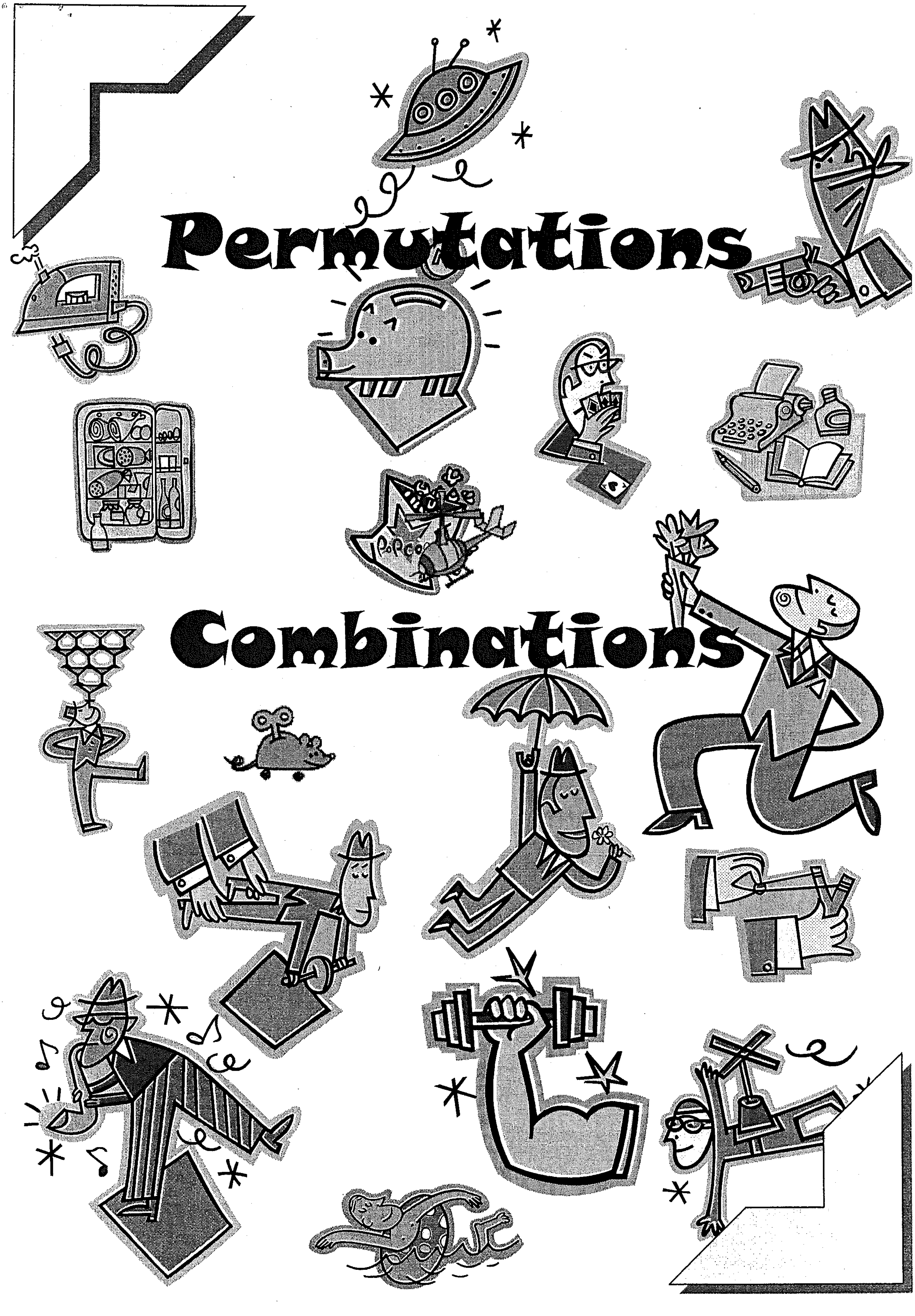
Permutations

Combinations



EXERCISES 28(a)

1. How many different arrangements can be made by taking 3 of the letters of the word *Sunday*?

2. In how many ways can 5 boys be arranged in a row?

3. In how many ways can a first, second and third prize be awarded in a class of 8 students?

4. How many 5-digit numbers can be formed from the digits 2, 3, 5, 6, 8, 9 if no digit can be used more than once in a number? How many even numbers can be formed?

5. Find the value of **(i)** ${}^6P_3$, **(ii)** ${}^{12}P_4$, **(iii)** ${}^8P_5$.

6. Find the value of **(i)** $6!$, **(ii)** $\frac{8!}{5!}$.

7. In how many ways can 4 consonants and 3 vowels be arranged in a row **(a)** so that the 3 vowels are always together, **(b)** so that the first and the last places are occupied by consonants?

8. In how many ways can four girls and three boys be arranged in a row so that **(a)** the boys are always together, **(b)** the men and boys occupy alternate places?

9. In how many ways can 6 people be seated in a motor car if only 2 can occupy the driver's position?

10. In how many ways can 6 people be arranged in a circle if 2 particular people are always **(a)** together, **(b)** separated?

11. Father, mother and 6 children stand in a ring. In how many ways can they be arranged if father and mother are not to stand together?

12. In how many ways can 5 boys and 3 women be arranged **(a)** in a row, **(b)** in a circle if in both cases the women are always to stand together?

13. Four men and four women are to be seated alternately **(a)** in a row **(b)** at a round table. In how many ways can this be done?

14. How many even numbers of 4 digits can be formed with the figures 3, 4, 7, 8 **(a)** if no figure is repeated, **(b)** if repetitions are allowed?

15. How many numbers greater than 4000 can be formed from the figures 3, 5, 7, 8, 9? (Repetitions not allowed.)

16. In how many ways can the letters of the word *permute* be arranged if **(i)** the first and last places are occupied by consonants, **(ii)** the vowels and consonants occupy alternate places?

17. If ${}^6P_r = 120$, find the value of r.

18. The number of arrangements of $2n + 2$ different objects taken n at a time is to the number of arrangements of $2n$ different objects taken n at a time as 14:5. Find the value of n.

19. How many numbers of 7 digits can be formed from the digits 1, 2, 3, 4, 5, 6, 7 if **(i)** the number begins with 1 and ends with 2, **(ii)** there are not more than 2 digits between 1 and 2?

20. If ${}^{2n}P_n = 8.\,{}^{2n-1}P_{n-1}$, find the value of n.

21. In how many ways can 5 different Mathematics books, 4 different Physics books and 2 different Chemistry books be arranged on a shelf if the books in each subject are to be together?

22. In how many ways can 3 men, 3 women and 3 boys be arranged in a row if the three boys are to remain together?

23. Find the number of arrangements of the letters in the word *pencils* if **(i)** *e* is next to *i*, **(ii)** *e* precedes *i*, **(iii)** there are three letters between *e* and *i*.

24. In how many ways can the letters of the word *principle* be arranged? In what proportion of these arrangements do the letters '*p*' come together?

25. In how many ways can the letters in *precision* be arranged? In how many of these arrangements do the vowels occupy even places?

26. How many arrangements can be made by the letters of the word *definition* **(i)** if the letters *i* do not occupy the first or last place, **(ii)** if the letters *i* are together?

27. How many arrangements of the letters in *tomato* are there, if the letters *o* are to be separated?

28. A car can hold 3 people in the front seat and 4 in the back seat. In how many ways can 7 people be seated in the car if 2 particular people must sit in the back seat and 1 particular person is the driver?

29. In how many ways can 4 people be accommodated if there are 4 rooms available?

30. In how many ways can 8 oarsmen be seated in an eight-oared boat if 3 can row only on the stroke side and 3 can row only on the bow side?

31. Prove **(i)** from the definition of ${}^nP_r$,
    (ii) the formula for ${}^nP_r$ that ${}^{n+1}P_r = {}^nP_r + r.{}^nP_{r-1}$.

32. Show that ${}^nP_r = {}^{n-2}P_r + 2r.{}^{n-2}P_{r-1} + r(r-1)\,{}^{n-2}P_{r-2}$.

33. In how many ways can 5 men and 5 women by arranged in a circle so that the men are separated? In how many ways can this be done if two particular women must not be next to a particular man?

34. How many arrangements of the letters of the word *PARRAMATTA* are possible?

PERMUTATIONS AND COMBINATIONS

SET 1A

1,, There are 6 arrangements : 6!
But we use 3 letters at a time : 3!
∴ the number of arrangements is $\frac{6!}{3!} = \frac{7}{6} = 120$

2,, $5! = 120$

3,, | 8 | 7 | 6 | ∴ $8 \times 7 \times 6 = 336$ ways

4,, | 6 | 5 | 4 | 3 | 2 | ∴ $6 \times 5 \times 4 \times 3 \times 2 = 720$

| 2 | 3 | 4 | 5 | 3 | ∴ $2 \times 3 \times 4 \times 5 \times 3 = 360$

5,, (i) ${}^6P_3 = 120$ (ii) ${}^{12}P_4 = 11880$ (iii) ${}^8P_5 = 6720$

6,, (i) $6! = 720$ (ii) $\frac{8!}{5!} = 336$

7,, (a)

V	V	V				
	V	V	V			
		V	V	V		
			V	V	V	
				V	V	V

$5 \cdot 3!4! = 720$

(b) | 4 | | | | | | 3 | $4 \times 3 \times 5! = 144$

8,, (a)

B	B	B	G	G	G	G
	B	B	B			
		B	B	B		
			B	B	B	
				B	B	B

$5 \cdot 3!4! = 720$

(b) | G | B | G | B | G | B | G | $4!3! = 144$

9,, Driver = 2! passengers = 5! ∴ $2!5! = 240$

10,, (a) $\frac{5!2!}{5} = 48$ (b) separated = total number of arrangements minus when 2 are together

$\frac{6!}{6} - 48 = 72$

11,, When mother and father are together : $\frac{7!2!}{7} = 1440$
Total number of arrangements : $\frac{8!}{8} = 5040$
∴ when mother and father are separated = $5040 - 1440 = 3600$

12,, (a) $6!6 = 4320$ (b) $\frac{6!6}{6} = 720$

13,, (a) $4!4!2! = 1152$ (b) $\frac{4!4!2}{8} = 144$

14,, (a) | 1 | 2 | 3 | 2 | $1 \times 2 \times 3 \times 2 = 12$ (b) | 4 | 4 | 4 | 2 | $4^3 \times 2 = 128$

15,, | 4 | 4 | 3 | 2 | $4^2 \times 3 \times 2 = 96$ | 5 | 4 | 3 | 2 | 1 | $5! = 120$ ∴ $120 + 96 = 21$

16. In "permute" – 4 consonants, 3 vowels

(i). C | 4 | | | | | 3 | C $5!\,6 = 720$

(ii) | C | V | C | V | C | V | C | $4!\,3 = 72$

17. $^6P_r = \frac{6!}{(6-r)!} = 120$

$\therefore (6-r)! = 6$ but $6 = 3!$

$\therefore (6-r)! = 3!$ $\therefore r = 3$

18. $^{2n+2}P_n : {}^{2n}P_n = 14:5$

$\frac{(2n+2)!}{(n+2)!} \times \frac{n!}{2n!} = \frac{14}{5}$

$\frac{(2n+2)(2n+1)}{(n+2)(n+1)} = \frac{14}{5}$

$20n^2 + 30n + 10 = 14n^2 + 42n + 28$

$6n^2 - 12n - 18 = 0$

$n^2 - 2n - 3 = 0$

$n = 3$ or $n = -1$

$\therefore n = 3$ as it must be a positive integer

19. i) | 1 | 5 | 4 | 3 | 2 | 1 | 1 | $5! = 120$

ii)

1			2			
	1			2		
		1			2	
			1			2

$4 \times 2! = 8$

1		2				
	1		2			
		1		2		
			1		2	
				1		2

$5 \times 2! = 10$

1	2					
	1	2				
		1	2			
			1	2		
				1	2	
					1	2

$6 \times 2! = 12$

$10 + 8 + 12 = 30$ ways

$30 \times 5! = 3600$

20. $^{2n}P_n = 8 \cdot {}^{2n-1}P_{n-1}$

$\frac{2n!}{n!} = \frac{8(2n-1)!}{n!}$

$2n! = 8(2n-1)!$

$2n = 8$

$\therefore n = 4$

21. $5!\,4!\,3!\,2! = 34560$

22. 3 men 3 women (3 boys) – 1 u

$7!\,3! = 30240$

23. pencils – 7 letters

(i) $6!\,2! = 1440$

(ii) $\frac{7!}{2!} = 2520$

(iii)

e				i		
	e				i	
		e				i

$3 \times 2! \times 5! = 720$

24. $\frac{9!}{2!\,2!} = 90720$ arrangements

When p's are together $\frac{8!}{2!} = 20160$

$\frac{20160}{90720} = \frac{2}{9}$

25. $\frac{9!}{2!} = 181440$

| | V | | V | | V | | V | |

$\frac{5!\,4!}{2!} = \frac{2880}{2} = 1440$

26. | 7 | | | | | | | | | 6 |

(i) $7 \times 6 = 42$ arrangements

$\frac{42 \times 8!}{2!3!} = 141120$

(ii) $\frac{8!}{2!} = 20160$

27. total arrangements $\frac{6!}{2!2!} = 180$

when the 'os' are together: $\frac{5!}{2!} = 60$

when separated: $180 - 60 = 120$

28. $4 \times 3 \times 4! = 288$

29. As all people can be in one room there are no restrictions. $4^4 = 256$

30. ${}^4P_3 \cdot {}^4P_3 \cdot 2! = 1152$

On the stroke side there are 4 positions to fill with 3 oarsmen. On the bow side there are also 4 positions to fill with 3 oarsmen. The two remaining positions can be filled by either of the 2 remaining oarsmen.

31. ${}^{n+1}P_r = {}^nP_r + r \cdot {}^nP_{r-1}$

$$RHS = \frac{n!}{(n-r)!} + \frac{r\,n!}{(n-r+1)!}$$

$$= \frac{(n-r+1)n! + rn!}{(n-r+1)!}$$

$$= \frac{n!(n-r+1+r)}{(n-r+1)!}$$

$$= \frac{n!(n+1)}{(n-r+1)!} = \frac{(n+1)!}{(n-r+1)!} = {}^{n+1}P_r = LHS$$

32. ${}^nP_r = {}^{n-2}P_r + 2r \cdot {}^{n-2}P_{r-1} + r(r-1)\,{}^{n-2}P_{r-2}$

$$RHS = \frac{(n-2)!}{(n-2-r)!} + \frac{2r(n-2)!}{(n-1-r)!} + \frac{r(r-1)(n-2)!}{(n-r)!}$$

$$= \frac{(n-r)(n-r-1)(n-2)! + 2r(n-r)(n-2)! + r(r-1)(n-2)!}{(n-r)!}$$

$$= \frac{(n-2)!(n-r)(n-r-1+2r) + r(r-1)(n-2)!}{(n-r)!}$$

$$= \frac{(n-2)!\left[(n-r)(n+r-1) + r(r-1)\right]}{(n-r)!}$$

$$= \frac{(n-2)!\,n(n-1)}{(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r = LHS$$

33. 5 men, 5 women

a) men are separated there is only one arrangement. See figure A below.

$5!4! = 2880$ ways.

b) When 2 particular women must not be next to a particular man, the region remaining is like a row because of the restriction on the circle. See figure B below.

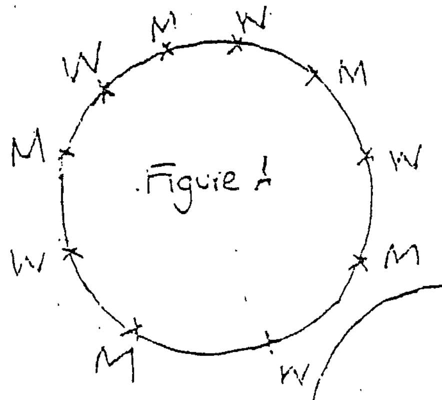
$3 \times 2 = 6$ arrangements

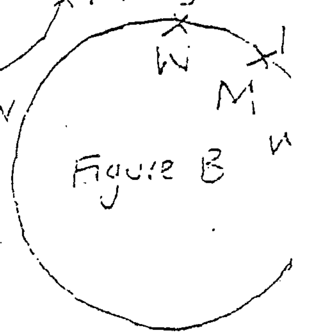
$6 \times 3! \times 4! = 864$

34. PARRAMATTA

$$\frac{10!}{4!2!2!} = 37800$$

There are ten letters in the word parramatta, therefore there are 10! arrangements. But since there are four 'A's', two 'R's' and two 'T's' we should divide by their numbers of arrangem





EXERCISES 28(b)

1. In how many ways can 3 books be selected from 8 different books?
2. In how many ways can a set of 2 boys and 3 girls be selected from 5 boys and 4 girls?
3. How many different selections can be made by taking 3 of the digits 4, 5, 6, 7, 8, 9?
4. In how many ways can a committee of 4 men and 5 boys be formed from 8 men and 7 boys?
5. From 8 soldiers, 7 sailors and 5 airmen how many sets can be formed each containing 5 soldiers, 4 sailors and 3 airmen?
6. In how many ways can 8 boys be divided into two sets containing 5 and 3 respectively?
7. A committee of 6 is to be selected from 10 people of whom A and B are two. How many committees can be formed **(i)** containing both A and B, **(ii)** excluding A if B is included?
8. In how many ways can 3 cards be selected from a pack of 52 playing cards if **(i)** at least one of them is an ace, **(ii)** not more than one is an ace?
9. In how many ways can a committee of 3 men and 4 women be chosen from 6 men and 7 women? What proportion of these committees contain a particular man and 2 particular women?
10. A committee of 7 politicians is chosen from 10 liberal members, 8 labor members and 5 independents. In how many ways can this be done so as to include exactly 1 independent and at least 3 liberal members and at least 1 labor member?
11. An eleven is to be chosen from 15 cricketers of whom 5 are bowlers only, 2 others are wicketkeepers only and the rest are batsmen only. How many possible elevens can be chosen which contain **(i)** 4 bowlers, 1 wicketkeeper and 6 batsmen, **(ii)** at least 4 bowlers and at least 1 wicketkeeper?
12. From 7 teachers and 5 pupils a committee of 7 is to be formed. How many committees can be selected if both teachers and pupils are represented and the teachers are in a majority?
13. From 4 oranges, 3 bananas and 2 apples, how many selections of 5 pieces of fruit can be made, taking at least one of each kind?
14. In Lotto, a frame contains the numbers 1 to 40 and we can select any 6 of these numbers. How many such combinations are possible?
15. In TAB betting, the 'trifecta' pays on the first three horses in correct order, the 'quinella' pays on the first two horses in either order. In a 10 horse race what is the possible number of **(a)** trifecta combinations **(b)** quinella combinations?
16. In how many ways can a jury of 12 be chosen from 10 men and 7 women so that there are at least 6 men and not more than 4 women on each jury?
17. In how many ways can a set of 3 or more be selected from 9 people?
18. In how many ways can n things be shared between 2 people.
19. In how many ways can a committee of 3 men and 4 boys be chosen from 7 men and 6 boys so as not to include the youngest boy if the eldest man is serving?
20. How many **(i)** selections, **(ii)** arrangements consisting of 3 consonants and 2 vowels can be made from 8 consonants and 4 vowels?
21. In how many ways can 4 Physics books and 3 Mathematics books be arranged on a shelf if a selection is made from 6 different Physics books and 5 different Mathematics books? In how many of these arrangements are the Physics books together?

22. In how many ways can 3 boys and 2 girls be arranged in a row if a selection is made from 5 boys and 4 girls? In how many of these arrangements does a boy occupy the middle position?

23. How many words (arrangement of letters), containing 3 consonants and 2 vowels, can be formed from the letters of the word *promise*?

24. From the definition of ${}^nC_r$ prove each of the following:
(a) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
(b) ${}^nC_k + 2\,{}^nC_{k-1} + {}^nC_{k-2} = {}^{n+2}C_k$
(c) ${}^{m+n}C_3 = {}^mC_3 + {}^mC_2\,{}^nC_1 + {}^mC_1\,{}^nC_2 + {}^nC_3$
(d) ${}^{n+3}C_r = {}^nC_r + 3\,{}^nC_{r-1} + 3\,{}^nC_{r-2} + {}^nC_{r-3}$
(e) ${}^nC_r = \dfrac{n-r+1}{r}\cdot{}^nC_{r-1}$

25. If ${}^nC_6 = {}^nC_4$, find the value of n.

26. The ratio of the number of combinations of $(2n + 2)$ different objects taken n at a time to the number of combinations of $(2n - 2)$ different objects taken n at a time is 99:7. Find the value of n.

27. In how many ways can 9 books be distributed amongst a man, a woman and a child, if the man receives 4, the woman 3 and the child 2?

28. In how many ways can 8 boys be divided into two unequal sets?

29. In how many ways can 8 girls be divided into 4 sets of 2?

SET 1B

1. ${}^8C_3 = 56$ 2. ${}^5C_2\,{}^4C_3 = 40$ 3. ${}^6C_3 = 20$ 4. ${}^8C_4\,{}^7C_5 = 1470$ 5. ${}^8C_5\,{}^7C_4\,{}^5C_3 = 1$

6. ${}^8C_5\,{}^3C_3 = 56$ 7. (i) ${}^2C_2\,{}^8C_4 = 70$ (ii) ${}^2C_1\,{}^5C_4 = 140$ 8. (i) ${}^4C_1\,{}^{48}C_2 + {}^4C_2\,{}^{48}C_1 + {}^4C_3\,{}^{48}C_0 = 4$

8. (ii) ${}^4C_1\,{}^{48}C_2 + {}^4C_0\,{}^{48}C_3 = 21808$ 9. ${}^8C_3\,{}^7C_4 = 700$; ${}^5C_2\,{}^5C_2 = 100$: $\frac{100}{700} = \frac{1}{7}$

10. ${}^5C_1\,{}^{10}C_3\,{}^8C_3 + {}^5C_1\,{}^{10}C_4\,{}^8C_2 + {}^5C_1\,{}^{10}C_5\,{}^8C_1 = 73080$ 11. (i) ${}^5C_4\,{}^2C_1\,{}^8C_6 = 280$

11. (ii) ${}^5C_4\,{}^2C_1\,{}^8C_6 + {}^5C_5\,{}^2C_2\,{}^8C_4 + {}^5C_5\,{}^2C_1\,{}^8C_5 + {}^5C_4\,{}^2C_2\,{}^8C_5 = 742$ 12. ${}^7C_4\,{}^5C_3 + {}^7C_5\,{}^5C_2 + {}^7C_6\,{}^5C_1 = 5$

13. ${}^4C_1\,{}^3C_2\,{}^2C_2 + {}^4C_1\,{}^3C_3\,{}^2C_1 + {}^4C_3\,{}^3C_1\,{}^2C_1 + {}^4C_2\,{}^3C_2\,{}^2C_1 + {}^4C_2\,{}^3C_1\,{}^2C_2 = 98$ 14. ${}^{40}C_6 = 38383$

15. a) ${}^{10}C_1\,{}^9C_1\,{}^8C_1 = 720$ b) ${}^{10}C_2 = 45$ 16. ${}^{10}C_{10}\,{}^7C_2 + {}^{10}C_9\,{}^7C_3 + {}^{10}C_8\,{}^7C_4 = 1946$

17. ${}^9C_3 + {}^9C_4 + {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9 = 466$ 18. $2^n - 2$ 19. ${}^1C_1\,{}^6C_2\,{}^5C_4 + {}^6C_3\,{}^6C_4 = 37$

20. (i) ${}^8C_3\,{}^4C_2 = 336$ (ii) $\frac{8!4!}{2!3!2!} = 40320$ 21. (i) ${}^6C_4\,{}^5C_3 = 150$ ∴ $150 \times 7! = 756000$

21. (ii) $4!\,4! \times 150 = 86400$ 22. (i) ${}^5C_3\,{}^4C_2\,5! = 7200$ (ii) ${}^5C_3\,{}^4C_2 \times 4! \times 3 = 4320$

23. ${}^4C_3\,{}^3C_2 = 12$; $5! \times 12 = 1440$

24. (a) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

$$LHS = \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r+1)!\,(n-r-1)!} = \frac{n!(r+1) + n!(n-r)}{(r+1)!\,(n-r)!} = \frac{n!(r+1+n-r)}{(r+1)!\,(n-r)!}$$

$$= \frac{n!(n+1)}{(r+1)!\,(n-r)!} = \frac{(n+1)!}{(r+1)!\,(n-r)!} = {}^{n+1}C_{r+1} = RHS$$

(b) ${}^nC_k + 2\,{}^nC_{k-1} + {}^nC_{k-2} = {}^{n+2}C_k$

$$LHS = \frac{n!}{k!(n-k)!} + \frac{2n!}{(k-1)!\,(n-k+1)!} + \frac{n!}{(k-2)!\,(n-k+2)!}$$

$$= \frac{n!(n-k+2)(n-k+1) + 2n!k(n-k+2) + n!k(k-1)}{k!\,(n-k+2)!}$$ Factorising by n, expanding and simplifying, we get

$$= \frac{n!(n^2+3n+2)}{k!(n+2-k)} = \frac{n!(n+1)(n+2)}{k!\,(n+2-k)!} = \frac{(n+2)!}{k!(n+2-k)!} = {}^{n+2}C_k = RHS$$

(c) ${}^{m+n}C_3 = {}^mC_3 + {}^mC_2\,{}^nC_1 + {}^mC_1\,{}^nC_2 + {}^nC_3$

$$RHS = \frac{m!}{3!(m-3)!} + \frac{m!}{2!(m-2)!} \cdot \frac{n!}{(n-1)!} + \frac{m!}{(m-1)!} \cdot \frac{n!}{2!(n-2)!} + \frac{n!}{3!(n-3)!}$$

$$= \frac{m(m-1)(m-2)}{6} + \frac{mn(m-1)}{2} + \frac{mn(n-1)}{2} + \frac{n(n-1)(n-2)}{6}$$

$$= \frac{m(m-1)}{2}\left[\frac{(m-2)}{3} + n\right] + \frac{n(n-1)}{2}\left[m + \frac{(n-2)}{3}\right]$$

$$= \frac{m(m-1)}{2} \cdot \frac{(m+n-2)}{3} + \frac{m(m-1)}{2} \cdot \frac{(2n)}{3} + \frac{n(n-1)}{2} \cdot \frac{(m+n-2)}{3} + \frac{n(n-1)}{2} \cdot \frac{2m}{3}$$

$$= \frac{m+n-2}{3}\left[\frac{m(m-1)}{2} + \frac{n(n-1)}{2}\right] + \frac{mn}{3}(m+n-2)$$

$$= \frac{m+n-2}{6}(m^2 - m + n^2 - n + 2mn) = \left(\frac{m+n-2}{6}\right)\left[(m+n)^2 - (m+n)\right]$$

$$= \left(\frac{m+n-2}{6}\right)(m+n)(m+n-1)$$ multiply by $(m+n-3)!$ Numerator & Denominator

$$= \frac{(m+n)!}{6(m+n-3)!} = \frac{(m+n)!}{3!\,(m+n-3)!} = {}^{m+n}C_3 = LHS$$

24(d) $^{n+3}C_r = {}^nC_r + 3\,{}^nC_{r-1} + 3\,{}^nC_{r-2} + {}^nC_{r-3}$

$$RHS = \frac{n!}{r!(n-r)!} + \frac{3n!}{(r-1)!(n-r+1)!} + \frac{3n!}{(r-2)!(n-r+2)!} + \frac{n!}{(r-3)!(n-r+3)!}$$

$$= \frac{(n+3)!}{r!(n-r+3)!}\left[\frac{(n-r+1)(n-r+2)(n-r+3)}{(n+1)(n+2)(n+3)} + \frac{3(n-r+2)(n-r+3)r}{(n+1)(n+2)(n+3)} + \frac{3r(r-1)(n-r+3)}{(n+1)(n+2)(n+3)} + \frac{3r(r-1)(r-2)}{(n+1)(n+2)(n+3)}\right]$$

After expanding and simplifying the numerators only, as the denominators are the same, we get

$$= \frac{(n+3)!}{r!(n-r+3)!} \times \frac{n^3+6n^2+2n+15}{(n+1)(n+2)(n+3)}$$

But $n^3+6n^2+2n+15 = (n+1)(n+2)(n+3) \neq 0$

$$= \frac{(n+3)!}{r!(n-r+3)!} \times 1 = \frac{(n+3)!}{r!(n-r+3)!} = {}^{n+3}C_r = LHS$$

e) $^nC_r = \frac{n-r+1}{r} \cdot {}^nC_{r-1}$

$$RHS = \frac{n-r+1}{r} \cdot \frac{n!}{(r-1)!(n-r+1)!} = \frac{n!(n-r+1)}{r(r-1)!(n-r+1)!} = \frac{n!}{r(r-1)!(n-r)!}$$

$$= \frac{n!}{r!(n-r)!} = {}^nC_r = LHS.$$

25) $^nC_6 = {}^nC_4 \quad \therefore \frac{n!}{6!(n-6)!} = \frac{n!}{4!(n-4)!} \Rightarrow 6!(n-6)! = 4!(n-4)!$

$$\therefore \frac{(n-4)!}{(n-6)!} = \frac{6!}{4!} \Rightarrow \frac{(n-4)(n-5)(n-6)!}{(n-6)!} = 30 \Rightarrow n^2-9n+20=30$$

$n^2-9n-10=0$; then using the quadratic formula we get $n=10$ and n but $n=-1$ is rejected as n must be a positive integer.

26) $^{2n+2}C_n : {}^{2n-2}C_n = 99:7$

so $\frac{(2n+2)!}{n!(n+2)!} \times \frac{n!(n-2)!}{(2n-2)!} = \frac{99}{7}$

$$\frac{(2n+2)(2n)(2n-2)!}{(n+2)(n+1)n(n-1)(n-2)!} \times \frac{(n-2)!}{(2n-2)!} = \frac{99}{7}$$

$$\frac{(2n+2)(2n+1)(2n)(2n-1)}{(n+2)(n+1)n(n-1)} = \frac{99}{7}$$

$$\frac{4(4n^2-1)}{(n+2)(n-1)} = \frac{99}{7}$$

$112n^2-28 = 99n^2+99n-198$

$13n^2 - 99n + 170 = 0$

Using the quadratic formula we get $n=5$ taking the positive integer only

27) $^9C_4\,{}^5C_3\,{}^2C_2 = 1260$

28) $^8C_7\,{}^1C_1 + {}^8C_6\,{}^2C_2 + {}^8C_5\,{}^3C_3 = 92$

29) $\frac{{}^8C_2\,{}^6C_2\,{}^4C_2\,{}^2C_2}{4!} = 105$

Note: Partition:
When a group of objects is divided into 2 equal sets, the normal work is done but we also divide.
Eg. Set of n objects divided into 2 equal sets

$$\frac{{}^nC_{n/2}\,{}^{n/2}C_{n/2}}{2!}$$

EXERCISES 28(c)

1. Five cards are drawn at random from a pack of 52 playing cards. What is the probability that they are all from the same suit?

2. A bag contains 5 red balls and 4 white balls. Three balls are withdrawn without replacement. Find the probability of drawing at least 2 red balls.

3. An urn contains 3 white balls, 4 red balls and 5 black balls. Three balls are drawn at random. What is the probability that they are
   (a) different colours **(b)** the same colour?

4. From 7 teachers and 5 pupils, a random selection of 7 is made. What is the probability that it contains at least 4 teachers?

5. A committee of 3 men and 4 women is to be chosen from 6 men and 7 women. What is the probability that it contains a particular man and a particular woman?

6. Five cards are drawn from a pack of 52 playing cards. What is the probability of drawing at least 3 aces?

7. Eight people are to be divided into two groups. What is the probability that there will be 4 in each group?

8. The letters of the word 'promise' are arranged in a row. What is the probability that there are 3 letters between p and r?

9. Six people arrange themselves at random in a circle. What is the probability that the tallest and the shortest are together?

10. Four men and three boys are arranged in a straight line. What is the probability that the men and the boys occupy alternate positions?

11. The number plates of a motor car contain 3 letters of the alphabet followed by 3 numerals. How many such number plates can be made? What proportion of these would contain 3 letters the same and 3 numerals the same?

12. The letters of the word 'independence' are arranged in a row. What is the probability of the letters 'e' being together?

13. A committee of 6 is to be selected from 10 people. What is the probability of the youngest and oldest being on the same committee?

14. A party of twelve, of whom A and B are two, are arranged at random in a straight line. What is the probability that A and B are not next to one another?

15. An urn contains 5 red cubes and 4 white cubes. Three cubes are drawn in succession without replacement. What is the probability that
   (a) the first two cubes are red and the third one white,
   (b) any two cubes are red, and one is white?

16. A carton contains 15 transistors of which 5 are defective. If a random sample of 6 transistors is drawn from the carton (without replacement), determine the probability of 0, 1, 2, 3, 4, 5 defective valves in the sample.

17. An urn contains 12 distinguishable cubes of which 5 are red and the remainder black. If a random sample of 6 cubes is drawn without replacement, calculate the probabilities of 0, 1, 2, 3, 4, 5 red cubes in the sample.

18. Two boxes each contain eight balls. In box A there are 3 black and 5 white balls, in box B there are 1 black and 7 white balls. For each box find the probability that two balls chosen at random without replacement will both be white.

19. A sample of 3 coins is selected without replacement from 8 coins, consisting of 4 five-cent coins and 4 ten-cent coins. What is the probability that the sample contains at least 2 five-cent coins?

20. A hand of 5 cards is dealt from a pack of 52 playing cards. What is the probability that it will contain at least one ace?

21. From a group of 12 people of whom 8 are males and 4 are females, a sample of 4 is selected at random. What is the probability that the sample contains at least 2 females?

22. Urn *A* contains 6 white and 4 black balls. Urn *B* contains 2 white and 2 black balls. From urn *A* two balls are selected at random and placed in urn *B*. From urn *B* two balls are then selected at random. What is the probability that exactly one of these two balls is white?

23. From a set of 10 cards numbered 1 to 10, two cards are drawn without replacement. What is the probability that **(a)** both numbers are even, **(b)** one is even and the other is odd, **(c)** the sum of the two numbers is 12, **(d)** both numbers are even and the sum of the two numbers is 12?

24. The letters of the word *tomato* are arranged in a row. What is the probability that **(a)** the two letters *o* are together, **(b)** the two letters *o* are not together?

25. The letters of the word *Tuesday* are arranged at random in a row. What is the probability that **(a)** the vowels and consonants occupy alternate positions **(b)** the vowels are together **(c)** the vowels are together and the letter *T* occupies the first place?

26. Four girls and four boys arrange themselves at random in **(a)** a row, **(b)** a circle. What is the probability in each case that the girls and the boys occupy alternate positions?

27. Six people, of whom *A* and *B* are two, arrange themselves at random in a row. What is the probability that
(a) *A* and *B* occupy the end positions,
(b) *A* and *B* are not next to each other,
(c) there are at least three people between *A* and *B*?

28. The digits 1, 2, 3, 4, 5, 6 are used to form numbers which contain two or more digits. (The same digit cannot be used more than once in a number). What proportion of such numbers are even numbers?

29. Five cards are selected without replacement from a pack of 52 playing cards. What is the probability of **(a)** exactly 3 hearts, **(b)** 4 aces, **(c)** no hearts?

SET 1C

1. $\dfrac{{}^{13}C_5}{{}^{52}C_5} \times 4 = \dfrac{33}{16660}$

2. $\dfrac{{}^5C_2\,{}^4C_1 + {}^5C_3\,{}^4C_0}{{}^9C_3} = \dfrac{25}{42}$

3. (a) $\dfrac{{}^3C_1\,{}^4C_1\,{}^5C_1}{{}^{12}C_3} = \dfrac{3}{11}$

3. (b) $\dfrac{{}^3C_3\,{}^4C_0\,{}^5C_0 + {}^3C_0\,{}^4C_3\,{}^5C_0 + {}^3C_0\,{}^4C_0\,{}^5C_3}{{}^{12}C_3} = \dfrac{3}{44}$

4. $\dfrac{{}^7C_4\,{}^5C_3 + {}^7C_5\,{}^5C_2 + {}^7C_6\,{}^5C_1 + {}^7C_7\,{}^5C_0}{{}^{12}C_7} =$

5. $\dfrac{{}^5C_2\,{}^6C_3}{{}^6C_3\,{}^7C_4} = \dfrac{2}{7}$

6. $\dfrac{{}^4C_3\,{}^{48}C_2 + {}^4C_4\,{}^{48}C_1}{{}^{52}C_5} = \dfrac{19}{10829}$

7. $\dfrac{{}^8C_4\,{}^4C_4}{2!} \cdot \dfrac{2}{{}^8C_1\,{}^1C_1 + {}^8C_2\,{}^2C_2 + {}^8C_3\,{}^3C_3 + {}^8C_4\,{}^4C_4} = \dfrac{35}{81}$

8. $\dfrac{6 \times 5!}{7!} = \dfrac{1}{7}$

9. Six people on a circle = 5! ; tall+short – one unit – 4!2! ; $\dfrac{4!2!}{5!} = \dfrac{2}{5}$

10. $\dfrac{4!\,3!}{7!} = \dfrac{1}{35}$

11.

26	26	26	10	10	10

; $26^3 10^3 = 17576000$

26	1	1	10	1	1

; $26 \times 10 = 260$; $\dfrac{260}{17576000} = \dfrac{1}{67600}$

12. $\dfrac{9!\,4!}{12!} = \dfrac{1}{55}$

13. $\dfrac{{}^1C_1\,{}^1C_1\,{}^8C_4}{{}^{10}C_6} = \dfrac{1}{3}$

14. When A & B next to eachother : 11!2! ; when not next to eachother: 12! – 11!2!

$\dfrac{12! - 11!2!}{12!} = \dfrac{5}{6}$

15. (a) $\dfrac{{}^5P_2\,{}^4P_1}{{}^9P_3} = \dfrac{10}{63}$ (b) $\dfrac{{}^5C_2\,{}^4C_1}{{}^9C_3} = \dfrac{10}{21}$

16.

Defective	0	1	2	3	4	5
	$\dfrac{{}^{10}C_6\,{}^5C_0}{{}^{15}C_6}$	$\dfrac{{}^{10}C_5\,{}^5C_1}{{}^{15}C_6}$	$\dfrac{{}^{10}C_4\,{}^5C_2}{{}^{15}C_6}$	$\dfrac{{}^{10}C_3\,{}^5C_3}{{}^{15}C_6}$	$\dfrac{{}^{10}C_2\,{}^5C_4}{{}^{15}C_6}$	$\dfrac{{}^{10}C_1\,{}^5C_5}{{}^{15}C_6}$
	0.04195	0.2517	0.4105	0.2400	0.0450	0.0020

17.

Red	0	1	2	3	4	5
	$\dfrac{{}^5C_0\,{}^7C_6}{{}^{12}C_6}$	$\dfrac{{}^5C_1\,{}^7C_5}{{}^{12}C_6}$	$\dfrac{{}^5C_2\,{}^7C_4}{{}^{12}C_6}$	$\dfrac{{}^5C_3\,{}^7C_3}{{}^{12}C_6}$	$\dfrac{{}^5C_4\,{}^7C_2}{{}^{12}C_6}$	$\dfrac{{}^5C_5\,{}^7C_1}{{}^{12}C_6}$
	$\dfrac{1}{132}$	$\dfrac{5}{44}$	$\dfrac{25}{66}$	$\dfrac{25}{66}$	$\dfrac{5}{44}$	$\dfrac{1}{132}$

18. $\dfrac{{}^5C_2\,{}^3C_0}{{}^8C_2} = \dfrac{5}{14}$; $\dfrac{{}^7C_2\,{}^1C_0}{{}^8C_2} = \dfrac{3}{4}$

19. $\dfrac{{}^4C_2\,{}^4C_1 + {}^4C_3\,{}^4C_0}{{}^8C_3} = \dfrac{1}{2}$

20. $\dfrac{{}^4C_1\,{}^{48}C_4 + {}^4C_2\,{}^{48}C_3 + {}^4C_3\,{}^{48}C_2 + {}^4C_4\,{}^{48}C_1}{{}^{52}C_5} = 0.3412$

21. $\dfrac{{}^4C_2\,{}^8C_2 + {}^4C_3\,{}^8C_1 + {}^4C_4\,{}^8C_0}{{}^{12}C_4} = \dfrac{6}{16}$

22. Urn A – 6 white, 4 black Urn B – 2 white, 2 black

Case 1: Probability of 2 black from A placed in B ; $\dfrac{4}{10} \times \dfrac{3}{9} = \dfrac{2}{15}$

in Urn B – 2 white, 4 black : to get one white ; $P(W,B) + P(B,W) = \left(\dfrac{2}{6}\cdot\dfrac{4}{5}\right) + \left(\dfrac{4}{6}\cdot\dfrac{2}{5}\right) =$

Total prob. of getting 2 blacks then 1 white: $\dfrac{2}{15} \times \dfrac{8}{15} = \dfrac{16}{225}$ (1)

Case 2: Probability of BW or WB from Urn A : $\dfrac{4}{10}\cdot\dfrac{6}{9} + \dfrac{6}{10}\cdot\dfrac{4}{9} = \dfrac{8}{15}$

In Urn B · 3 white, 3 black : to get one white ; $P(W,B) + P(B,W) = \left(\dfrac{3}{6}\cdot\dfrac{3}{5}\right) + \left(\dfrac{3}{6}\cdot\dfrac{3}{5}\right) = \dfrac{3}{5}$

Total prob. of getting BW then white : $\dfrac{8}{15} \times \dfrac{3}{5} = \dfrac{8}{25}$ (2)

22. (cont.)

Case 3: Probability of 2 white from Urn A: $\frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$

In Urn B – 4 white, 2 black: to get one white;

$P(W, B) + P(B, W) = \left(\frac{4}{6} \cdot \frac{2}{5}\right) + \left(\frac{2}{6} \cdot \frac{4}{5}\right) = \frac{8}{15}$

Total probability of getting BW then white: $\frac{8}{15} \times \frac{1}{3} = \frac{8}{45}$ ③

$\therefore$ Total = ① + ② + ③

$= \frac{16}{225} + \frac{8}{25} + \frac{8}{45}$

$= \frac{128}{225}$

23. (a) $\frac{{}^5C_2 \, {}^5C_0}{{}^{10}C_2} = \frac{2}{9}$ (b) $\frac{{}^5C_1 \, {}^5C_1}{{}^{10}C_2} = \frac{5}{9}$ (c) $\frac{{}^2C_1 \, {}^2C_1}{{}^{10}C_2} = \frac{4}{45}$ (d) 2 pairs: $\frac{2}{45}$

24. (a) $\frac{5!\,2!}{6!} = \frac{1}{3}$ (b) $\frac{6! - 5!\,2!}{6!} = \frac{2}{3}$

25. (a) $\frac{4!\,3!}{7!} = \frac{1}{35}$ (b) $\frac{5!\,3!}{7!} = \frac{1}{7}$ (c) $\frac{4!\,3!}{7!} = \frac{1}{35}$

26. (a) $\frac{4!\,4!\,2!}{8!} = \frac{1}{35}$ (b) $\frac{4!\,3!}{7!} = \frac{1}{35}$

27. (a) $\frac{4!\,2!}{6!} = \frac{1}{15}$

(b) When A and B are next to eachother: $5!\,2!$

When A and B are not next to eachother: $6! - 5!\,2!$

$\frac{6! - 5!\,2!}{6!} = \frac{2}{3}$

*(c) $\frac{6 \times 4!}{6!} = \frac{1}{5}$

* Note: There are only 6 positions in which A and B can sit, and 4! ways in which the other people can sit for each of the six positions.

28. Full probability is 1. The chances to get an one digit number is $\frac{1}{2}$. Therefore the chance to get an even number from 2 or more digits is $1 - \frac{1}{2} = \frac{1}{2}$

29. (a) $\frac{{}^{13}C_3 \, {}^{39}C_2}{{}^{52}C_5} = \frac{2717}{33320}$

(b) $\frac{{}^4C_4 \, {}^{48}C_1}{{}^{52}C_5} = \frac{1}{54145}$

(c) $\frac{{}^{13}C_0 \, {}^{39}C_5}{{}^{52}C_5} = \frac{2109}{9520}$