Mrs Hickey
Mrs Quarles
Miss Lau
Mrs Stock
Mrs Choong

PYMBLE LADIES' COLLEGE

YEAR 12

MATHEMATICS EXTENSION 1

HSC TRIAL EXAMINATION 2002

Time Allowed: 2 hours + 5 mins reading time

INSTRUCTIONS

- **All questions should be attempted**
- **Write your name and your teacher's name on each page**
- **Start each question on a new page**
- **DO NOT staple the questions together**
- **Only approved calculators may be used**
- **A standard integral sheet is attached**
- **Marks might be deducted for careless or untidy work**
- **Hand this question paper in with your answers**
- **ALL rough working paper must be attached to the back of the last question**
- **Staple a coloured sheet of paper to the back of each question**
- **There are seven (7) questions in this paper**
- **All questions are of equal value**

Question 1 — **Marks**

(a) Evaluate $\lim_{x \to 0} \dfrac{\sin 3x}{x}$ — 1

(b) The point P (7, -1) divides the interval AB externally in the ratio 3 : 2.
If A is (-2, 5) find the coordinates of B. — 2

(c) Solve for x

$$\frac{x+1}{x-2} < 2$$ — 2

(d) Find the gradient of the tangent to the curve $y = \tan^{-1}(2x)$ at the point where $x = \dfrac{1}{2}$. — 2

(e) Evaluate $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$ — 2

(f) On the same number plane, sketch the graphs of

(i) $y = |2x-1|$ and $y = |x+1|$ — 2

(ii) Hence, or otherwise, solve $|2x-1| \le |x+1|$ — 1

Question 2 **(Start a new sheet of paper)**

(a) Prove that $\dfrac{\sin 2\theta}{\sin\theta} - \sec\theta = \dfrac{\cos 2\theta}{\cos\theta}$ **2**

(b) Evaluate $\int_{\frac{1}{2}}^{1} 4t(2t-1)^5\,dt$ by using the substitution $u = 2t - 1$ **4**

(c) The angle between the lines $y = 3x$ and $y = mx$ is $45°$.
Find the value(s) of m. **3**

(d) Solve $\tan 2\theta - \cot\theta = 0$ where $0 \le \theta \le \pi$ **3**

Question 3 **(Start a new sheet of paper)** **Marks**

(a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\left(\frac{x}{2}\right)dx$ **2**

(b) Use Mathematical Induction to prove that

(i) $4(1^3 + 2^3 + 3^3 + \cdots + n^3) = n^2(n+1)^2$, for $n = 1,2,3\ldots$ **3**

(ii) Hence find the value of $\lim\limits_{n\to\infty}\left(\dfrac{1^3 + 2^3 + 3^3 + \cdots + n^3}{n^4}\right)$ **1**

(c) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$
where $R > 0$ and $0 \le \alpha \le \dfrac{\pi}{2}$ **2**

(ii) Hence sketch $y = \sin x + \sqrt{3}\cos x$ for $-2\pi \le x \le 2\pi$ showing any x and y intercepts. **2**

(iii) Find the general solution to $\sin x + \sqrt{3}\cos x = \sqrt{2}$ **2**

Question 4 (Start a new sheet of paper) Marks

(a) α, β *and* γ are the roots of the equation $x^3 + 2x^2 - 3x + 5 = 0$

(i) State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ **2**

(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$ **2**

(b) If a polynomial $P(x)$ is divided by $(x+1)$ the remainder is 5 and when $P(x)$ is divided by $(2x+1)$ the remainder is 3. Find the remainder when $P(x)$ is divided by $(x+1)(2x+1)$. **3**

(c) From a point S the bearings of two points P and Q are found to be 331° T and 011°T respectively. From a point F, 7 km due north of S, the bearings of P and Q are 299° T and 020° T respectively.

(i) Show that $PF = \sin 29^\circ \times \dfrac{7}{\sin 32^\circ}$ **2**

(ii) By considering the triangle FPQ, show that if the distance between P and Q is d metres, then

$$d^2 = 49\left(\frac{\sin^2 29^\circ}{\sin^2 32^\circ} + \frac{\sin^2 11^\circ}{\sin^2 9^\circ} - 2\frac{\sin 29^\circ \sin 11^\circ \cos 81^\circ}{\sin 32^\circ \sin 9^\circ}\right)$$ **3**

Question 5 (Start a new sheet of paper) Marks

(a) Consider the function $f(x) = \dfrac{x-1}{x^2}$

(i) Show that there is only one stationary point and determine its nature **3**

(ii) Determine the point of inflexion. **1**

(iii) What happens to $f(x)$ as $x \to \pm\infty$? **1**

(iv) What happens to $f(x)$ as $x \to 0$? **1**

(v) Sketch the curve showing all its essential features. (Use at least half a page.) **2**

(b) (i) Prove that $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \ddot{x}$ **2**

(ii) An object moving in a straight line has an acceleration given by $\ddot{x} = x(8-3x)$ where x metres is its position relative to a fixed point 0.

At $x = 0$, it has a speed of 4 m/s. Find its speed when it is 1 m on the positive side of 0. **2**

Question 6 **(Start a new sheet of paper)** **Marks**

(a) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin is given by the equation $\frac{d^2x}{dt^2}=-16x$ where t is time in seconds.

(i) Show that $x=a\cos(4t+\alpha)$ is a solution of motion for this particle. (a and α are constants). **1**

(ii) When $t=0$, $v=4$ m/s and $x=5$ m. Show that the amplitude of the oscillation is $\sqrt{26}$ metres. **2**

(iii) What is the maximum speed of the particle? **1**

(b) $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ are two points on the parabola $x^2=4ay$. The tangents at P and Q meet at T which is always on the parabola $x^2=-4ay$.

(i) Derive the equation of the tangent at P. **2**

(ii) Hence write down the equation of the tangent at Q. **1**

(iii) Show that T is the point $(a(q+p),apq)$. **1**

(iv) Show that $p^2+q^2=-6pq$ **1**

(v) Find M, the midpoint of PQ. **1**

(vi) Hence, or otherwise, find the locus of M. **2**

Question 7 **(Start a new sheet of paper)** **Marks**

(a) (i) On the same number plane, sketch the graphs of $y=\cos^{-1}x$ and $y=\sin^{-1}(\frac{x}{2})$. Label the important features. **2**

(ii) Show $y=\cos^{-1}x$ and $y=\sin^{-1}(\frac{x}{2})$ intersect at $x=\frac{2}{\sqrt{5}}$. **2**

(iii) Find the inverse function of $y=\sin^{-1}(\frac{x}{2})$ **1**

(iv) Hence or otherwise find the area bounded by the $x-axis$ and the graphs $y=\cos^{-1}x$ and $y=\sin^{-1}(\frac{x}{2})$ (answer correct to 2 decimal places.) **3**

(b) Wheat is the only crop grown on Sandy's property in outback NSW. Per hectare the amount of water, W, in kilolitres, used during irrigation times is given by

$$W=Cg^2+\frac{D}{g}$$

where g is the amount of grain produced in tonnes per hectare and C and D are positive constants. There is a limited amount of water available for irrigation.

(i) Show that, for maximum hectares under irrigation, production of grain per hectare, g, is given by

$$g=\left(\frac{D}{2C}\right)^{\frac{1}{3}}$$ **2**

(ii) Show that for maximum grain produced on Sandy's property, grain production per hectare needs to be about 59% more than that given in part (i) above. **2**

THE END

1.

a) $\lim_{x\to 0} \frac{\sin 3x}{x} = 3 \lim_{x\to 0} \frac{\sin 3x}{3x} = 3$ (1)

b) B is $(1, 3)$. (2)

or

$(-2,5)$ (x, y)

$-3 : 2$

$\frac{-4-3x}{-3+2} = 7$ $\qquad$ $\frac{10-3y}{-3+2} = -1$

$-4-3x = -7$ $\qquad$ $10-3y = 1$

$-3x = -3$ $\qquad$ $-3y = -9$

$x = 1$ $\qquad$ $y = 3$

c) $\frac{x+1}{x-2} = 2$ $\qquad$ $x \neq 2$

$x+1 = 2x-4$

$x = 5$

$x > 5$ or $x < 2$

d) $y = \tan^{-1} 2x$ (2)

$\frac{dy}{dx} = \frac{1}{1+(2x)^2}$

$= \frac{1}{1+4x^2}$

when $x = \frac{1}{2}$

$m_{tang} = \frac{1}{1+4.\frac{1}{4}}$

$= \frac{1}{2}.$

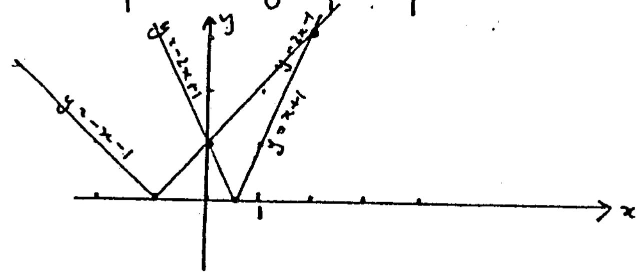
Q1

e) $\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx = \left[\sin^{-1}\frac{x}{3}\right]_0^3$ (2)

$= \sin^{-1}\frac{3}{3} - \sin^{-1} 0$

$= \frac{\pi}{2}.$

f) $y = |2x-1|$ and $y = |x+1|$ (2)



$2x-1 = x+1$ $\qquad$ $-2x+1 = x+1$

$x = 2$ $\qquad$ $-3x = 0$

$x = 0.$

$\therefore |2x-1| \leq |x+1|$ when $0 \leq x \leq 2.$ (1)

Q2.

$$\frac{\sin 2\theta}{\sin\theta} - \sec\theta = \frac{\cos 2\theta}{\cos\theta}$$

a) $LHS = \dfrac{2\sin\theta\cos\theta}{\sin\theta} - \dfrac{1}{\cos\theta}$ ②

$= 2\cos\theta - \dfrac{1}{\cos\theta}$

$= \dfrac{2\cos^2\theta - 1}{\cos\theta}$

$= \dfrac{\cos 2\theta}{\cos\theta}$.

b) $u = 2t - 1$ when $t = 1$ $u = 1$

$\dfrac{du}{dt} = 2$ when $t = \frac{1}{2}$ $u = 0$. ④

$$\int_{\frac{1}{2}}^{1} 4t(2t-1)\,dt = \int_0^1 2(u+1)u^5 \times \frac{1}{2}du$$

$$= \int_0^1 u^6 + u^5\,du$$

$$= \left[\frac{u^7}{7} + \frac{u^6}{6}\right]_0^1$$

$$= \frac{1}{7} + \frac{1}{6}$$

$$= \frac{13}{42}$$

c) $\tan\theta = \left|\dfrac{3-m}{1+3m}\right|$ ③

$1 = \dfrac{3-m}{1+3m}$ $\qquad -1 = \dfrac{3-m}{1+3m}$

$1+3m = 3-m$ $\qquad -1-3m = 3-m$

$4m = 2$ $\qquad -2m = 4$

$m = \frac{1}{2}$ $\qquad m = -2$

d) $\tan 2\theta - \cot\theta = 0 \quad 0 \le \theta \le \pi$ ③

$$\frac{2\tan\theta}{1-\tan^2\theta} - \frac{1}{\tan\theta} = 0$$

$2\tan^2\theta = 1 - \tan^2\theta$

$3\tan^2\theta = 1$

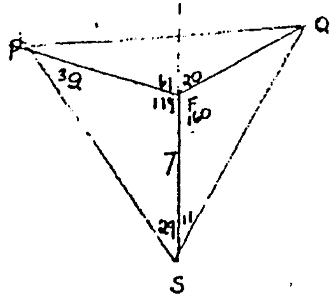
$\tan\theta = \pm\dfrac{1}{\sqrt{3}}$

$\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Check $\theta = \dfrac{\pi}{2}$ $\quad 0 - 0 = 0$ True.

$\therefore$ Solns $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$

-4

c. 

(i). Consider Δ PFS.

$$\frac{PF}{\sin 29} = \frac{7}{\sin 32}$$

$$PF = \sin 29 \times \frac{7}{\sin 32}$$

2

(ii) Similarly considering Δ QFS

$$QF = \sin 11 \times \frac{7}{\sin 9}$$

3

Now considering Δ PQF

$$PQ^2 = PF^2 + QF^2 - 2\,PF \times QF \cos 81°$$

i.e. $$d^2 = \sin^2 29 \times \frac{49}{\sin^2 32} + \sin^2 11 \times \frac{49}{\sin^2 9} - 2 \times \sin 29 \times \frac{7}{\sin 32} \times \sin 11 \times \frac{7}{\sin 9} \times \cos 81$$

$$= 49\left[\frac{\sin^2 29}{\sin^2 32} + \frac{\sin^2 11}{\sin^2 9} - \frac{2\sin 29 \sin 11 \cos 81}{\sin 32 \sin 9}\right.$$

Q4

$$x^3 + 2x^2 - 3x + 5 = 0$$

(i) $\alpha + \beta + \gamma = -\frac{b}{a} = -2$ (2)

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = -3$

(ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$ (2)

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 4 - 2 \times -3$

$= 10$

b) $P(x) = Q(x)(x+1)(2x+1) + ax + b$ (3)

$P(-1) = -a + b = 5$ ①

$P(-\frac{1}{2}) = -\frac{1}{2}a + b = 3$ ②

①-② $-\frac{1}{2}a = 2$

$a = -4$

$b = 1$

$\therefore$ Remainder $= -4x + 1$

Q3.

(ii) $\lim_{n\to\infty}\left(\frac{1^3+2^3+3^3+\dots+n^3}{n^4}\right) = \lim_{n\to\infty} \frac{n^2(n+1)^2}{4n^4}$ (1)

$= \lim_{n\to\infty} \frac{n^4+2n^3+n^2}{4n^4}$

$= \lim_{n\to\infty} \frac{1+\frac{2}{n}+\frac{1}{n^2}}{4}$

$= \frac{1}{4}$

c. let $\sin x + \sqrt{3}\cos x = R\sin(x+\alpha)$

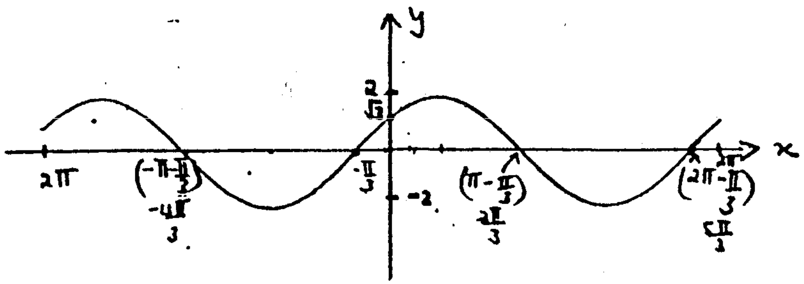
$R\cos\alpha = 1$ ①

$R\sin\alpha = \sqrt{3}$ ② (2)

$\frac{②}{①}$ $\tan\alpha = \sqrt{3}$ $\quad R^2 = 4$

$\alpha = \frac{\pi}{3}$ $\quad R = 2.$

$\therefore \sin x + \sqrt{3}\cos x = 2\sin\left(x+\frac{\pi}{3}\right)$



(2)

(iii) $2\sin\left(x+\frac{\pi}{3}\right) = \sqrt{2}$ (2)

$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

$x+\frac{\pi}{3} = n\pi + (-1)^n\frac{\pi}{4}$

$\therefore x = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$

Q3

a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\left(\frac{x}{2}\right)\,dx$ (2)

$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos x + 1)\,dx$

$= \frac{1}{2}\left[\sin x + x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[1+\frac{\pi}{2} - \left(\frac{1}{2}+\frac{\pi}{6}\right)\right]$

$= \frac{1}{2}\left[\frac{1}{2}+\frac{\pi}{3}\right]$

$= \frac{1}{4}+\frac{\pi}{6}.$

b) let $S_n = 4(1^3+2^3+\dots+n^3)$ (3)

Required to prove $S_n = n^2(n+1)^2$

For $n=1$ $\quad$ LHS $= 4(1^3) = 4$

RHS $= 1^2(1+1)^2 = 4$

Statement is true for $n=1$.

Assume statement is true for $n=k$

$S_k = 4(1^3+2^3+\dots+k^3) = k^2(k+1)^2$

when $n=k+1$

$S_{k+1} = 4(1^3+2^3+\dots+k^3+(k+1)^3)$

$= 4(1^3+2^3+\dots+k^3) + 4(k+1)^3$

$= k^2(k+1)^2 + 4(k+1)^3$

$= (k+1)^2(k^2+4(k+1))$

$= (k+1)^2(k+2)^2$

Thus if it is true for $n=k$ it is true for $n=k+1$

It is true for $n=1$ & hence it is true for $n=2$ & so on.

Hence it is true for all n

Q6.

$$\frac{d^2x}{dt^2} = -16x$$

(i) $x = a\cos(4t+\alpha)$ ①

$\dot{x} = -4a\sin(4t+\alpha)$

$\ddot{x} = -16a\cos(4t+\alpha)$

$= -16x$

$\therefore x = a\cos(4t+\alpha)$ is a soln

(ii). When $t=0$ $v=4$ ②

$4 = -4a\sin\alpha$

i.e. $-1 = a\sin\alpha$ ①

when $t=0$ $x=5$

$5 = a\cos\alpha$ ②

i.e ①^2+②^2 $(\sin^2\alpha+\cos^2\alpha=1)$

$a^2 = (-1)^2+(5)^2$

$a^2 = 26$

$a = \sqrt{26}$.

OR

using $v^2 = n^2(a^2-x^2)$

$16 = 16(a^2-25)$

$1 = a^2-25$

$a^2 = 26$

$a = \sqrt{26}$

(iii) Max speed occurs when $\sin(4t+\alpha)=1$. ①

$\dot{x} = -4\times\sqrt{26}\sin(4t+\alpha)$

$= -4\sqrt{26}$

$\text{speed}_{max} = |-4\sqrt{26}|$

$= 4\sqrt{26}$

Q6

b. $x^2 = 4ay$

$y = \frac{1}{4a}x^2$

(i) $\frac{dy}{dx} = \frac{1}{2a}x$ ②

$m_{\text{tang}_P} = \frac{1}{2a}\times 2ap$

$= p$

Eq^n of tangent at P

$y - ap^2 = p(x-2ap)$

$y = px - ap^2$. ①

(ii) $y = qx - aq^2$ ② ①

(iii) Solving eqns for tangents simult. ①

$0 = (p-q)x - a(p^2-q^2)$

$x = \frac{a(p^2-q^2)}{p-q} = \frac{a(p+q)(p-q)}{p-q}$

$x = a(p+q)$

① $y = ap(p+q) - ap^2$

$= apq$

$\therefore T$ is $(a(p+q), apq)$

(iv) T lies on $x^2 = -4ay$ ①

i.e. $a^2(p+q)^2 = -4a^2pq$

$(p+q)^2 = -4pq$

$p^2+q^2+2pq = -4pq$

$p^2+q^2 = -6pq$

(v) M is $\left(\frac{2a(p+q)}{2}, \frac{a(p^2+q^2)}{2}\right)$ ①

$= \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

Q 5

$f(x) = \dfrac{x-1}{x^2}$

(i) $f'(x) = \dfrac{x^2 . 1 - 2x(x-1)}{x^4}$ (3)

$= \dfrac{x^2 - 2x^2 + 2x}{x^4}$

$= \dfrac{x(2-x)}{x^4}$

$= \dfrac{2-x}{x^3}$

$f'(x) = 0$ when $x = 2$.

$f''(x) = \dfrac{x^3 . -1 - 3x^2(2-x)}{x^6}$

$= \dfrac{-x^3 - 6x^2 + 3x^3}{x^6}$

$= \dfrac{2x^2(x-3)}{x^6}$

$= \dfrac{2(x-3)}{x^4}$

when $x = 2$ $f''(x) < 0$

$\therefore$ Only stationary pt $(2, \frac{1}{4})$ which is a max

(ii) $f''(x) = \dfrac{2(x-3)}{x^4} = 0$ when $x = 3$ (1)

x	3^-	3	3^+
$f''(x)$	$-$	0	$+$

Change in concavity

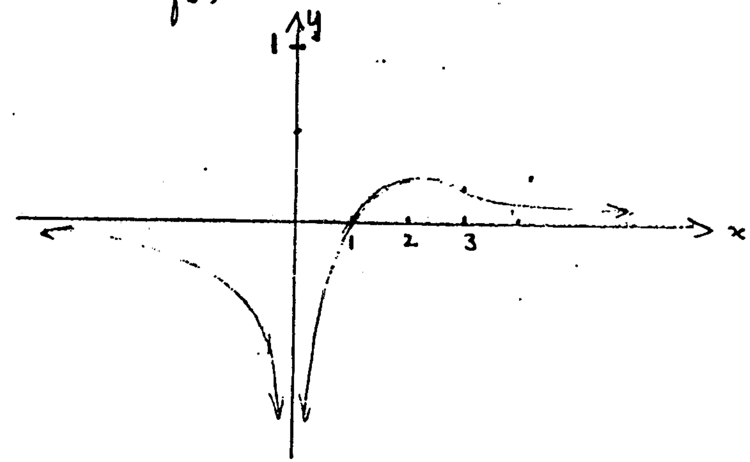
$\therefore$ there is a pt of inflexion at $(3, \frac{2}{9})$

Q5.

(iii) As $x \to \infty$ $f(x) \to 0$ (1)

As $x \to -\infty$ $f(x) \to 0$

(iv) As $x \to 0$ $f(x) \to -\infty$ (1)



(2)

b) $\dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = \dfrac{d}{dv}\left(\frac{1}{2}v^2\right)\dfrac{dv}{dx}$

$= v . \dfrac{dv}{dx}$ (2)

$= \dfrac{dx}{dt} . \dfrac{dv}{dx}$

$= \dfrac{dv}{dt}$

$= \ddot{x}$

$\dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = 8x - 3x^2$ (2)

$\frac{1}{2}v^2 = \int 8x - 3x^2\, dx$

$\frac{1}{2}v^2 = 4x^2 - x^3 + c$

when $x = 0$ $v = 4$ $\therefore c = 8$

$v^2 = 8x^2 - 2x^3 + 16$

when $x = 1$ $v^2 = 8 - 2 + 16$

Q6

For M

$x = a(p+q)$

$x^2 = a^2(p^2+q^2+2pq)$

$= a^2(-6pq+2pq)$

$x^2 = -4a^2pq$ (1)

(2)

$y = \frac{a}{2}(p^2+q^2)$

$= \frac{a}{2} \times -6pq$

$y = -3apq$

$\therefore \frac{y}{-3a} = pq$

Sub in (1)

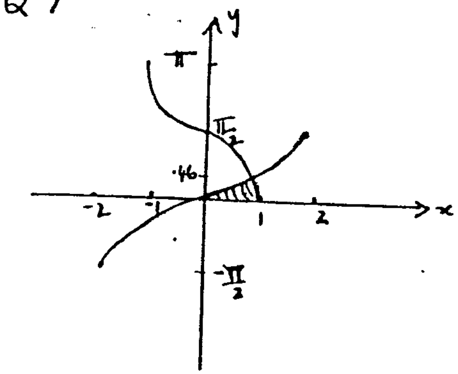
$x^2 = -4a^2 \times \frac{y}{-3a}$

$x^2 = \frac{4}{3}ay$

$3x^2 = 4ay$

Q 7

(i)



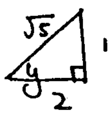
2

(ii) $y = \cos^{-1} x$

when $x = \frac{2}{\sqrt{5}}$

$y = \cos^{-1}\frac{2}{\sqrt{5}}$

i.e. $\cos y = \frac{2}{\sqrt{5}}$.



Also $\sin y = \frac{1}{\sqrt{5}}$.

Consider $y = \sin^{-1} x$

$y = \sin^{-1}\left(\frac{\frac{2}{\sqrt{5}}}{2}\right)$

i.e. $\sin y = \frac{1}{\sqrt{5}}$ True

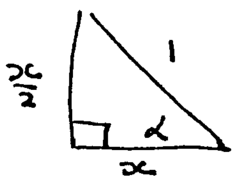
∴ Curves intersect at $x = \frac{2}{\sqrt{5}}$.

Or ....

2

let $\cos^{-1} x = \sin^{-1}\frac{x}{2} = \alpha$.

$\cos\alpha = x$ $\sin\alpha = \frac{x}{2}$



$x^2 + \frac{x^2}{4} = 1$

$5x^2 = 4$

$x = \pm\frac{2}{\sqrt{5}}$ but

$x = \frac{2}{\sqrt{5}}$ $x > 0$ from graph

(iii) $y = \sin^{-1}\frac{x}{2}$

Inv. fn is $\sin y = \frac{x}{2}$

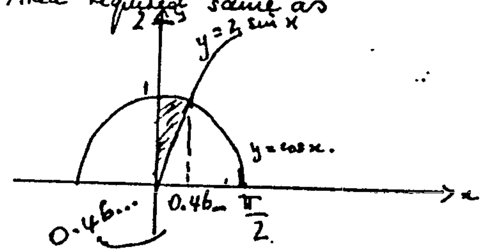
i.e. $x = 2\sin y$

Q7.

(iv) $y = \cos^{-1} x$

Inv $x = \cos y$.

Area required same as



Area = $\int_0^{0.46...}(\cos x - 2\sin x)\, dx$

$\left[\sin x + 2\cos x\right]_0^{0.46...}$

$[0.443.. + 1.792.. - 0 - 2]$

$= 0.24$ units^2.

7 b
i) To maximise hectares irrigated, we need to minimise Water per hectare. (W)

$$\frac{dW}{dg} = 2Cg - \frac{D}{g^2} \quad \Big\} \ \tfrac{1}{2}$$

$= 0$ for stationary pts.

ie $g^3 = \frac{D}{2C}$ $\quad \Big\} \ \tfrac{1}{2}$

$$\boxed{g = \left(\frac{D}{2C}\right)^{\frac{1}{3}}} \quad ①$$

$$\frac{d^2W}{dg^2} = 2C + \frac{2D}{g^3} \quad \Big\} \ \tfrac{1}{2}$$

> 0 since $C, D, g > 0$ ∴ min^m TP. $\quad \Big\} \ \tfrac{1}{2}$

ii) Let G = tonnes of grain per kL water

We need to maximise G

G = tonnes of grain per hectare × hectares per kL water $\quad \Big\} \ \tfrac{1}{2}$.

$$= g \times \frac{1}{Cg^2 + \frac{D}{g}}$$

$$\frac{dG}{dg} = \frac{Cg^2 + \frac{D}{g} - g\left(2Cg - \frac{D}{g^2}\right)}{\left(Cg^2 + \frac{D}{g}\right)^2}$$

$= 0$ for stationary pts

ie $g^3 = \frac{2D}{C}$ $\quad \Big\} \ \tfrac{1}{2}$.

$$\boxed{g = \left(\frac{2D}{C}\right)^{\frac{1}{3}}} \quad ②$$

$$\frac{dG}{dg} = -\frac{1}{g}\left(Cg^3 - 2D\right) \times \frac{1}{\left(Cg^2 + \frac{D}{g}\right)^2}$$

g	$\sqrt[3]{\frac{2D}{C}} - \epsilon$	$\sqrt[3]{\frac{2D}{C}}$	$\sqrt[3]{\frac{2D}{C}} + \epsilon$
$\frac{dG}{dg}$	~~neg × neg × pos = pos.~~	0	~~neg × pos × pos = neg~~

∴ Max^m T.P. $\quad \Big\} \ \tfrac{1}{2}$

Now from ① & ② above comparing results

$$\frac{\sqrt[3]{2D/C}}{\sqrt[3]{\frac{D}{2C}}} = \sqrt[3]{4}$$

$= 1.587\ldots$

$\approx 59\%$ more than before $\quad \Big\} \ \tfrac{1}{2}$.