$

$(2x+3)(x-6) = 0$

$x = -3/2$ (1) or $x = 6$ (given)

when $x = -3/2$ $\quad x^2 = -6y$

$(-\frac{3}{2})^2 = -6y$

$y = -3/8$ (1) $\qquad X = \underline{(-\frac{3}{2}, -\frac{3}{8})}$

d)

y_0	y_1	y_2	y_3	y_n
0	13	10	9	13 (1)

$\frac{h}{3}\{y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots)\}$

where $h = \frac{b-a}{n}$

Interval is 6m $\quad h = 6$ $\quad (\frac{24}{4})$

Area $\approx \frac{6}{3}\{0 + 13 + 4(13+9) + 2\times 10\}$

4

$= 2(13 + 88 + 20)$

$= 2 \times 121$

$= \underline{242 \text{ sq.m.}}$

8 a) $y = x^3 - 2x^2$

$y' = 3x^2 - 4x$ (1)

at $(1,-1)$ $\quad y' = 3\times 1 - 4\times 1$

$\Rightarrow m = -1$ (1)

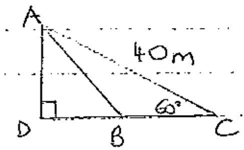
3

$(y - ^{-}1) = -1(x-1)$

$y + 1 = -x + 1$

$\underline{x + y = 0}$ (1)

b) i)



SHOW

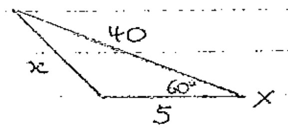
$\sin 60° = \frac{AD}{40}$

2

$\frac{40\times\sqrt{3}}{2} = AD$

$AD = 20\sqrt{3}$ metres

ii)



$x^2 = 40^2 + 5^2 - 2\times 40\times 5\times \cos 60°$

2

$= 1625 - 200$ $\qquad$ SHOW

$x = \sqrt{25\times 57}$

$x = 5\sqrt{57}$

c) $\sec x \cdot \cot x$

1

$= \frac{1}{\cos x} \times \frac{\cancel{\cos x}}{\sin x}$

$= \frac{1}{\sin x}$ $\quad (\frac{1}{2})$

$= \underline{\operatorname{cosec} x}$

8. (cont.)

d) $\int \frac{2x}{x^2+5}\,dx$ $\quad f'(x) = 2x$, $\quad f(x) = x^2+5$

$= \underline{\underline{\ln(x^2+5) + C}}$

no brackets ($\frac{1}{2}$) ... ignore if missing

e) $\pi \int_0^8 x^2\,dy$ $\quad y = x^3 \Rightarrow x = \sqrt[3]{y}$ $\quad \sqrt[3]{y^2} = x^2 \Rightarrow x^2 = y^{2/3}$

$= \pi \int_0^8 y^{2/3}\,dy$

$= \pi \left[\frac{3}{5} y^{5/3}\right]_0^8$

$= \pi \left[\frac{3}{5} 8^{5/3} - 0\right]$

$= \underline{\underline{\frac{96\pi}{5}}}$ cu. units (19.2π) 3

9 a) $\frac{d}{dx}\left(\frac{x}{e^{2x}}\right)$ $\quad u = x$, $u' = 1$ $\quad v = e^{2x}$, $v' = 2e^{2x}$ 2

$y' = \frac{vu' - uv'}{v^2} = \frac{e^{2x}.1 - x.2e^{2x}}{(e^{2x})^2} = \frac{\cancel{e^{2x}}(1-2x)}{(e^{2x})^{\cancel{2}}} = \frac{1-2x}{e^{2x}}$

b) $A = \frac{1}{2}r^2\theta = \frac{3\pi}{10} \Rightarrow \theta = \frac{3\pi}{5\cancel{10}} \times \frac{2}{r^2}$ $\quad \theta = \frac{3\pi}{5r^2}$

$l = r\theta = \frac{\pi}{5} \Rightarrow \theta = \frac{\pi}{5} \div r$ $\quad \theta = \frac{\pi}{5r}$ $\quad \Rightarrow \frac{3\pi}{5r^2} = \frac{\pi}{5r}$

9 b) (cont.) $\frac{3\cancel{\pi}}{\cancel{5}r^{\cancel{2}}} = \frac{\cancel{\pi}}{\cancel{5}r}$

$\underline{\underline{3 = r}}$

sub. in $l = r.\theta$

$\frac{\pi}{5} = 3.\theta$

$\underline{\underline{\frac{\pi}{15} = \theta}}$

c) i) $y = 9x(x-2)^2$

$y' = 9x.2(x-2) + (x-2)^2.9$

$= 18x^2 - 36x + 9x^2 - 36x + 36$

$= 27x^2 - 72x + 36$

SP's $0 = 9(3x^2 - 8x + 4)$

$0 = (3x-2)(x-2)$

$x = 2/3$ or $x = 2$

at $x = 2/3$ $\quad y = 9 \times \frac{2}{3}\left(\frac{2}{3} - 2\right)^2$ $\quad \underline{(2/3, 10\frac{2}{3})}$ $\underline{max}$

$x = 2$ $\quad y = 9 \times 2(2-2)^2$ $\quad \underline{(2, 0)}$ $\underline{min}$

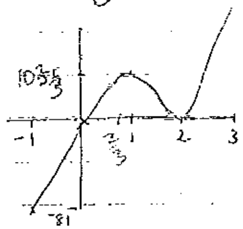
ii) $y'' = 54x - 72$

Possible P.o.I $0 = 54x - 72 \Rightarrow x = 4/3$

x	1	4/3	5/3	$\Rightarrow$ change in concavity
y''	−ve	0	+ve	

at $x = 4/3$ $\quad y = 9 \times 4/3(4/3 - 2)^2 = 5\frac{1}{3}$ $\quad$ P.o.I $\underline{(4/3, 5\frac{1}{3})}$

iii)



8

iv) $x = 3$ $\quad \underline{y = 27}$

10 a) $\frac{d}{dx}\left(\ln(x^2-9)\right) = \frac{2x}{x^2-9}$ 2

b) $\frac{2\sqrt{3}}{(\sqrt{5}+2\sqrt{6})} \times \frac{(\sqrt{5}-2\sqrt{6})}{(\sqrt{5}-2\sqrt{6})} = \frac{2\sqrt{15}-4\sqrt{18}}{5-4\times 6} = \frac{2\sqrt{15}-12\sqrt{2}}{-19} = \frac{12\sqrt{2}-2\sqrt{15}}{19}$

$\frac{2\sqrt{15}-12\sqrt{2}}{-19} = \frac{12\sqrt{2}-2\sqrt{15}}{19} = \frac{2(6\sqrt{2}-\sqrt{15})}{19}$ 2

c) i) $T_n = a + (n-1)d$

$T_{79} = 1 + 78\times 3$

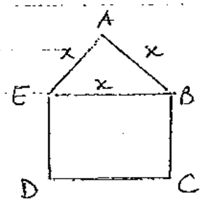
$= \underline{235}$

ii) $S_n = \frac{n}{2}(a+l)$ or $\frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{25}{2}\left[2 + 24\times 3\right]$

$= \underline{\underline{925}}$

d) i)



SHOW

$AB = EB = EA = x$ sides of an equilateral $\triangle$

$DC = EB = x$ opposite sides of a rectangle

$P = x + x + BC + x + DE$

sub $BC = DE$

$\Rightarrow P = 3x + 2BC$

$P - 3x = 2BC$

$\frac{P-3x}{2} = BC$

10 d) (cont.)

ii)

Area $\triangle ABE = \frac{1}{2}\times x \times x \times \sin 60°$ using ½ab sin C

SHOW.

$= \frac{1}{2}x^2 \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}x^2$ ①

Area of ▭ BCDE $= \left(\frac{P-3x}{2}\right)\times x$ ②

Total area ① + ② $\frac{\sqrt{3}}{4}x^2 + \frac{(P-3x)}{2}\times x = \frac{\sqrt{3}x^2 + 2Px - 6x^2}{4}$

$= \frac{1}{4}\left[2Px - (6-\sqrt{3})x^2\right]$

iii) $\frac{dA}{dx} = \frac{1}{4}\left[2P - 2(6-\sqrt{3})x\right]$

max/min $0 = \frac{1}{4}\left[2P - 2(6-\sqrt{3})x\right]$

$2P = 2(6-\sqrt{3})x$

$\frac{P}{x} = \frac{\cancel{2}(6-\sqrt{3})}{\cancel{2}}$

check $\frac{P}{x} = 6-\sqrt{3}$ is a max.

$\frac{d^2A}{dx^2} = \frac{1}{4}\left[-2(6-\sqrt{3})\right] < 0 \therefore$ max.