Student's name: Student's number: Teacher's name:

Presbyterian Ladies' College, Sydney
Mathematics Department

HSC Assessment Task 1, 2008
Instruction, Notification and Reporting Sheet

Course:	**Mathematics (2 unit)**
Topics:	Plane Geometry
	Linear Functions
	Locus and the Parabola
Date:	Friday November 16 (period 4)
Time allowed:	40 minutes
Weighting:	20%

The outcomes being assessed are printed overleaf.

Instructions:

- Approved calculators may be used.
- Bring a stapler and staples.
- Write your student number on every page you hand in.
- All questions may be attempted.
- Start each Section on a new sheet of paper.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- The questions will be collected with your answers.

Marks awarded:

Section	Mark
1	/10
2	/10
3	/10
Total	/30

Teacher's comment:

Textbook references:

Topic:
Plane Geometry
Linear Function
Locus and the Parabola

Student's comment:

This sheet will be re-printed as the front cover of the question booklet.

Student Number.................................

Section 1 (10 marks) Remove pages 5,6,7 and 8 and use this as your writing paper.

Marks

a. A trapezium has vertices $A(0,0)$, $B(2,3)$, $C(7,4)$ and $D(3,-2)$

Sketch this information on the number plane.

i. Show that AB is parallel to DC. **1**

ii. Show that the equation of CD is $3x-2y-13=0$. **1**

iii. Show that the perpendicular distance between the parallel sides of the trapezium is $\sqrt{13}$ units. **2**

b. i. Show that the triangle with vertices $A(2,3)$, $B(2,8)$ and $C(5,7)$ is isosceles. **1**

ii. Find the size of all the angles of the triangle. **2**

iii. Find the co-ordinates of D so that **BACD** is a rhombus. **1**

iv. Find the area of the rhombus BACD. **2**

End of Section 1

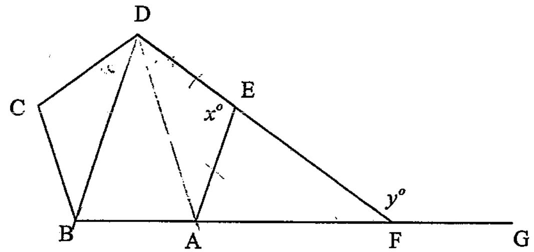
Student Number..................................

Section 2 (10 marks) Remove pages 9,10,11 and 12 and use this as your writing paper.

Marks

a. The diagram below shows a regular pentagon ABCDE.

Two of the sides have been extended and meet at F.

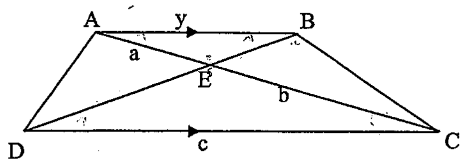


i. Find the value of x. **1**

ii. Find the value of y, giving reasons. **2**

iii. Find the size of $\angle$BDA, giving reasons. **2**

b. ABCD is a trapezium, with AB parallel to CD.
The diagonals meet at E.



i. Prove that ΔABE is similar to ΔCDE, giving reasons. **2**

ii. Give an expression for y, in terms of a, b and c. **1**

iii. Rachel says: "Triangle ADC and triangle BCD have the same area, but they are not necessarily congruent, but they might be congruent." **2**

Explain why Rachel is correct.

End of Section 2

Student Number..................................

Section 3 (10 marks) Start a new sheet of writing paper.

Marks

a. Find the gradient of the tangent to the parabola $x^2 = 6y$ at the point $(1, \frac{1}{6})$. **2**

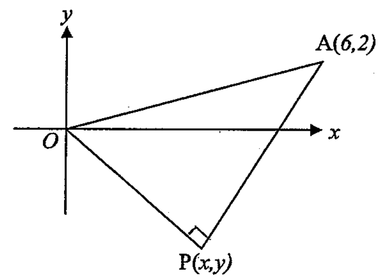
b. Consider the parabola with equation $x^2 = 8(y+1)$

i. Find the coordinates of the vertex of the parabola **1**

ii. Find the focal length **1**

iii. Find the equation of the directrix of the parabola **1**

c.



i. Find an expression for the gradient of AP in terms of x and y. **1**

ii. Hence, or otherwise, find the equation of the locus of all points P such that OP is perpendicular to AP. **2**

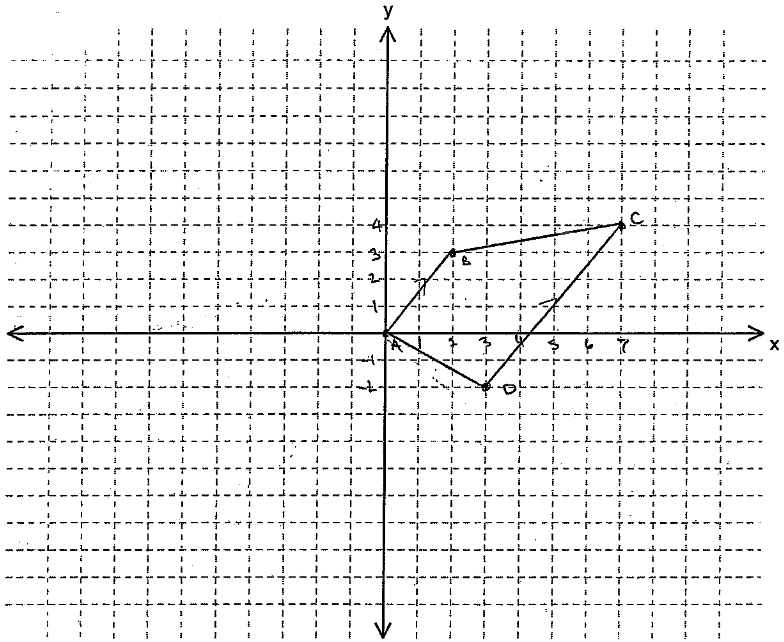
iii. Hence, or otherwise, draw a graph of the locus of point P. **2**

(There is no need to show x and y intercepts on your graph).

End of Assessment Task

5

Section 1:
Part a. Write your solution here and on the reverse side of this sheet.



a) i. Show $m(AB) = m(DC)$

$m(AB) = \frac{3-2}{2-0} = \frac{3}{2}$

$m(DC) = \frac{4+2}{7-3} = \frac{6}{4} = 3/2$ 1/1

$m(AB) = m(DC)$ $\therefore$ $AB \parallel DC$

ii. use ~~$y - y_1 = m(x - x_1)$~~

use $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$\frac{4+2}{7-3} = \frac{y+2}{x-3}$

$y + 2 = \frac{3}{2}(x - 3)$

$2(y+2) = 3(x-3)$

$2y + 4 = 3x - 9$

$2y = 3x - 13$

$3x - 2y - 13 = 0$

[illegible] equation

1/1

iii. $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

~~pick a point on CD : D(3, -2)~~

~~sub into equation~~

~~$d = \frac{|3(3) - 2(-2) - 13|}{\sqrt{3^2 + 2^2}}$~~

~~$= 0$~~

$B(2, 3)$

$CD: 3x - 2y - 13 = 0$

$d = \frac{|3(2) - 2(3) - 13|}{\sqrt{9+4}}$

$= \frac{13}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$

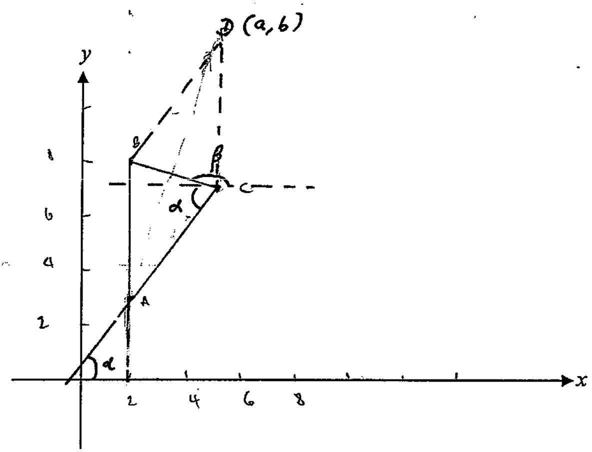
$= \frac{13\sqrt{13}}{13}$

$= \sqrt{13}$

$\therefore \perp d = \sqrt{13}$ units

2/2

Section 1:
Part b. Write your solution here and on the reverse side of this sheet.



(i) Show two sides are equal in length.

$AB = \sqrt{(2-2)^2 + (3-8)^2} = 5$

$BC = \sqrt{(2-5)^2 + (8-7)^2} = \sqrt{10}$

$CA = \sqrt{(5-2)^2 + (7-3)^2} = 5$ 1

$AB = CA$ $\therefore$ $\triangle ABC$ is isosceles
(two sides of an isosceles $\triangle$ are equal)

(ii) NA

$m_{AC} = \tan\alpha = \frac{7-3}{5-2} = \frac{4}{3}$

$\therefore \alpha = 53°8'$

$\therefore \angle BAC = 90° - 53°8'$ (Vert.opp∠s)
$= 36°52'$

$m_{BC} = \tan\beta = \frac{8-7}{2-5} = -\frac{1}{3}$

$\beta = 161°34'$

$\therefore \angle BCA = (180° - 161°34') + \alpha$
$= 71°34'$

$\therefore \angle CBA = 180° - 71°34' - 36°52'$
$= 71°34'$

(iii) Since DC is a vertical line
$D(5, 12)$

(iv) Area of rhombus BACD

$= \frac{1}{2} \times BC \times AD$ (product of diagonals $\times \frac{1}{2}$)

$= \frac{1}{2} \times \sqrt{10} \times AD$

but $AD = \sqrt{(5-2)^2 + (12-3)^2}$

$= \sqrt{9 + 81}$

$= \sqrt{90}$

$\therefore$ Area $= \frac{1}{2} \times \sqrt{10} \times \sqrt{90}$

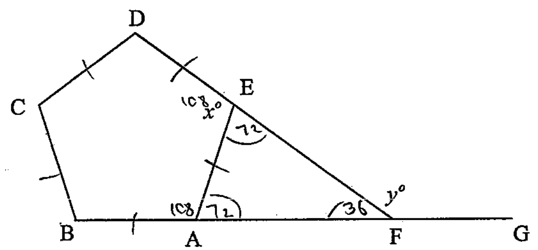
$= \frac{1}{2} \times 30$

$= 15$ sq. units

8

Student Number 120038

Section 2
Part a. Write your solution here and on the reverse side of this sheet.



(a) i. $S = (n-2) \times 180$
$= 3 \times 180$
$= 540$
$540 \div 5 = 108$
$\therefore x = 108°$ $x = 108°$ ✓ 1/1

(ii) $\angle DEA + \angle AEF = 180$ (straight angle, 180°)
$\therefore \angle AEF = 72°$ ✓
$\angle EAF = 72°$ (similarly) ✓
$\angle AEF + \angle EAF + \angle EFA = 180°$ ($\angle$ sum of Δ, 180)
$\therefore \angle EFA = 180 - 2(72)$
$= 36°$ ✓ Angle sum of st line
$\angle GFE + \angle AFE = 180°$ (straight angle, 180°)
$\therefore y = 180 - 36$
$= 144$ ✓
$y = 144°$ 2/2

Student Number.................................

(iii) Construct lines BD and AD
If DE = EA (given), Δ DAE is isosceles Δ.
If $x = 108°$,
$\angle ADE + \angle DAE + x = 180$ ($\angle$ sum of Δ, 180)
$\angle ADE = \angle DAE$ (base $\angle$s of isosceles Δ)
$\therefore 2\angle ADE = 180 - 108$
$\therefore \angle ADE = 36°$
$\therefore \angle CDB = 36°$ (similarly) ✓
If $\angle CDE = x = 108°$,
$\angle CDB + \angle ADE + \angle BDA = 108°$ 2.
$\therefore \angle BDA = 36°$ ✓ 5/5

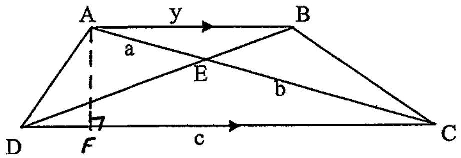
(b) (i) $\angle AEB = \angle DEC$ (vertically opposite $\angle$'s) ✓
$\angle BAC = \angle ACD$ (alternate $\angle$s on parallel lines) ✓
$\angle ABD = \angle BDC$ (similarly) ✓
$\therefore \Delta ABE \parallel\!| \Delta CDE$ (equiangular) ✓ 2

(ii) Since $\Delta ABE \parallel\!| \Delta CDE$

$\therefore \frac{AE}{CE} = \frac{AB}{CD} \Rightarrow \frac{a}{b} = \frac{y}{c} \Rightarrow y = \frac{ac}{b}$

Student Number........120038..........

Section 2
Part b. Write your solution here and on the reverse side of this sheet.



Note : Part b was started on previous page

(iii) Since area of $\triangle ADC = \frac{1}{2} \times DC \times AF$
" " $\triangle BDC = \frac{1}{2} \times DC \times AF$

$\therefore$ Areas are equal.

Not necessarily congruent because we have only 1 common side but no other information to confirm if it is

S.S.S
S.A.S
A.A.S
or
R.H.S test.

3/5

$8\frac{1}{2}$/10

12 0038

Section 3

(a) $x^2 = 6y$

$\therefore y = \frac{x^2}{6}$

$y' = \frac{x}{3}$

where $x = 1$

$m = \frac{1}{3}$

2/2

(b) (i) $(0, -1)$

(ii) $a = 2$

(iii) $(0, -3)$

2/3

(c) (i) $m(AP) = \frac{2-y}{6-x}$

1/1

(ii) $m(OP) = \frac{y}{x}$

show $m(OP) \times m(AP) = -1$

$\therefore \frac{y}{x} \times \frac{2-y}{6-x} = -1$

$\frac{2y-y^2}{6x-x^2} = -1$

$2y - y^2 = -6x + x^2$

$x^2 - 6x + y^2 - 2y = 0$

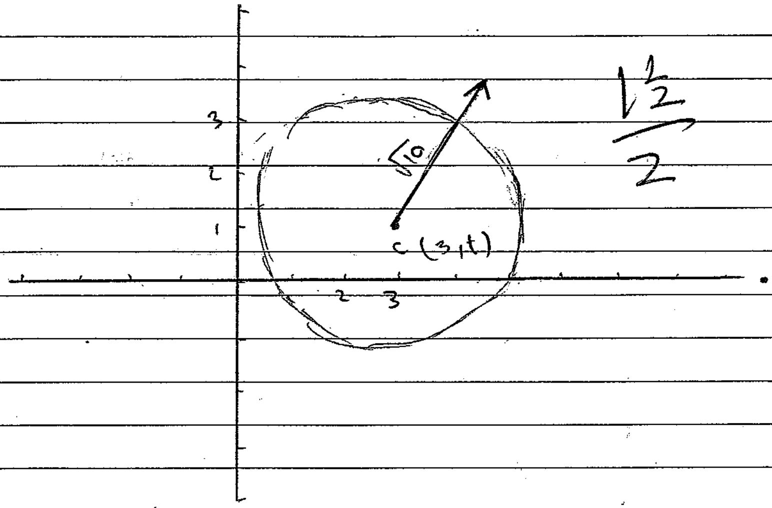
2/2

120038

(iii) $(x-3)^2 - 9 + (y-1)^2 - 1 = 0$

$(x-3)^2 + (y-1)^2 = 10$

$c(3;1)$ $\therefore r = \sqrt{10}$



$\frac{1\frac{1}{2}}{2}$